

Problem Solving and the Initial Practical and Theoretical Education of Teachers in Spain

Lorenzo Blanco

University of Extremadura, Badajoz.

Several developments during the last decades have provoked spectacular changes in both Mathematics Teaching and the Education of Mathematics Teachers. From this perspective of renovation, we wish to define a new context in mathematics teacher education in which we consider that "pedagogical content knowledge" (Shulman, 1993, Mellado, 1998; Blanco & Otano, 1999) has to be the knowledge base for the education of the future teachers. In the present work, using problem solving as reference, we explain our program and different types of activities which help to create learning environments referred to "learning to teach mathematics". These activities allow the prospective teachers to build up their own pedagogical content knowledge.

In the 1960s and 1970s the *modus operandi* of most teacher education programs was to attempt to make the future teachers competent mathematicians and, only in passing, introduce a little pedagogy (Cooney, 1994, 225). The changes that have taken place in mathematics teaching since then have led to an evolution in the education of mathematics teachers.

We find ourselves in a situation of seeking new challenges in which we have to respond to the demands of our prospective teachers on the basis of experience or specific investigations. We also have to work from the new perspective suggested by the curricular proposals that set out the fundamental aspects of these changes.

From this perspective of renovation, we are now defining a new context in mathematics teacher education in which we consider that "pedagogical content knowledge" (Blanco & Otano, 1999; Mellado, 1998; Shulman, 1993) has to form the knowledge base for the education of future teachers.

Consequently, as teacher educators we must consider three basic aspects of our professional activity:

- (a) A new view of school mathematics and of the teachers' different classroom roles and conceptions.
- (b) Teacher education research that refers to basic knowledge required for mathematics teaching (Bromme, 1994; Tamir, 1991) and to the difficulties that preservice and novice teachers have in their teaching practice (Ball & Wilson, 1990; Blanco, 2001; Brown & Borko, 1992; Cooney, 1985)
- (c) We assume that every educational experience has to involve a creative process based on student activity, and that the students' pre-existing knowledge and conceptions must be taken into account in the learning process. In the present case, we have to start from the student teachers' conceptions about mathematics and its teaching and learning.

Professional orientations suggest to us the creation of environments where the students can explore mathematical ideas. Thus, the prospective teachers will have

to be taught in a similar way to how they will have to teach - exploring, formulating conjectures, communicating, and reasoning.

Nevertheless, while these activities are necessary, they are not a sufficient condition for the prospective teachers to acquire the pedagogical knowledge required for the effective development of primary education in the classroom (Blanco, 1996). Student teachers encounter different difficulties in transferring the knowledge of mathematics education acquired in the initial teacher education centres to the primary school classroom.

Clearly, the above references suggest a new curricular framework for teacher education. They point to another knowledge base and a new methodological framework for mathematics teacher education which would determine a new concept of mathematics teaching/learning in education: learning to teach mathematics.

In this light, our objective would be to enable prospective primary teachers (PPTs) to acquire and develop the capability of translating into the classroom all this new mathematics cultural baggage which we want to communicate from a renovatory perspective. In sum, the aim would be for them to acquire the capacity for pedagogical reasoning (Brown & Borko, 1992; Shulman, 1993), and to attain cognitive schemes which would allow them to analyze and handle specific teaching contexts.

Pedagogical Content Knowledge (PCK).

In Blanco, Mellado, and Ruiz (1995), there is given a description of PCK that we take as a necessary referent in the process of learning to teach (See Figure 1). We specified different aspects related to mathematics teaching/learning, distinguishing two components that are well differentiated although closely related to each other: the *static component* and the *dynamic component*.

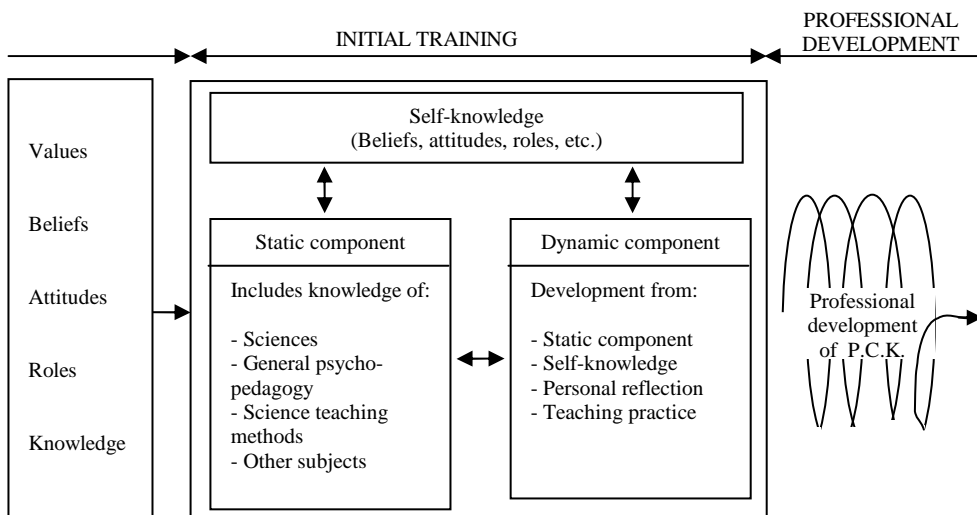


Figure 2. Schematic diagram of Pedagogical Content Knowledge (Blanco, 1999; Blanco, Mellado & Ruiz, 1995; Mellado, 1998;).

We shall take the *static component* to include those aspects of interest that are independent of the specific person who is teaching and of the specific context in which the teaching activity is being performed. It is impersonal, and can be found, and therefore studied and acquired, in written or audiovisual materials without having to develop any specific classroom practice. We could say that it would constitute the theoretical body of the teachers' knowledge. Thus, there is reference to mathematical content knowledge, specific knowledge about mathematics teaching/learning, and general psycho-pedagogical knowledge, *inter alia*. Static knowledge is necessary for students teachers, but it is insufficient for them to learn how to teach (Blanco, 1996; Mellado, 1998).

Furthermore, the study of their own conceptions, knowledge, and attitudes concerning mathematics and its teaching/learning is especially important as a first step towards generating new, more suitable, conceptions and practices. There is a need, therefore, for "self-knowledge" in relation to each of the items noted for the static component, to allow them to become aware of their explicit or implicit theories both relative to theoretical points of view that they might hold and to teaching in practice.

Such a study does not in itself automatically guarantee the transfer of knowledge of mathematics education to practice in the primary school classroom if the teachers have not also acquired practical schemes of classroom action (Mellado, 1998).

This part of knowledge, called the *dynamic component*, is generated and evolves from personal knowledge, beliefs, and attitudes. It requires a personal involvement, and develops by means of a dialectic process between the assimilated theory and actual experience.

By way of example, a group of teachers might study and apply to problems of mathematics the problem-solving scheme proposed by Polya (1957) or Schoenfeld (1985). But when they are in front of a primary class of 10 or 12-year-olds, working specific topics of the school curriculum, they need to readapt that scheme to their pupils' level and capabilities. This process of adaptation will be influenced by the conceptions and beliefs they hold about mathematics and mathematics teaching/learning, and above all by their previous teaching experience and reflection during and about the actions they had developed. Evidently, not all teachers will carry out this process in the same way, even though they might have studied the same content. Each one will give it a personal touch, coming down to a greater or lesser degree in favour of one or another heuristic device, creating an atmosphere that is more or less participative and collaborative, choosing different examples, etc.

The PCK is dynamic in so far as teaching practice and reflection-action allow the teachers to reconsider their knowledge, modifying or reaffirming part of the same. It only becomes visible through personal involvement, through reflection

and observation, and requires teaching practice in the subject within a specific classroom context.

From this perspective, practice teaching constitutes an obligatory reference in mathematics teacher education. It represents a privileged opportunity to investigate the process of learning to teach, and to give prospective teachers the chance to analyze critically their teaching strategies and compare them with their previous conceptions. Practice teaching is, in sum, the appropriate time to unite teaching theory and practice.

Qualitative methods constitute a suitable instrument for the initial and ongoing education of teachers, since they are then allowed to reflect on and go deeper into "their" thoughts and "their" teaching activity. Hence, some of the most commonly used techniques have been interviews, classroom observations, diaries, etc., which have served in some cases to understand and learn from others' experience, and in other cases to reflect on their own.

Development and Construction of PCK on Problem Solving

When PPTs enter an Initial Teacher Education Centre they bring with them the educational baggage of many years in school. They thus naturally have conceptions and beliefs concerning mathematics and the teaching/learning of mathematics that derive from their own learning experience. These conceptions are most often very traditional and self-contradictory, and, consciously or unconsciously, resurface during their preservice practice teaching.

We believe, therefore, that it is necessary to take into account this body of knowledge, beliefs, and attitudes concerning mathematics and the teaching/learning of mathematics because "learning involves an interaction between new and existing conceptions, with the outcome being dependent on the nature of the interaction" (Hewson & Hewson, 1989, p. 193).

In our teaching activity we set out a four-stage process in which we develop different kinds of activities that allow us to work on both components (static and dynamic) of the PCK, permitting their integral development.

Evidently, these actions are bound to the curriculum proposal which we must, as educators of teachers, put into practice in order to help insert problem solving into the curriculum of obligatory education. We consider that the ideal stage for this proposal to be effective is during teacher education.

Thus, in the program of the subject "Problem Solving in Primary Education", which is part of the curriculum for prospective teachers, we consider different phases, which we shall briefly describe in the following.

Stage 1. At the beginning of the school year we analyze the conceptions, beliefs, and attitudes of the PPTs about problem solving in order to determine their pre-existing ideas. We use group interviews with 2, 3, or 4 students. These are semi-open interviews: the items of a set script are modified in real-time according to the line the dialogue is taking, thus allowing the students to be more spontaneous. The tapes of the interviews are transcribed by the students themselves, and the results are analyzed in the group so that the PPTs may gain awareness of their own conceptions and beliefs.

The results of the interviews are contrasted with interviews and their corresponding analyses of students of previous years. This has allowed us to observe that the PPTs' conceptions and beliefs are consistent in so far as the results vary little from year to year. Similarly, the results are compared with the orientations established in the curricular proposals.

We shall show some general results from the interview analyses. The PPTs manifest a conception about problem solving which is linked to the idea of applying previously learnt knowledge. This forms part of a perspective of "teaching for problem solving" (Schroeder & Lester, 1989) which comes from their experiences as school students.

Thus, we would note the following, picked out from the interview transcripts:

Qu. What importance does problem solving have in the mathematics classroom?

M. One gives an explanation, a theorem, or a definition that has to be learnt, and then one goes on to the problem.

J. The normal thing is that you explain the theme, you give them certain concepts, properties, ... Then, with the problems the students find a usefulness for the theme.

The PPTs' activity as problem solvers during their years in obligatory schooling has usually been closely bound up with what we know as type-problems. This led them to learn the examples done in class by memory, especially as preparation for exams. Consequently, the students did not learn to solve problems, but to establish "tricks" of substitution which gave them correct results starting from the original problems. Their ideas concerning problem solving strategies are clearly conditioned by their problem-solving experience based on these type-problems and by the constant reference to written text problems. They do not conceive of a general procedure, but of a strategy linked to the wording of the problem, centring their attention on recalling earlier analogous problems and the type-problem.

G. The problems were always of the same type. There was no variety, ... they resolved them the same always. It was basically repeat and repeat, learning by repetition.

M. It is true that in some way we started from a model, and often we learnt the model beforehand. Instead of understanding it, we set straight out to see how you did this or that example.

This form of working problem solving provokes a contradiction between their personal experience, which they judge as having been negative and monotonous, and their conception of mathematics as linked to reasoning and rigour. Consequently they feel that problem solving in the mathematics classroom has been of little help in their studies of other subjects, except for some of the content of science, and almost none at all in real life, except for the typical examples connected to commercial mathematics which are the ones they identify with the usefulness of mathematics.

M. Mathematics is to learn to think and to learn to reason. And yet it is a very mechanical learning. It is just learning the procedure and then knowing how to carry it out.

As a consequence of their teaching experience, they find it hard to assimilate a different outlook on problem solving, or to approach and solve the other types of activities (connected to problem solving) indicated in the curricular proposals.

Stage 2. We believe that the PPTs' relative ignorance of problem solving and the difficulties they manifest as solvers is also one of the causes of their resistance to considering problem solving as a suitable context for learning mathematics.

That is why we include a stage centred on the PPTs themselves as problem solvers that allows us to analyze several variables associated with problem solving. The tasks put to the students are situations that encourage different forms of reasoning, and such processes as experimenting, discussing, conjecturing, representing, justifying, and so forth. We can hence describe problem solving as constituting a working environment.

This way of approaching mathematics enables the students to use the new personal experience that they gain as the basis for a critique of the view they had of themselves as mathematics learners and problem solvers, and hence create a suitable context for their conceptions and beliefs to change. It also encourages a new and more positive attitude towards problem solving in particular and mathematics itself in general.

As was observed in the work of Civil (1996), we also find that resistance, unease, fear, disbelief, and scepticism are some of the reactions and emotions that students start out with. But, as in the cases described by Civil, when the students share their feelings and ideas, and communicate their conjectures and results, they are able to learn about their mathematical world and to reflect on their reactions and behaviour.

In this stage, starting from the PPTs' own experience as solvers, we work up different aspects of the topic, related to problem-solving strategies, the analysis of affective and cognitive factors, problem classification, etc. We strive for a parallelism between the teaching received by the PPTs and how we would like them later to teach in the obligatory education levels of primary and secondary schools.

Stage 3. Thirdly, prior to practice teaching, we have introduced a stage in which we present activities that originate from specific mathematics teaching/learning contexts that arise in the primary classroom (Blanco, 1996; Blanco, 2001).

The description and presentation of the teaching problems about problem solving, their analysis, and the possible alternatives are presented from cases which were prepared on the basis of our teaching and research experience.

In this regard, we would recall that cases can be seen as:

- (a) providing precedents for practice teaching;
- (b) as a means to develop the process of reflection and generate critical approaches to teaching situations;
- (c) as situations which present questions rooted in the new approaches to teaching mathematics that one is trying to generate; and (d) as a form of presenting teaching problems situated in a specific teaching/learning context.

The studies we have carried out over the last ten years or so have allowed us to prepare a series of materials which serve as a starting point to encourage positive learning environments, and permit reflection on real or simulated classroom situations, whether on general aspects or on specific topics (Blanco, 1996; Blanco, 2001).

Stage 4. Finally, we take advantage of the practice teaching to make observations and video recordings of the prospective teachers in classroom problem solving activities at the primary level. This material, together with the interviews and/or questionnaires, the logbooks, artefacts, and recall stimulation interviews are analyzed in groups subsequent to the practice teaching (which the students perform in the second trimester).

This stage permits us to pull all the knowledge developed throughout the course together, using the framework of practical teaching situations.

An Arithmetic Problem Lesson: Posing the Problem and the Subsequent Difficulties of a PPT

As an example, in Blanco (1996) we presented a case study of a student in arithmetic problem solving, giving an analysis of the work developed in a class of the third year of primary school (9-10 year olds). This example forms part of the material presented to students in Stage 3. We centre on the relationship between the theoretical and practical knowledge concerning problem solving in arithmetic and the class activity developed during the practice teaching.

Domingo, the PPT, put to the pupils a situation which he considered to be an open and practical problem. He provoked student activity by means of blackboard diagrams (Figure 1) and the use of scrip money to lend realism to working through the problems, along the lines described by Harvey (1994).

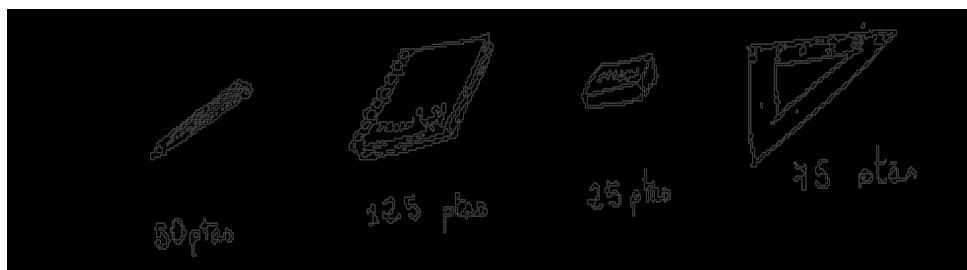


Figure 1. Drawings made by the PPT to serve as support for the different activities proposed.

His idea was to set a real situation, well known to the students, and which could serve to motivate them. The drawing suggested different activities and, therefore, could provoke a context where we might observe some of Domingo's

qualities as teacher in relation to arithmetic problem solving. In his program, he had foreseen three different activities, which we copy from his documents:

- "Hand over a 500 peseta note, buy what we want and get the change."
- "With a 100 peseta note, try to buy at least three things."
- "With the 100 peseta note, take another 500 peseta note and buy what we like, even if these are several things which are the same. But we have to spend all the money with nothing left over."

As our informant noted in his logbook:

These were problems that I had been keen to do, since the use of the material (scrip and real objects) not only seemed to me useful as a solving strategy, but as a motivating element for the students, something that I had verified on other days (Domingo's logbook).

The setting up of the activity seemed correct and appropriate, in line with what had been suggested in class, and coherent with the activity developed by the teacher-tutor in the practice teaching with these same pupils on previous days.

Nevertheless, the analysis of his performance revealed, as in the work of Cooney (1985), a contradiction between Domingo's reflections as expressed in his logbook and the reality seen on the video recording, i.e., between the statements of our informant and his actual teaching practice in the primary education classroom.

We shall briefly take up the third of these activities where one can appreciate certain elements of Domingo's practical knowledge relating to arithmetic problem solving.

Algorithmic Process and Relationship with the Pupils.

The third of the problems put forward, "we have to spend 600 pesetas exactly", presented an open situation with multiple solutions, and which could be solved using diverse calculation procedures.

Domingo sets this activity by assigning to two pupils the roles of cashier at the check-out and shopper. Addressing the latter and pointing to him:

Dom. ...You have to buy yourself the things you want, but you have to spend the exact money, the check-out clerk must not give you any change.

The analysis of the development of this activity was interesting because it showed us the relationship set up between the teacher and the pupils in solving problems.

The difficulties in managing the set activity were manifest from the moment of the presentation of this new problem. The reason was the imprecise indications our PPT gave the pupils about how to approach the problem. The situation obliged him to repeat the content of the problem on various occasions. It also obliged him to give the pupils hints about the procedure they could use to solve the problem, even though these hints, which were in a certain sense contradictory, induced some of the pupils to modify the procedure of their calculations.

In the analysis of the observations, we fixed on the pupils' calculational procedures. Thus, one sees two girls at the blackboard, and the rest of the class solving the problem. They are trying partial solutions according to the marked prices, grouping together first quantities 100 pesetas at a time (50 + 50, or 25 + 25 + 25 + 25). Then, seeing that they are using many numbers, they try other larger quantities (125 + 125), and so forth until the required quantity is completed. I.e., the pupils were using the different numbers until they had arrived at 600, without taking any notice of how many objects of each type they were buying. This procedure was used by a majority of the pupils and was thus reflected in their workbooks.

Domingo, in spite of the above, presented a more formal result on the blackboard (Figure 2). His choice was not at all a common result amongst the pupils. He had picked it from the only pupil who had obtained it, because it came the closest to the result that he had obtained in planning the lesson.

3 pencils are	150 ptas
2 rulers are	150 ptas
2 India rubbers are	50 ptas
2 notebooks are	250 ptas
Total	600 ptas

Figure 2. Domingo's result on the blackboard.

In our analysis we highlighted the vacuum produced between the general procedure followed by the pupils and the result that Domingo presented which corresponded to a more developed stage of mathematical ability.

When he chose that particular pupil's result, he told the rest of the class that he would consult with them about the strategies they had followed. However he had chosen a specific solution, the most elaborate, and expressly noted his interest and concern in checking that the result of the operations was indeed exactly 600. I.e., our informant confirmed the preoccupation that PPTs manifest for algorithmic procedures as a fundamental aspect of problem solving.

Dom. Come on José Ramon. You have already got it.

Dom. Let's see what we have bought. ... see if the 600 pesetas has really been spent. Let me see, if any of you haven't got it, carry on doing it now.

The result that he considered for the problem was not at all close to that obtained by the pupils. They, probably as a result of this last indication ("if any of you haven't got it, carry on doing it now"), set themselves down to rubbing out in their workbooks the "sums" they had done, and then to substitute them with the new operations that the PPT had written up on the board.

Likewise, some pupils got up to show different solutions to Domingo on the blackboard. Our informant held a brief dialogue with some of them individually, but never addressed the larger group to analyze the different solution procedures that the problem offered by being an open problem.

This fact passed wholly unnoticed by the PPT. He did not become aware of the situation until we expressly pointed it out from the video that we had recorded.

We observed that the open situation that the PPT aimed to use had not been in fact considered as such. The teacher had shown a preoccupation with exact calculation, for a certain algorithmic procedure, without any of the student participation that could have taken place as the result of using the open situation posed by the problem.

As the informant himself remarked when we analyzed the video:

It was a very open situation which was unconsciously converted into a closed one when I wrote up one solution on the board as if it were the only and definitive solution.

Everything that our future teacher knew about the need to start from the pupils themselves, from their knowledge and skills, etc., seemed to have been forgotten. But similarly, the theoretical referents established in relation to strategies that should be developed for problem solving, especially with respect to retrospection, i.e., analysis of the process that was followed, examination of the different strategies used, transfer of the result and of the procedure to similar problems, etc., that the PPT had experienced in the corresponding seminars, were not taken into account on this occasion. I.e., we found a misfit between Domingo's knowledge in this respect, which we had previously evaluated positively in our classes, and the actual teaching activity developed in his practice teaching.

This is an example of the difference between learning to solve problems (heuristic knowledge, control of factors that affect problem solving, etc.) and

learning to manage a problems class so that the primary level pupils involved will indeed learn mathematics. It is also a clear example that being able to do the first does not imply being able to do the second. Evidently, PPTs must acquire the knowledge and experience needed to assimilate both skills.

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Author

Lorenzo J. Blanco, Faculty of Education, University of Extremadura, Badajoz, Spain. Email: <ljblanco@teleline.es>