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One Teacher's Journey to Change Her Mathematics Teaching

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After 15 years of teaching and four weeks' participation in an inservice program designed to help teachers use student thinking to teach more meaningful mathematics, Teresa started the new school year by saying, " I feel like a first year teacher!" (Teresa, summer 1993).

This is a story about Teresa, a sixth-grade mathematics teacher who holds a master's degree in mathematics education, but still believes she has a long journey ahead of her as she changes her understanding of what teaching mathematics for understanding means. The story begins with a National Science Foundation (NSF) project in which she was directly involved for two years. The story continues with Teresa's concerns, questions, reflections, and ideas about improving her own mathematics instruction at the middle school level. The story does not end, however, because Teresa is continually struggling with change; yet, the story is optimistic because Teresa has discovered a new direction for her role as teacher. This new direction provides motivation for her to change her teaching methods and create a learning environment in which students reason and make sense of the mathematics they encounter. The purpose of our work is to provide information about one teacher's struggle to change her mathematics classroom in order that others may find consolation in the realization that change takes time; and, worthwhile, meaningful change takes years.

In mathematics, as well as science and other subject areas, teachers are being asked to examine teaching methods to help students develop reasoning skills to compete in the technological world (National Council of Teachers of Mathematics, 2000; Australian Education Council, 1991). Results from the Third International Mathematical and Science Study further confirm the need for effective methods of teaching (Peak, 1996). Research indicates that the process of change is difficult. "Understanding the internal processes individual teachers go through as they change is critical to researchers and educators during this vital era of reform" (Senger, 1999). When teachers are asked to implement new approaches and methods in teaching, they encounter conflicts, dilemmas and sometimes problems, many with no obvious solutions (Steinberg, 1994). For any enduring change in teaching practices, the process must become generative (Franke, Carpenter, Fennema, Ansell, & Behrend, 1998). This type of self-sustaining change focuses on assisting teachers in constructing their own perspective on change and providing them information from research on how students learn (Lubinski & Jaberg, 1997). As teachers develop their understanding of how students learn, they are encouraged to reflect on how their instructional decision making impacts student understanding. Organizing, sequencing, and planning mathematics instruction is then based on assessing and using students' thinking.

In his study on teachers' beliefs and practices about the teaching and learning of mathematics, Clarke (1997) identified seven components of a teacher's role. These components were developed from projects connected with mathematics education reforms. They include teachers' (a) use of non-routine problems, (b) knowledge of students' interests and needs, (c) use of a variety of classroom organizational styles, (d) development of a mathematical discourse community, (e) identification and focus on the big ideas of mathematics, (f) use of informal assessment methods to inform instructional decisions, and (g) facilitation of student reflection. Clarke described the practices of two teachers based on these seven components. The teachers were in self-contained classrooms using NSF project-developed materials. In a follow-up study, Clarke concluded that teachers can become comfortable with resisting the temptations to tell and that more experience with reform-based materials encouraged "greater emphasis on assessing processes of problem solving, communication and so on." (Clarke, 1999, p. 21)

Our perceptions of Teresa's role as a mathematics teacher using reform-based ideas are closely connected to Clarke's findings. Teresa routinely used non-routine problems and her decisions considered students' interests and needs. She used a variety of classroom organizational styles to create a learning community that fostered mathematical discourse. She focused on the big ideas of mathematics and she used informal assessment methods to inform her instructional decisions.

Our Story Begins with an NSF Project

Teresa was a participant for two years in an NSF project, "Influences on Pre-service Teachers' Instructional Decision Making" (Lubinski, Otto, & Rich, 1992). Although she was an experienced teacher, she was encouraged to develop a classroom learning environment that reflected current findings from research on children's mathematical reasoning. For two years she attended biweekly seminars during the school year and a total of 8 weeks of seminars over three summers. During the second year of her involvement, she worked with student teachers for two consecutive semesters. Topics for discussion at the seminars were often generated by teachers' questions or remarks, which appeared weekly in the journals they were asked to maintain. These journal entries encouraged the teachers to reflect on their classroom practices, on teaching issues important to them, and on the discussions occurring at the biweekly seminars. Grading and assessment, inclusion, and addressing parents' and students' perception of teaching and learning mathematics were among the concerns that were raised by Teresa. Her journal entries along with transcripts of interviews conducted with her during the initial two years of the project will tell the story of change from Teresa's viewpoint.

As Teresa began the project, she indicated a desire to develop her role as a teacher. During one interview, she said:

> Well, believe it or not, I've been at this for 15 years and I have never been happy with my role in that classroom. I think it's because my role has always been like an information giver and because that's what I've done, I guess that's why I continue to do it, but I've always looked for ways to get out of that role. I really feel that my role is one of presenting information but more than that, it's kind of guiding them and that's the role that I would like to develop, more of a guide than a presenter.

She indicated that she didn't like teacher-centred lessons but sometimes had difficulty getting students to interact during group work. Teresa wanted her students to understand, but she routinely chose the lesson and manipulatives based on the suggestions in the textbook. She taught procedures and liked to use word problems so students could apply the computational skills they were learning to real-life situations. When she was asked if there were any barriers that kept her from teaching mathematics the way she wanted, she responded that the biggest obstacle to implementing new ideas was herself.

During the first summer of the project, Teresa attended a four-week seminar. She was exposed to research-based information on students' mathematical reasoning through articles, videotapes, speakers, and interactions with elementary-age students. The teachers were asked to solve problems themselves and share their solutions. These tasks encouraged Teresa to develop her own strategies and understanding. The topics involved addition, subtraction, multiplication and division of whole numbers and fractions. Materials from the Cognitively Guided Instruction (CGI) Project (Fennema, Carpenter, & Peterson, 1991) and from the fraction research of Nancy Mack (Mack, 1990) were used. (Mack was also a project consultant and met several times with the teachers during the initial two years of the project.) The philosophy of constructivism (children construct knowledge; they do not receive knowledge) permeated the experience.

After the first summer of the project, as Teresa approached the beginning of a new school year, she was full of questions (at least 1000, she said!) and felt like a new teacher. However, she reported a sense of direction. She was developing a new approach to teaching and learning, a theoretical framework based on how children make sense of mathematics. She now wrote that she wanted to break away from the textbook: "We tend to think that what is printed in the sixth-grade book is what is important in sixth grade. While I don't want to take away the importance of guidance I do want to break down barriers that inhibit us from moving beyond." She had gained an appreciation for the ability of students to solve problems that she had not previously considered. She wanted to plan according to the information she gained by listening to students, and not be guided only by the structure of the mathematics textbook or her own understanding of mathematics.

As Teresa continued to change her beliefs and practices over the next two years, three recurring themes were evident:

- she began to develop a philosophy of teaching as she reflected on her teaching practices;
- she struggled with her own understanding of mathematics, particularly of fractions, an important content area for sixth grade in the U.S.; and
- she learned to better cope with inclusion and other issues as her practices changed.

Reflecting on Teaching and Learning

Developing a Philosophy of Teaching

After her initial exposure to a cognitively guided approach to teaching, Teresa realized that this approach does not provide a step-by-step set of instructions, prescribed materials, nor a set of activities to be completed. She now believes that the knowledge brought to class by the students should determine what activities to use, how to sequence instruction, and how to approach lesson planning. She described her lessons as frequently starting with a problem to solve and having students work in groups:

> They brainstorm several ways to approach the problem. The way they choose to solve the problem will depend on the group's discussion and the procedure everyone can understand. The procedure chosen will not necessarily be suggested by the smartest pupil in the group, nor will the procedure be the easiest one. The direction the group takes will reflect the understanding of the entire group. Since everyone is responsible for the strategy and the solution in their reflections, it is essential that everyone understands the methods chosen and is able to discuss the solution.

As Teresa developed her philosophy, changes in her teaching practice occurred. She:

- began to focus on students' thinking,
- encouraged her students to use symbols that represented their reasoning process,
- developed the social skills of her students so group work was productive and beneficial,
- gave more responsibility for learning to her students, and
- emphasized individual accountability by asking students to write about mathematics.

The following teaching sequence was observed toward the end of her first year in the project. Teresa asked her students how they thought about solving the following problem: "If you make five connected houses out of toothpicks, how could you determine the number of toothpicks needed for all five houses?" (See

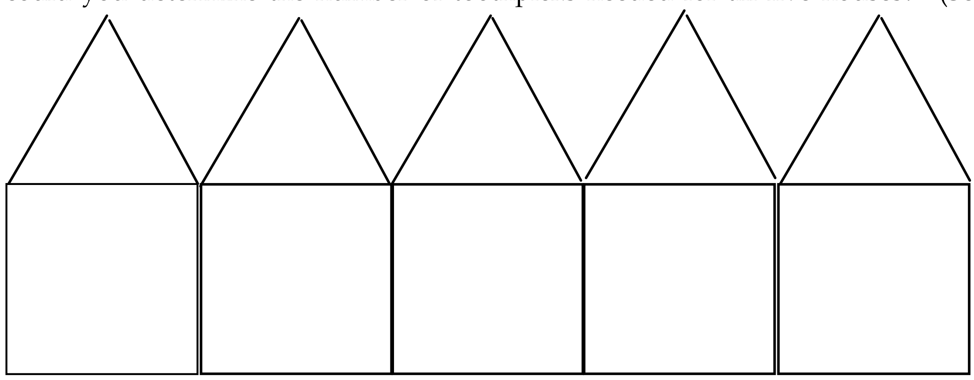

Figure 1.)

Figure 1. Five connected houses.

During the lesson, one student said he would count the four toothpicks needed for each house, excluding the vertical toothpick on the right side, and multiply the 4 by the number of houses and add one for the vertical toothpick on the right side. Teresa asked him to illustrate this on the board. He circled groups of 4 toothpicks. She then asked her student how he would determine how many toothpicks are needed for 10 houses? and then, many houses? Her objective was to have her students recognize a pattern and come to a symbolic generalization to represent the total number of toothpicks needed no matter what the number of houses.

The instructional decisions Teresa made in this lesson were significantly different from those in the initial lesson that we observed. After one year in the project, her role as teacher was that of questioner and the students' roles were that of problem solvers. The mathematics content was typical of a sixth grade class in the United States; however, the level of reasoning and sense making was not typical. Because the representation for each solution process could be matched to a student's reasoning, the mathematics had meaning for Teresa's sixth graders.

The topics for the second summer seminar included problem solving, geometry, assessment, and curriculum planning. This summer, Teresa wrote that she discovered that her thinking was too narrow, especially in relation to planning and activity selection. She began looking at the "big" picture. She decided to look at her objectives and sort them into groups of related ideas and concepts and use these connected ideas to develop activities and lessons:

> It has taken me over a year to get this idea through my head. I used to find an activity and apply it to an objective. Now I want to look at the objectives and use them as a springboard to create activities. Wow! What a difference in thinking! Instead of 'Here is an objective, what activity would best teach the objective?' I say, "Here are objectives that have this [idea] in common. What activity(ies) would best promote understanding of these concepts?

Teresa began to use students' thinking for planning in two ways. She related that as the class was discussing and sharing ideas, she recognized errors in their thinking and then developed tasks in response to her assessment. For purposes of long-range planning, she used students' thinking to make decisions about which concepts to cover and how to approach those concepts. Teresa's ever-increasing awareness of how mathematics instruction can be organized and taught in relation to students' thinking continued to generate questions and doubts as reflected in her journal a few weeks before the start of that new school year: "Do you ever start doubting yourself? Am I really cut out to be the kind of teacher I know I should be?"

As the school year progressed, Teresa found her understanding of teaching and learning mathematics developing as she interacted with other teachers. She valued her discussions with colleagues: "When our ideas match, I feel supported but I am most excited when ideas vary. Listening to other opinions and ways of thinking about mathematics makes me re-examine my own thinking." By interaction and then reflection, Teresa developed a philosophical framework and an understanding of what it means to teach and learn mathematics as a sense-making enterprise.

Struggling with her Own Understanding of Fractions

Initially, Teresa reported that operations with fractions were a large part of her sixth-grade curriculum. As part of the first summer seminar, Teresa interviewed a fifth-grade student about his understanding of fractions and the basic operations on them. She reported that he seemed to have a clear understanding of fraction concepts and the operations of addition, subtraction and multiplication with fractions as well as good reasoning skills. With regard to dividing fractions, she felt that he had such a good beginning that she believed that understanding would come with an extension of his "intuitive sense." However, here she indicated some of her uncertainty in teaching division of fractions: What does it mean to 'invert and multiply'? She wanted to extend her own understanding of division of fractions. However, as we read her reflections throughout the year, we noticed she realized that in addition to gaining a better understanding of division of fractions, there was a need for her to develop a richer understanding of fractions in general, including fraction equivalencies and the other operations with fractions.

After working with her students on adding and subtracting fractions and encouraging the use of different representations (fraction pieces, drawing pictures, etc.) and seeing how the different groups of students thought about the situations, she asked herself in a reflection: "I'm going to need to explore the 'overflow' idea some more. Also how does all this work with mixed numbers?" 'Overflow' for Teresa refers to the idea that when using pictures and joining two fraction representations such as those for 5/8 and 3/4, you exceed a whole. Allowing students to express their own thinking had Teresa looking more carefully at her own understanding and representation of fractions. This extended her reflection, "Do we really need to have the kids know how to reduce fractions? If a child can reason and move through a problem situation and arrive at an answer that is meaningful to him or her, isn't that enough?" She struggled with the emphasis that's put on "expressing the answer in simplest form." Then she observed that as students worked in groups, they often generated different fraction representations for the same numbers and concluded, "there has to be a point where kids understand equivalency with respect to reducing."

She continued to develop her own understanding of division of fractions. This often took place in the context of classroom discussions with her students as they tried to develop a procedure to carry out the division of fractions that made sense to them. The approach that was developed focused on rewriting the fractions with common denominators and then looking at how many groups of the size of the second numerator could be made from the size of the first numerator. For example, 3/4 ÷ 1/3 could be rewritten with common denominators as 9/12 ÷ 4/12. Then, students would look at the numerators and ask, 'How many groups of 4 are in 9?' It would be two and one-fourth. Teresa commented that this method translated well using whole and mixed numbers. She wrote in her journal: "My students really understand common denominators and I think they see the link between the fraction materials and the answers they get . . . I would say continue with common denominator division until 8th grade. Then take a look at the whole idea behind 'invert and multiply'."

In an interview conducted during the second summer seminar, Teresa reflected on how she had previously taught multiplication of fractions:

> Well, I used to just put it on the board and say it is the top times the top and the bottom times the bottom. Isn't that fun? And that is all there is to it, and I knew that wasn't good enough. And we would spend two months on fractions and my students would get to seventh grade and they wouldn't have a clue what a fraction was and it kind of drove me crazy because we had just worked for all that time and so I began asking myself why. You know, why isn't this working? Well, I knew deep down when you are presented with a lot or rules and regulations that if you don't remember the rule, then forget it. You are not going to remember how to do this, and so I knew there had to be a different way of doing it.

As part of her continual struggle during year two, Teresa wrote about her ideas on dividing fractions and the impact of conducting an individual fraction interview with a student. This opportunity to explore fraction understanding had a long-term effect on Teresa's conception of teaching fractions:

> The concept of just teaching the rule 'invert and multiply' was great until my interview with an 11-year-old student who said he knew nothing about dividing fractions. One component of the CGI philosophy that has fascinated me since the beginning is just how much children bring with them mathematically and how much we seem to squash that thinking assuming they 'are not ready for that.' I have seen kindergarten children (not 'gifted' either!) solve not only addition and subtraction problems but multiplication and division problems as well. How did they do it? By not being 'bogged down' by the terms and working with the concepts to which they could relate. This particular interview was just like that. The student and I talked a great deal about fractions and he told me what he felt he knew and what he didn't know. When it came to dividing fractions and looking at a problem such as 3/4 divided by 1/4, he flatly stated, 'I can't do that.'
>
> CGI encourages you to take a situation such as this and relate the problem to something with which the student can work. So I said, 'You have a personal pan pizza. You have eaten one piece and so you have three-fourths of a pizza left. If you gave one-fourth of the pizza to each of your friends, how many friends could you share the remaining pizza with? He didn't hesitate when saying '3.' What made the setting so different that he solved the problem easily? For one, the setting was real to him and he could relate to the problem in a personal way. But I feel that was not all that was going on. By placing the fraction in a context such as that, the question became 'How many one-fourths are there in three-fourths?' Isn't that what the basic idea of division of whole numbers encompasses? Simply shifting to division of fractions does not change the interpretation of the problems. While this idea was not new to me, the meaning of division had been lost along the way when denominators changed and invert and multiply became the rule.
>
> Then we switched to fractions that wouldn't come out so nicely. 'If I have 9/10 of a pizza, how many 2/10 servings do I have?' The interpretation remains 'How many 2/10 are there in 9/10?' Manipulatives were always available and the student chose to use them. His answer, after showing 9/10 and partitioning it into servings of 2/10, was that we would have four servings and this piece left over.' Since it is half of what we wanted a serving to be, we have four and one-half servings.
>
> By this time, we had talked about interpretation of division problems and used common denominators enough that he was comfortable with his interpretations and was solving the equations correctly. Now was the time to throw him a curve. What would he do with something like 5/8 divided by 1/2? He didn't hesitate to say, 'I know you are trying to find how many halves are in five-eighths. Since I always changed fractions like this to common denominators, I'll do it here, too. That is exactly what he did. His problem became 5/8 divided by 4/8 and he asked how many 4/8 were in 5/8. He chose to use manipulatives again, partitioning 5/8 into groups of 4/8. He decided he had 1 group and 1/4 of what it would take to make another group. His answer was 1 and 1/4.

> All this from a student who originally decided he didn't know anything about dividing fractions. I have taught division of fractions with story problems like this since this conversation. The meaning of the problems is apparent to the students and their understanding of the division process is enhanced.

More recently, Teresa reflected:

> This year I did not use manipulatives as much as I should. I tried to construct meaning by talking about division 'situations' with respect to the symbols and having students draw what was happening in the problem. What a bust! I had learned before that trying to develop meaning with symbols is a waste of time. Why did I try it again? I had the idea that connecting the symbols with drawings would work. Maybe it did--- a little, a very little bit. I am more sure that from the situation comes manipulative work, then discussion of patterns . . . and <u>then</u> comes a record of what is happening, the symbol form. Every time I try for understanding through the back door, it doesn't work. It is very hard to break that habit at sixth grade when so much emphasis is placed on symbol manipulation.

So, Teresa continues to struggle to make good decisions about the content and how to promote understanding of that content. However, her struggles have provided her with new insights into her role as teacher and more opportunities for her students to reason and make sense about the mathematics they encounter in their classroom.

Coping with Inclusion and Assessment

Two issues that generated interest, concern, and discussion during the seminars were inclusion and assessment. Teachers were expected to integrate students with physical and/or learning disabilities into their "regular" classes. Philosophically many of the teachers believed this to be a positive and sound educational decision, but "how to do this effectively" became the issue. Teresa's journal writing reflected many teachers' concerns: "What am I going to do with these kids? They probably don't know any facts! Do they have a foundation?" The inclusion of all students coincided with Teresa's involvement with the project:

> I think I would have lost my mind this year with inclusion students had I not been a part of [the project]. Learning disabilities are many and quite diverse. The other side of the coin is true as well. Learning abilities are many and quite diverse. Do you know what I used to do? I had all levels in my class and I had a hard time teaching 'down the middle.' Sure, it was easier to make one set of plans but I was losing so many kids. Everyone attacks a problem in a way that is meaningful for them. The variety of methods given to solve a particular problem is so rich that it adds another dimension to the class that cannot be 'staged.' Do you know what I decided was one main obstacle in dealing with the inclusion kids? Getting them to realize they have ideas. They don't have to wait for someone to tell them what to do first, second, third, etc. They can explore their own ideas. The problem is, they

> don't know they can explore their own ideas. They sit and wait for someone to tell them what to do.

Teresa's ideas have continued to evolve. She related that the attitude of the teacher is of greatest concern when dealing with inclusion classes. If students are viewed as having little to contribute to the class and have little ability, they are often taught "isolated pieces of information" that they're expected to "regurgitate." When facts and rules are taught in isolation, Teresa believes that mathematics has no meaning for the student. She wrote, "No wonder kids hate math claiming that it doesn't make sense." She believes inclusion students have the ability to think and reason: "I have watched them perform at levels that astonished their special education teachers."

The following incident illustrates how she facilitates the inclusion of all students in her mathematics classes and helps these special-needs students realize they generate valuable ideas. Although she generally uses heterogeneous grouping, she presented a task to her class with inclusion students grouped together. All students received the same initial instructions and Teresa sat at the table with her inclusion students:

> We literally stared at each other silently! They were all waiting for me to tell them the steps and I was bound and determined I wasn't going to! So we just sat and looked at each other for a very long five minutes. Finally, one of the kids said, 'Well... we could . . .' The other kids' heads just about snapped off they turned so fast to look at the speaker. But someone had an idea, a starting place. As she explained her idea not one student looked in my direction, so intent they were on <u>her</u> idea. I stood up and said, 'Sounds like a plan,' and walked away. Actually, I don't think anyone heard me.

The task involved data from a survey. Each group had to explain their survey data and how they decided to graph the results. The results from the inclusion group were excellent and their "verbal presentation was exceptional because they were the ones who took charge of the project. It was theirs every step of the way, not some teacher's ideas that they tried to carry out."

Another important issue that evolved during the project was assessment. Many teachers involved in the project were asked to assess students, not only for grading purposes, but also as "proof of learning" for their district and the state. As Teresa and other teachers began helping students develop their own understanding, the use of worksheets decreased and more communication with parents was necessary to explain why pages of computation were not being sent home. As more open-ended problems were used, grading rubrics were developed. Teresa is beginning to examine the assessment issue and has not come to any conclusion except that she has much to learn:

> It is so easy for our discussion to focus on assessment of what the students have done. While that is important, we must find new ways of assessing for the future. How much time have school teachers had in developing that skill? Not much. We

> go through a unit and then assess. We go through the next unit and assess. We have new skills to learn.

The Ongoing Struggle with Change

Why Change?

In a very vivid way, Teresa's story illustrates the difficulty, the excitement, and the rewards of changing beliefs and practices in respect to the teaching of mathematics. At the end of the project, Teresa reflected:

> Nothing comes close to placing the power of learning where it belongs--with the students. Almost every time we share ideas and strategies, I learn something new about what they are thinking. Their sharing may clarify a point that had been twisted in their minds and is not mathematically correct or it reinforces that the task was effective and accomplished the goals set forth in the beginning. As I look for ways to develop understanding for my students I realize that my own understanding of mathematics is greatly enhanced. My students reveal so much when they share their ideas I find myself shaking my head at the depth of reasoning that goes on. We underestimate their abilities to think and reason in the classroom. What can we expect though with traditional teacher-centred, lecture-oriented, symbol-based curriculum? I won't sit back and watch my first love [mathematics] being stuffed into minds that are closed to new ways of thinking only because they have never been offered the opportunity to [learn in this way].

Discussion

A New Direction and New Role for Teresa

Teresa continues to struggle to make sense of her own understanding of mathematics. She continues to examine the best way to teach mathematics to her sixth graders. She also realizes that students need time to struggle with and reflect upon mathematical ideas. She remains intrigued and interested in her students' thinking. These findings are interesting in light of Clarke's (1999) follow-up study of his two teachers and their perception of the teacher's role. His more recent findings suggest that over time, teachers had become so familiar with the material they taught that they forgot the students' need to struggle, reduced the use of small group work, and lost the excitement of soliciting and hearing the variety of solution strategies presented by students. These findings do not apply to Teresa at this point. However, some of Clark's findings seem to resonate with Teresa's evolving role as teacher. Clark found that the two teachers in his study condensed unit content, broadened their views on the big ideas of mathematics, and incorporated student time to reflect throughout the lesson. Teresa's school is departmentalised. Thus, she teaches five periods of sixth-grade mathematics each day and has 125 students. We conclude that Teresa is a particularly reflective individual who continues to be challenged by the difficulty of teaching within the reform vision of mathematics. Do the differences that we observe between Teresa and Clarke's teachers exist because Teresa teaches only mathematics in a departmentalised situation? or because she has a strong background in mathematics? or because she is a particularly reflective individual?

Implications for All Teachers

After following Teresa and observing her change in beliefs about how mathematics should be taught, how students learn mathematics, and how to sequence and plan mathematics instruction, questions remain. What does a call for change mean for other teachers who do not share her experiences and background? Teresa began the project with a good mathematics background, good rapport with her students, and an interest and open mind to change. Teresa's reflective ability has led her to a "self-sustaining" level. Even after her formal involvement with the project had ended, Teresa's decision-making processes continued to be affected by her students' thinking. To develop this level of reflection, Teresa shared these suggestions:

- keep a journal,
- have an open mind and willing attitude to try new ideas, and
- communicate about teaching with colleagues.

The journal provides teachers with opportunities to make observations about their students. Observations about their own teaching, as well as insights gained from these observations, can be recorded. Furthermore, a journal can provide a place for teachers to write down questions they ask themselves.

Teresa's changes in beliefs and practices continue. Teresa has relinquished the role of "answer-giver" and so she is comfortable with the questions that have no simple answers. She focuses on ways to develop student understanding in a middle school mathematics classroom. She continues to generate questions and concludes, "I don't have all the answers." Thus, Teresa's journey to change her mathematics teaching continues!

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