Prospective Indonesian Secondary Mathematics Teachers' Epistemological Obstacles in Designing Project-Based Numeracy Tasks: A Case Study

Vivi Suwanti
Universitas Negeri Malang

Cholis Sa'dijah
Universitas Negeri Malang

Makbul Muksar
Universitas Negeri Malang

Received: October 2023 II Accepted: May 2024

This study aims to describe the epistemological obstacles of prospective Indonesian secondary mathematics teachers' in designing project-based numeracy tasks (PbNTs). Three case studies focused on qualitative analyses of prospective Indonesian secondary mathematics teachers' PbNTs from responses to a semi-structured interview based on indicators of epistemological obstacles. It was found that epistemological knowledge was not the main cause of the obstacles they experienced, which is inconsistent with findings described in previous studies. It was revealed the prospective secondary mathematics teachers were too focused on the mathematical aspects and did not pay attention to the role of contextual knowledge in the process of PbNT design.

Keywords: epistemological obstacles • prospective mathematics teachers • secondary education • project-based tasks • numeracy

Introduction

Numeracy is one of the fundamental components in learning that has now attracted the attention of educational experts and practitioners in various countries such as Australia (Fisher et al., 2017; Forgasz & Hall, 2019; Goos et al., 2014; Hall & Zmood, 2019; Sellings et al., 2018), United States of America (Wake, 2015), and Canada (Liljedahl, 2015). As well, Indonesia was one of the countries that began to introduce numeracy as one of the fundamental components in its general education curriculum (Kemendikbud, 2017; Machromah et al., 2021; Purnomo et al., 2022).

Numeracy refers to abilities that involve applying mathematical knowledge, skills and understanding in non-mathematical contexts, and specifically how mathematics is used in the workplace and society (Goos et al., 2015; Wake, 2015). For example, scholars have noted the numeracy practices in the construction of stone walls holding terraced rice fields in the northern Philippines (Geiger et al., 2020). In the mathematics education literature, numeracy includes a range of areas such as financial numeracy (Turineck & Cavalcante, 2021), adult numeracy (Tout et al., 2020), and scientific numeracy (Tegeh et al., 2021). Numeracy involves the ability to make sense of non-mathematical contexts through a critical orientation of mathematical thinking (Geiger et al., 2015). The 21st century numeracy model developed by Goos et al., places context at the heart of numeracy and is anchored by the key role of mathematical knowledge, tools, and mathematical dispositions embedded in a critical orientation in evaluating the appropriateness of selecting mathematical thinking to analyse complex context information (Goos et al., 2014; Gorman et al., 2023). This model supports students to be experienced in using mathematics in real-world contexts across all subjects at school (Geiger, 2018; Goos et al., 2013, 2014; Wake, 2015). Among the learning methods aligned with the principles of 21st century skills that take into account the across-curriculum approach advocated, is project-based learning (Zakiah et al., 2020).

Project-based learning, one of the cross-curriculum learning methods that carries the context of an authentic problems, involves a product as a solution (Condliffe et al., 2017; Haatainen & Aksela, 2021; Zakiah et al., 2020), and starts with a project-based task (Craig & Marshall, 2019). This differentiates the strategy from other learning methods (Guo et al., 2020). For example, in a geometry project entitled *Neighbourhood Renewal Project*, students were asked to design two-dimensional scale plan of a neighbourhood with the driving question, "How do you design your neighbourhood plan?" (Ubuz & Aydınyer, 2019). Project-based tasks and 21st century numeracy models that share aspects of context, content knowledge, and the active student engagement are compatible integrations. A project-based numeracy task (PbNT) in this study is a project-based task that is designed using the elements of the 21st century numeracy model. Therefore, the combination of learning involving numeracy tasks and authentic projects has promising prospects for future research.

Several studies have identified difficulties and discrepancies in the design and use of project-based tasks by teachers and prospective teachers (Aldabbus, 2018; Haatainen & Aksela, 2021; Tamim & Grant, 2013). Scholars have identified that prospective teachers' ability in designing numeracy tasks is low (Kohar et al., 2022), and they experience classroom management difficulties during task delivery. These difficulties and discrepancies may be caused by the interaction between new knowledge about project and numeracy tasks and their prior knowledge of learning, which we refer to as *epistemological obstacles* (Minaidi & Hlapanis, 2005).

Previous studies on the obstacles occurring in prospective teachers' ability to design tasks have been conducted in the realm of mathematical knowledge (Moon et al., 2012; Murniasih et al., 2020; Mutambara & Tsakeni, 2022), pedagogical knowledge that examines the obstacles in designing lesson plans (Prabowo et al., 2022), and the implementation of new learning methods (Ceolim & Caldeira, 2017; Murniasih et al., 2022). The knowledge required by prospective teachers in the design process of higher-order thinking skills tasks, such as project-based numeracy tasks, includes mathematical knowledge and pedagogical knowledge (Sa'dijah et al., 2023). Therefore, the epistemological obstacles referred to in this study are the integration of mathematical epistemological obstacles (e.g., the fraction sense epistemological obstacle [Susiswo et al., 2021]), and pedagogical epistemological obstacle (e.g., epistemological obstacles to the application of technology in education [Minaidi & Hlapanis, 2005]).

In an Indonesian educational context, numeracy began to be included as one of the aspects in the national assessment in 2021 (Purnomo et al., 2022), for which project-based learning utilisation was recommended (Kemendikbud, 2017). Since studies on the ability of Indonesian prospective secondary mathematics teachers (PSMT) on numeracy tasks (Kohar et al., 2022; Machromah et al., 2021; Yustitia et al., 2021) have not examined epistemological obstacles, the study reported in this article aimed to describe the mathematical and pedagogical epistemological obstacles that arise for Indonesian PSMTs when designing PbNTs. This study has the potential to provide insights into teacher education programs that support prospective teachers to design critical numeracy tasks. It is advantageous to assist teacher educators in identifying epistemological obstacles that arise for PMSTs when designing PbNT so teacher educators can use that knowledge to support PMSTs' learning.

Literature Review

Numeracy

Numeracy can be interpreted as the ability to use mathematics with critical assessment in everyday life. It also involves the interpretation of existing quantitative information by paying attention to non-mathematical elements (Forgasz & Hall, 2019; Geiger, 2018; Hall & Zmood, 2019). Numeracy tasks are often characterised by structured word problems; have many solutions, strategies or even no complete solution; and are connected to a variety of non-mathematical factors. Therefore, to foster students' sense of critical numeracy, numeracy tasks must be designed based on local conditions and environments that involve various knowledge domains, are related to student performance, and have both relevance and application in everyday life (Fisher et al., 2017; Geiger, 2018; Turineck & Cavalcante, 2021). Teachers can use the 21st century numeracy model as it encompasses the design principles of rich numeracy tasks (Geiger, 2018; Getenet, 2022; Goos et al., 2013). This model outlines four key dimensions of the broad numeracy definition, namely: contexts (the capacity to use mathematical knowledge in a variety of contexts both in and out of school), mathematical knowledge (mathematical concepts and skills; problem solving strategies; and estimation skills), tools (the use of material, representational and digital tools to mediate and shape thinking), and dispositions (confidence and willingness to use mathematical approaches to engage in life-related tasks; readiness to use mathematical knowledge flexibly and adaptively). These dimensions are activated through critical orientation (analytical and evaluative skills) (Geiger et al., 2020).

According to Tout et al. (2020) the complexity of numeracy tasks can be viewed from textual and mathematical aspects. The level of context use in mathematical problems can be classified into zero order (the given problem instruction can be executed directly mathematically without interpretation), first order (the context is used to identify variables and relationships for the mathematical formulation of the problem), and second order (the context is a source for defining variables, relationships, and assumptions for the mathematical formulation of the problem) (Salgado, 2016). The level of mathematical cognitive processes in numeracy tasks can be classified into understanding/knowing (covering facts, concepts, and procedures that students need to know), applying (focusing on the ability of students to apply knowledge and concepts understanding to problem solving), and reasoning (goes beyond the solution of routine problems to encompass unfamiliar situations, complex contexts, and multistep problems) (Kemendikbud, 2017; Martin & Mullis, 2019). The same numeracy task, however, can be perceived as having different difficulties for different students, depending on factors such as familiarity with the context of the task, knowledge of mathematical concepts, or problem-solving and reasoning abilities (Tout et al., 2020). Therefore, PSMTs are also expected to design numeracy tasks that suit the conditions and the needs of the students.

Project-based Learning

Project-based learning activities provide opportunities for students to learn by doing, engaging in deep learning that raises questions through authentic problems and challenges (Condliffe et al., 2017; Ubuz & Aydınyer, 2019). Authentic projects begin with a task that provides context and direction in line with the learning objectives; facilitates deep inquiry into the content; provides opportunities for choice, opinion, and critique; and comes with a final product presentation to audiences (Craig & Marshall, 2019). Based on a synthesis of project-based learning elements (Haatainen & Aksela, 2021) in the design of project tasks, it is necessary to pay attention to compatibility with student learning objectives, project centrality in the learning process, context (authentic, meaningful, related to real-world contexts or important issues, and related to student attention and interest), project artefacts (produce a final product), collaboration, construction of knowledge, and activation of student involvement. Barak and Assal (2018) developed a P3 taxonomy of tasks that distinguishes between three levels of tasks including Practice (exercises and close ended tasks where the solution is known in advance and students can check if they answered correctly), Problem solving (small-scale, open-ended tasks where students may use different solution methods and arrive at different answers), and Projects (ill-structured, challenging tasks where the problem is not clear, and students take part in defining the problem, setting goals, identifying constraints and choosing solution methods).

Epistemological Obstacles

Learning obstacles are obstacles to the thinking process that can be caused by three factors, namely: ontogenic obstacles (students' mental age readiness to learn), didactic obstacles (education system/curriculum), and epistemology obstacles (student knowledge that has limited context application) (Hariyani et al., 2022; Murniasih et al., 2020; Prabowo et al., 2022). Ontogenic and didactic obstacles can arise due to errors and mismatches of curriculum-derived learning processes to students' characteristics and mental age (Cesaria & Herman, 2019; Murniasih et al., 2020). Meanwhile, epistemological obstacles can arise due to the dialectical construction of knowledge where new knowledge is derived from prior knowledge that simultaneously supports and conflicts with new schemas (Minaidi & Hlapanis, 2005).

Epistemological obstacles can be seen as a person's knowledge that is limited to a certain context. When faced with a different context, the knowledge becomes unusable and difficulties are experienced (Cesaria & Herman, 2019). Murniasih et al. (2020) examined PSMTs' epistemological obstacles in solving mathematical problems taking the form of language representations and found a tendency to generalise and to rely on intuition. Minaidi and Hlapanis (2005) found that epistemological obstacles in teaching can arise from prior experience and verbal expression. Furthermore, these authors divided prior experience into two factors, namely former education (influenced by the prior knowledges and various teaching experiences possessed by teachers to consolidate the teaching model used) and pre-performed learning methods (teachers adopt ways and methods that help them learn more effectively). Prior experience refers to Zambrano and Noriega's (2011, as cited in Cesaria & Herman, 2019) view of important characteristics, namely, they are unable to respond and understand new knowledge obtained and they are only able to respond to a few concepts that have been previously understood. Verbal expression arises from linguistic expression where words do not always transfer the same meaning for everyone depending on prior knowledge and experience.

Conceptual Framework

Numeracy tasks and authentic projects have something in common in terms of real-world problem contexts that promote positive dispositions through active student engagement and involving multidisciplinary knowledge as the focus of learning (Geiger et al., 2020; Haatainen & Aksela, 2021; Zakiah et al., 2020). Therefore, teaching critical numeracy through project-based learning will be viewed as beneficial. To start project-based numeracy learning, there is a need to first design PbNT (Condliffe et al., 2017; Grant, 2009; Zakiah et al., 2020). PbNTs are designed by taking into account 21st century numeracy models including: context, mathematical knowledge, dispositions, tools and critical orientations (Goos et al., 2014), a synthesis of project-based learning elements including: 1) learning objectives, 2) centrality of project, 3) context, 4) project artefacts, 5) collaboration, 6) construct knowledge, and 7) activate student involvement (Haatainen & Aksela, 2021), and, a project taxonomy that includes ill-structured challenging tasks (Barak & Assal, 2018).

The PbNT elements of the 21st century numeracy model and project-based learning synthesis have similarities in the use of real-world contexts, the presence of content according to learning objectives (in this case mathematical content covered by numeracy), and the disposition that facilitates the student' active engagement with the task. The constituent elements of project-based numeracy tasks included in Figure 1.

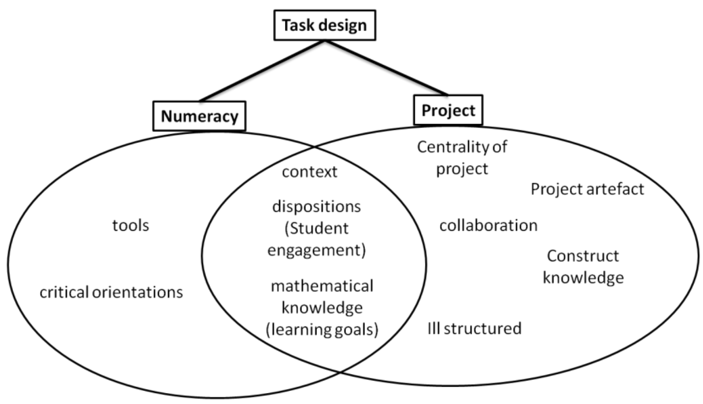


Illustration of the constituent elements of PbNT in diagram form

Figure 1. Constituent elements of PbNT in diagram form.

International Commission on Mathematical Instruction (ICMI) Study Group developed five dilemmas that should be considered in task design process, including: context as a dilemma, language as a dilemma, structure as a dilemma, distribution as a dilemma, and level of interaction as a dilemma (Sullivan et al., 2021). The study approach on PSMTs' knowledge in task design can be seen from two approaches, namely mathematical knowledge for teaching, and mathematical didactic knowledge (Scheiner et al., 2023). Mathematical knowledge for teaching is seen as practice-based knowledge with multidimensional constructs consisting of mathematical and pedagogical content knowledge domains. Conversely, mathematical didactic knowledge is regarded as theory-based knowledge resulting from the elaboration of basic principles in mathematical didactics, serving as guidelines and directions for teaching and teacher education. Therefore, the approach used by PSMTs in the PbNT design process can be analysed from both perspectives.

The design process of numeracy tasks and projects is complex because it requires a wider range of skills than routine mathematics tasks. Aldabbus (2018) found that more than 75% of teacher participants from Bahrain were unable to apply project-based tasks due to many challenges including selection of significant content of tasks, time management, and lack of practical activities. Haatainen and Aksela (2021) determined that teachers were unable to provide clear project-based tasks to focus student discovery and learning motivation. Tamim and Grant (2013) discovered that teachers used project learning models either to reinforce, extend, or initiate learning, or to direct student learning according to the teacher's comfort level in creating a balance between curriculum and testing needs and aspirations for the use of constructivist strategies on the continuum of student learning processes.

Kohar et al. (2022) analysed PSMTs' ability to design numeracy tasks, and focused on two domains (namely, level of context use, and mathematical cognitive process). These researchers found that the level of context used in tasks varied between zero and first order with cognitive processes scored at the understanding/knowing level. Goos et al. (2013) found that teachers experienced difficulties in determining how long a numeracy task investigation should be carried out so that students have enough time to explore without losing motivation, and how much scaffolding should be given. They also discovered that teachers argued that numeracy tasks should be broken down into subtasks to make it easier for students to achieve learning objectives. These difficulties and modifications show the PSMTs' approach as a practitioner to connect between their mathematical knowledge for teaching and the required mathematical didactic knowledge (Scheiner et al., 2023). The occurrence of discrepancies in the PSMTs' chosen approach could be caused by epistemological obstacles.

There are many important considerations in task design besides mathematical knowledge, especially when PSMTs want to use certain pedagogical choices (Sullivan et al., 2021). Mathematical knowledge for teaching is conceptualised as a multidimensional construct consisting of mathematical and pedagogical content knowledge domain (Scheiner et al., 2023). Hart (2014) developed a pedagogical content analysis framework for task design with two main steps, namely: analyse the mathematics and analyse the mathematics pedagogically. Therefore, in this study, the PSMT epistemological obstacles in the PbNT design process are analysed through the integration of mathematical epistemological obstacles (Murniasih et al., 2020) and pedagogical epistemological obstacles (Minaidi & Hlapanis, 2005) modeled in Figure 2.

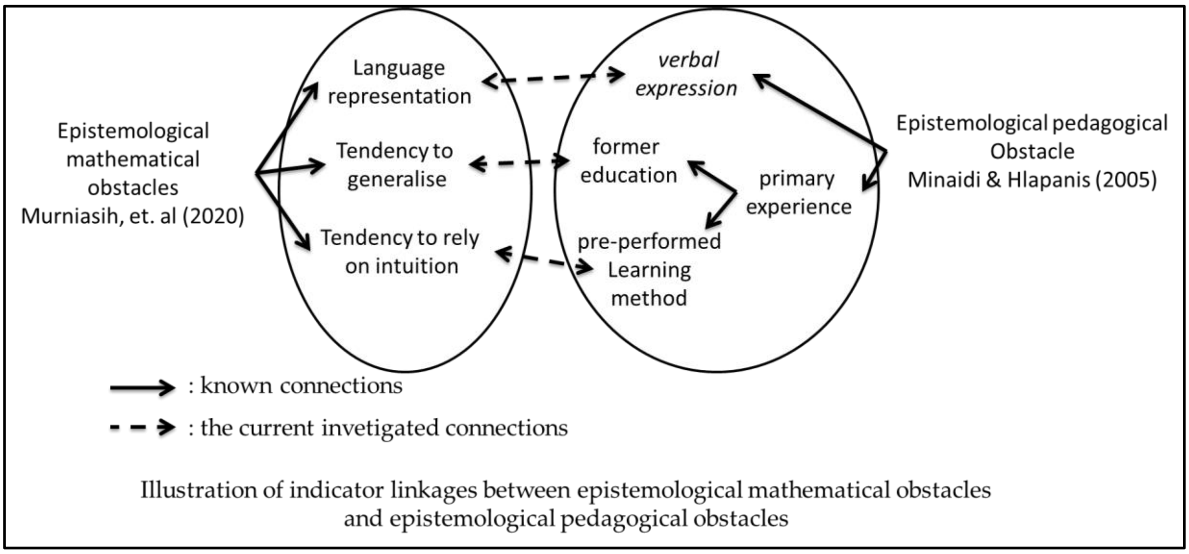


Figure 2. Factors of epistemological obstacles.

Mathematical epistemological obstacles of *language representation* appear through transition errors from written language to mathematical symbols (Murniasih et al., 2020). The reason for the emergence of this type of obstacle is in accordance with the causal factors of *verbal expression* pedagogical epistemological obstacles. This is because information from a linguistic expression is not always interpreted with the same meaning by each individual (Cesaria & Herman, 2019). In the context of this study the information in question focuses on PbNT. The *tendency to generalise* in mathematical epistemological obstacles is caused by PSMT's lack of mathematical knowledge when solving new problems so they generalise old knowledge and skill to solve new problems (Bishop et al., 2014; Moru, 2009; Murniasih et al., 2020). It is also resistant to the introduction of new knowledge and skill (Bishop et al., 2014). This type of obstacle is in accordance with the *former education* of pedagogical epistemological obstacles in which prior knowledge and skills from teaching experience consolidate the pedagogical knowledge used by PMSTs (Minaidi & Hlapanis, 2005), in the context of this study, the focus is on knowledge of PbNT. The *tendency to rely on intuition* in mathematical epistemological obstacles is caused by PSMTs' belief in concepts they have always considered to be correct for solving mathematical problems (Purnomo et al., 2014). However, this established intuition cannot be used for new problems, so PSMT cannot evaluate the correctness of the solution and tend to rely on deceptive intuition (Moru, 2009). This corresponds to the *pre-performed learning method* factor of the pedagogical epistemological obstacle where PSMT believe that the learning methods that are effective for themselves when studying will also be effective for their students (Minaidi & Hlapanis, 2005). Based on the linkage between the forms of mathematical epistemological obstacles and the causal factors of pedagogical epistemological obstacles, we compiled epistemological obstacle factors focusing on the PSMTs' PbNT design process and aimed to explore the forms and factors of epistemological obstacles. Therefore, the key research question that guided our study is:

> *What and how do epistemological obstacles arise for Prospective Indonesian Secondary Mathematics Teachers when designing project-based numeracy tasks?*

Methods

A qualitative approach with descriptive data to describe the epistemological obstacles that occurred when PSMT designed PbNT was used in this study. This study is an explanatory type (Cohen et al., 2018) based on the epistemological obstacles theory by Minaidi and Hlapanis (2005) and Murniasih et al. (2020). This study was limited by the number of participants, but each case was described in depth. We adapted Gronow et al.'s (2022) case study method and data analysis for their study about teachers' understanding of mathematical structures and their application to learning.

Participants

The participants of this study were three prospective secondary mathematics teachers at one of the private universities in Malang City, Indonesia, who had practical experience in schools. In accordance with the Indonesian government's policy on national assessment in 2021 (Purnomo et al., 2022) numeracy was being introduced to PSMTs through formal courses. The participants in this study, however, were PSMTs who had taken all the formal courses before learning how to teach numeracy in schools. The participants had already completed theoretical instruction about project-based learning from learning models and lesson-planning courses. All three participants gleaned an understanding about numeracy when completing practice teaching in schools. As an ethical consideration for participants who are still student teachers, we secured informed consent from them and use initials to maintain their anonymity. The three participants are denoted as initials IAB, NR, and IM. Further details of the participants are given in Table 1.

Table 1
Participant Information

Participant initials	Demographics			
	Major	Gender	Practical teaching experience	School context
NR	Mathematics education 4th year	Female	Primary schools	Rural school (purely rural culture)
IAB	Mathematics education 4th year	Female	Junior high schools	Urban and rural border schools (mix of urban and rural cultures)
IM	Mathematics education 3rd year	Female	Junior high schools	Rural schools that already have digital technology facilities

Data Collection

The data sources included the PbNT design test, semi-structured interviews with PSMT on obstacles to design process of PbNT, and observation field notes of PSMTs' behaviours and external knowledge sources. The PbNT design test instrument provides instructions for participants to design PbNT according to their knowledge. Participants can seek information and ideas from various sources during the test. The interview guidelines of the epistemological obstacle-based interview are divided into the initial numeracy and project knowledges interview, and the PbNT design process and implementation test-based interview (see Appendix A). The initial numeracy and project knowledge interview was conducted to find out what knowledge PSMT already had about numeracy and project learning. Meanwhile, the PbNT design process and implementation test-based interview was conducted to collect data on the process that PSMTs go through when designing PbNT and imagining how it will be implemented in the classroom.

The epistemological obstacle-based interview guide was developed based on Minaidi and Hlapanis' (2005) pedagogical epistemological obstacle factors and Murniasih et al.'s (2020) mathematical epistemological obstacle forms, as seen in Figure 2. The factors of epistemological obstacles in the PSMTs' PbNT design are described in Table 2.

Table 2
Factors of PSMT Epistemological Obstacles in PbNT Design

Epistemological obstacles indicators	Sub indicators	Descriptor
Verbal expression	Verbal expression (VE)	Differences in word meanings between those conveyed by sources of information about numeracy and project-based tasks and PSMT's understanding.
Prior experience	Former education (FE)	Tendency to generalise project-based learning and numeracy knowledge based on incomplete theoretical background and teaching experience.
	Pre-performed learning method (PL)	PSMT intuitively adopt ways and methods that help them learn more effectively.

Before being used to collect data, instruments were piloted on three other PSMTs in the same class to check the validity of the data obtained. This practice is in accordance with Gronow et al.'s (2022) case study on teachers' understanding of mathematical structures.

Procedures

The data collection process for this study was carried out by the researcher (first author) individually with each participant during different periods of time. Data collection procedures carried out in this study included:

1) providing initial stimulation to PSMT to activate knowledge about numeracy and project-based learning through prompting questions in design test:

> Try to remember about project-based learning that you have learned! Try to remember about numeracy that you know when teaching practice at school! Imagine that you will teach numeracy using project-based learning to junior high school students. Please, design a project-based numeracy task that you will use as a problem to be investigated during learning!

2) the researcher giving participants time to find information and ideas about PbNT design, 3) analysed PbNT designed by participants, 4) interviews, and 5) analysed the forms and factors of epistemological obstacles in PbNT design.

Analysis

For the analysis of qualitative data, the following steps were employed: data reduction, data display, data analysis and interpretation, and conclusion drawing and verification (Cohen et al., 2018). In Indonesia, the Ministry of Education has provided specific guidance on the aspects of numeracy that teachers need to pay attention to, namely 1) context covers personal, socio-cultural, and science; 2) mathematical content covers number, geometry, algebra, and data and uncertainty; and 3) thinking process covers the levels of knowing, applying, and reasoning (Kemendikbud, 2017). Thus, for the PbNT elements, the context and mathematical content in Table 3 are adapted to the conditions in Indonesia, which is the background context of this study. The coding used refers to the factors that cause the emergence of epistemological obstacles from the interview transcripts (associated with epistemological obstacles in Table 2) and the form of PbNT generated from the task design process (associated with elements of PbNT in Table 3 adapted from Haatainen and Aksela [2021] and Geiger [2018]).

Table 3
Elements of PbNT

Elements	Description
Context (N,P)	Projects should be authentic, meaningful, related to real-world contexts or important issues (personal, cultural, and scientific).
Mathematical content (N,P)	Mathematical concepts and skills (knowing level), problem solving strategies (applying level), estimation and reasoning skills (reasoning level).
Disposition/student enggagement (N,P)	Encourage students' confidence to engage and use mathematical knowledge from start to finish.
Tools (N)	Use of physical (models, measuring instruments), representational (symbol systems, graphs, maps, diagrams, drawings, tables), and digital (computers, software, calculators, internet) tools to mediate and shape thinking.
Critical orientation (N)	The use of mathematical information and activities to make decisions and judgments, form opinions, add support to arguments, and challenge arguments or positions.
Centrality of project (P)	Projects are not the end point of learning as in most standard classes, but projects in project-based learning are seen as a process throughout the learning.
Collaboration (P)	It requires social negotiation of knowledge, working collaboratively in groups, to build possible solutions for the project.
Project artefact (P)	Project activities should involve creating a useful end product that addresses the driving question and provides a representation of student learning.
Construct knowledge (P)	Project-based learning involves students in the construction of knowledge through deep inquiry, critical thinking, the use of problem solving, and through the revision of what is currently known and what needs to be understood before moving on to the next stage.
Ill-structured (P)	Students take part in defining the problem, setting goals, identifying constraints and selecting solution methods.

Notes: N,P: numeration and project shared element; N: numeration element; P: project element

We conducted interrater coding by cross-checking the interpretation and results of data analysis with experienced experts in study of cognitive obstacles (Cheung & Tai, 2023). There were discrepancies in the interpretation of the epistemological obstacle factors coding of former education and pre-performed learning method, due to the presence of similar features as causes. These discrepancies were resolved through discussion where agreement was reached on adjusting the description of "pre-performed learning method".

Results

Derived from the PbNT design test results, test-based interviews, and field notes, we present data from the three participants focusing on the emergence of epistemological obstacles.

Epistemological Obstacles: Case Study IAB

(i) Original

Ilham memberikan tantangan kepada Ika untuk menghitung jumlah uang koin yang diperlukan, untuk memenuhi papan catur. Pada kotak pertama diberi 1 uang koin, kotak kedua 2 uang koin, 4 uang koin untuk kotak ketiga, 8 koin untuk kotak keempat demikian berlanjut memenuhi 64 kotak.

a. Bantu Ika menentuka susunan banyak koin pada tiap-tiap kotak papan catur tersebut. Nyatak dalam bentuk perpangkatan

b. Jika berat tiap-tiap uang koin adalah 16 gr, hitunglah berat uang koin pada tiap tiap kotak. Nyatakan dalam bentuk perpangkatan

(ii) Translation

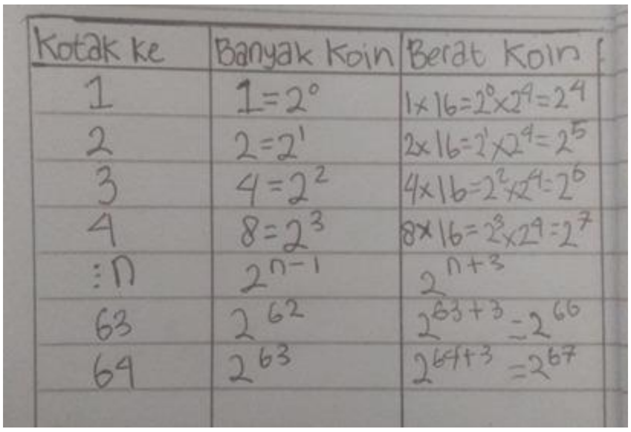
Ilham challenged Ika to calculate the number of coins needed to fill the chessboard. The first square is given 1 coin, the second square is 2 coins, 4 coins for the third square, 8 coins for the fourth square, and so on to fulfill 64 squares.

a. Help Ika determine the arrangement of the many coins in each chessboard square. Express it in multiplication form

b. If each coin weighs 16 grams, calculate the weight of the coins in each square. Express it in multiplication form.

Figure 3. PbNT designed by IAB.

Based on the Figure 3, IAB designed three tasks focused on multiplication including exponents. Her PbNT was not ill-structured, as she used a close-ended practical task approach. This is also supported by the alternative task solution made by IAB in Figure 4.



Kotak ke	Banyak Koin	Berat Koin
1	$1=2^0$	$1\times16=2^0\times2^4=2^4$
2	$2=2^1$	$2\times16=2^1\times2^4=2^5$
3	$4=2^2$	$4\times16=2^2\times2^4=2^6$
4	$8=2^3$	$8\times16=2^3\times2^4=2^7$
: n	2^{n-1}	2^{n+3}
63	2^{62}	$2^{63+3}=2^{66}$
64	2^{63}	$2^{64+3}=2^{67}$

Figure 4. IAB alternative task solution.

IAB has shown ideas that lead to the use of mathematical tools such as the chessboard and coins. The mathematical tools, however, were not actually used or illustrated in the form of pictures. In Table 4, the elements of PbNT designed by IAB are presented along with the analysis of their conformity with the elements of PbNT.

Table 4
Element Analysis of PbNT Designed by IAB

Elements	Description
Context (N,P)	Authenticity based on the personal context of the chessboard.
Mathematical content (N,P)	Math content about multiplication (number) with the cognitive level of the task at the applying level.
Disposition/student enggagement (N,P)	Command words provide the impetus for students to engage in solution finding.
Tools (N)	The use of chessboards and coins as physical tools for the representation of mathematical ideas, but not in the form of pictures that could help shape students' thinking.
Critical orientation (N)	Mathematical information and activities in task are not open enough. The task command sentence directly asks students to use the concept of exponents. There is not enough opportunity for students to form critical opinions.
Centrality of project (P)	Tasks are not central to learning as they are exercises to reinforce the concept of exponents used in solving problems in the tasks.
Collaboration (P)	There is no sign of collaboration in the task.
Project artefact (P)	The final product is only a problem solution in a close ended question. Representation of student learning in the form of the ability to apply the concept of exponents to the problem in the task.
Construct knowledge (P)	The task involves students in constructing the ability to solve the problem of exponentiation.
Ill-structured (P)	The task is not ill-structured because the definition of the problem, goal, and solution method have been set directly in the task.

To clarify whether the deviation of IAB's PbNT form is due to epistemological obstacles, an interview was conducted regarding the process of how IAB designed the PbNT. IAB looked for task ideas by browsing the internet. IAB did not construct tasks based on teaching practice experience. She was, however, suspected of experiencing obstacles from the former education factor due to a lack of teaching experience. To understand IAB's intentions in utilising the task in Figure 3, we provide excerpts from her interview transcripts.

R: What do you know about numeracy skills for students?

IAB: Numeracy is counting, especially for students due to the COVID pandemic, their numeracy skills are still poor. I have encountered a school that has 7th and 8th grade students with very poor numeracy skills, especially 8th grade students who will carry out the minimum competency assessment test, they are required to understand numeracy and literacy.

R: What do you know about project task in math learning?

IAB: Project tasks are tasks for making props such as in the material of three-dimensional space, for example teacher gives tasks to make a block or cube.

R: How did you imagine implementing this task in the classroom?

IAB: First, divide students into groups of 4. Second, ask the students to plan how to arrange the coins and the weight of each coin in the row. Third, give the students 2 days to work. Fourth, ask each group to submit the report according to the deadline. Fifth, ask each group to present the group work result. [...] Students are guided only on the introduction of concepts at the first meeting, then students will work by themselves in groups. [...] What I assessed was the criteria for the accuracy of the calculations and the formulas they used.

IAB's initial knowledge of numeracy and project-based learning can be said to be incomplete (not based on inquiry or critical numeracy). She faced epistemological obstacles from previous educational factors due to incomplete knowledge of project tasks and numeracy. The tasks did not serve as the core of learning, lacked structural clarity, failed to foster knowledge construction, and project artefacts were limited to reports. The tasks designed by IAB do not construct knowledge because the inquiry was not in-depth (only limited to arranging coins) and did not encourage critical thinking (did not provide opportunities for students to justify what was presented in the task). She taught the concepts of multiplication and exponentiation directly to students through traditional teaching methods that were not conducive to critical numeracy and inquiry, which she had experienced during her time as a student. She intuitively adopted traditional teaching methods, which caused pre-performed learning method factor of epistemological obstacles. However, collaboration that was not mentioned in her task did emerge when she explained her PbNT implementation process. Based on IAB's suspected former education factor of epistemological obstacles, we conducted further interviews to trace IAB's history of numeracy and project knowledge.

R: Which teaching methods do you know best??

IAB: I have understood guided inquiry method the most, where teachers give tasks to work in groups so that students can draw conclusions from the task.

R: What learning methods made you learn more effectively when you were a student?

IAB: I think using problem-based, which is usually given by the teacher through a problem to solve.

R: Where did you learn the concept of numeracy learning and project-based learning?

IAB: I learned numeracy and project-based learning from the internet and YouTube (we asked IAB to send us the internet link where she learned numeracy and project-based learning).

IAB tended to enjoy inquiry-based learning and problem-based learning, which were embedded in her understanding. She experienced the pre-performed learning method factor since she intuitively covered the deficit of project knowledge with problem-based knowledge. This led to her PbNT structure approach distortion, because she failed to recognise the difference between these learning approaches. Based on the search for internet sources where IAB studied numeracy and projects, she did not experience verbal obstacles. However, the reference source for IAB explained project-based learning as similar to problem-based learning without emphasising the main characteristics of the project and seemed to agree that the project artefacts were learning media.

Epistemological Obstacles: Case Study NR

(i) Original

MATA PELAJARAN: MATEMATIKA
KELAS/SEMESTER: VIII/1
BAB: BANGUN RUANG SISI DATAR
TUGAS: BUATLAH SUATU BENTUK BANGUNAN ATAU BENDA BERDASARKAN SUSUNAN BANGUN RUANG SISI DATAR (KUBUS, BALOK,PRISMA DAN LIMAS)

PETUNJUK UMUM :

1. Baca secara cermat sebelum melaksanakan tugas proyek
2. Pelajari materi bangun ruang sisi datar
3. Kerjakan tugas sesuai petunjuk
4. Melakukan kegiatan mencari data atau referensi dari manapun
5. Konsultasi dengan guru apabila ada yang tidak dipahami
6. Catat hasil semua pencarian
7. Laporkan juga hasil kerja dan presentasikan

(ii) Translation

Subject: Mathematics
Grade/semester: VIII/1
Chapter: flat-sided three-dimensional spaces
Task: Make a building or object based on the arrangement of flat-sided three-dimensional spaces (cubes, blocks, prisms, and pyramids).

GENERAL INSTRUCTIONS:

1. Read carefully before carrying out project tasks
2. Learn about flat-sided three-dimensional spaces
3. Do the task as directed
4. Conduct activities to find data or references from anywhere
5. Consultation with the teacher if something is not understood
6. Record the results of all searches
7. Also, report your work and present it.

Figure 5. PbNT designed by NR.

NR chose flat-sided three-dimensional space as PbNT mathematical content. The task did not show an authentic context. It was unclear on what concept of flat-sided three-dimensional spaces she wanted to construct. Based on Figure 6, the former education factor of epistemological obstacles due to mathematical content knowledge deficit of flat-sided three-dimensional spaces or language of geometric terms was detected. She classified cones and tubes as one of the flat-sided three-dimensional spaces.

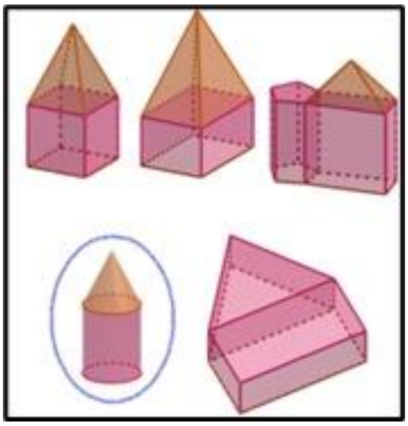


Figure 6. Alternative task solutions by NR.

The PbNT did not show any use of "tools" either in writing or drawing. In fact, the instruction she used in the main task was difficult to understand. Like IAB, NR also did not show any need for student collaboration in the PbNT. Analysis of the PbNT elements is presented in more detail in Table 5.

Table 5
Element analysis of PbNT designed by NR

Element	Description
Context (N,P)	Does not present a real-world/authentic problem.
Mathematical content (N,P)	The task is at the knowing level where students are asked to identify, recall, and classify the combination of flat-sided three dimensional space.
Disposition/student enggagement (N,P)	The command sentences used in the task encourage students to remember the shapes of flat-sided three dimensional spaces and combine them into new shapes.
Tools (N)	The use of mathematical idea representation tools both in writing and drawing was not detected.
Critical orientation (N)	Task provides an opportunity for students to make decisions and arguments about the creativity of combinations of flat-sided three dimensional shapes. The prompts in the task are open-ended.
Centrality of project (P)	The designed task serves as the central part of learning about combinations of flat-sided three dimensional shapes.
Collaboration (P)	The sentences in the task do not indicate a form of group work/collaboration.
Project artefact (P)	Final product of the task is the creation of a combination of flat-sided three dimensional shapes. It can expand students' understanding that the shape of a space does not always refer to one type of space.
Construct knowledge (P)	Knowledge construction on the concept of a combination of flat-sided three dimensional shapes. The initial concept has been given directly.
Ill-structured (P)	The task is ill-structured. Students define for themselves what shapes will be combined. However, the problem given by the task is only solved in one or two steps.

Further investigation into NR's prior knowledge and how PbNT was implemented was conducted.

R: What do you know about numeracy skills for students?

NR: Numeracy skills for students is the ability where math is not only using numbers for calculations, but also for reasoning to solve or anticipate problems in everyday life, for example, there is garbage, and we can calculate how long this garbage can decompose.

R: What do you know about project task in math learning?

NR: Project learning in mathematics is a learning process where students produce a product at the end of the learning process. Teachers usually give problems for students to solve. The teachers' role is as a facilitator for students. Project task is not only done in groups but also individually.

R: Please explain what is the purpose of the task you designed?

NR: Students form flat-sided three-dimensional spaces like buildings. There are cubes, blocks, pyramids, prisms. Later after the students are explained the concepts of flat-sided three-dimensional spaces, the teacher gives project task to students. So, the existing flat-sided three-dimensional spaces are creatively turned into buildings around students which are a combination of new shapes.

R: What are some examples?

NR: For example, I ask students to search the classroom and school area. I ask them to look for objects that are a combination of flat-sided three-dimensional spaces. For example, the classroom and the roof are a combination of cubes and pyramids. After that, I will direct the learning to find the volume of the combination of the shapes.

R: Where did the idea for this task come from?

NR: I searched for ideas on the internet with the keyword utilisation of flat-sided three-dimensional spaces. Then, I had the idea that a project could be made from a combination of flat-sided three-dimensional shapes.

R: Do you think this task is appropriate for numeracy and project-based tasks?

NR: It is appropriate because numeracy is not only about counting numbers but also about how to apply math in daily life. Therefore, this task can be used to train their application in daily life. I understand the project as producing a product. I ask students to produce a product in the form of a creation of a combination of flat-sided three-dimensional shapes that are around them.

NR's knowledge of numeracy and project-based tasks supports an inquiry/guided approach (structured inquiry). NR did not experience obstacles due to misunderstanding the verbal expression because she described numeracy and projects in accordance with what was written in the reference. NR's intention/desire, however, was not well conveyed through the verbal representation of the task designed.

Based on the interview, the PbNT designed by NR featured a genuine context relevant to the students' surrounding environment (e.g., NR imagines the combination of cubes and pyramids that characterise traditional buildings in Indonesian villages), yet the written verbal description of the task did not reflect this authenticity. The idea of connecting the shapes of buildings familiar to students and finding the area of the combinations can be done with the help of tools (such as picture, ruler, or models). NR, however, did not yet have the knowledge that numeracy tasks required the involvement of mathematical tools. This hindered the emergence of ideas about these tools explicitly for completing the task. Thus, NR experienced the former education factor of epistemological obstacles due to a lack of knowledge about the elements of numeracy tasks.

Just like IAB, NR was suspected of experiencing pre-performed learning method factor of epistemological obstacles because she intuitively combined the direct preaching method instead of concept construction. To confirm this assumption, we conducted interviews about the alleged causes.

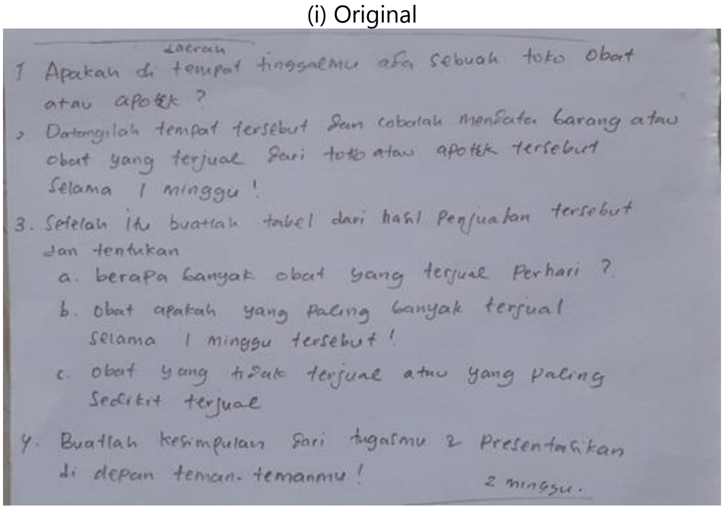
R: What learning methods made you learn more effectively when you were a student?

NR: I prefer the practicum-based learning method because, in addition to getting the knowledge about concept, I can also get direct knowledge of its implementation. Whereas when I was a student, I understood best when explained by direct preaching method.

NR was interested in learning methods that involve practicum with directly provided material or concepts rather than self-constructing them. This led to the occurrence of pre-performed learning method factor of epistemological obstacles, wherein NR instinctively employed the teaching approach to present the content prior to assigning the project. In NR's case, the practicum learning approach she used for the project led to more of her PbNT element approaches being in line with the literature.

Epistemological Obstacles: Case Study IM

(i) Original

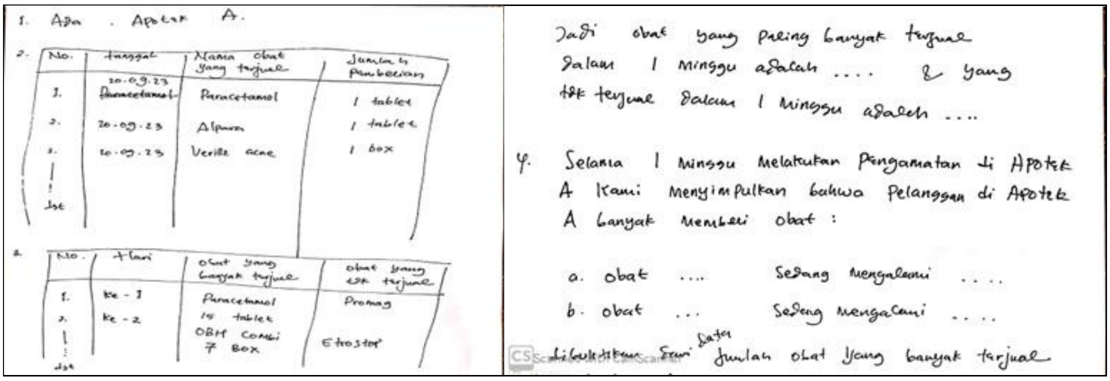

1 Apakah di tempat tinggalmu ada sebuah toko obat atau apotek ? (daerah)
2 Datangilah tempat tersebut dan cobalah mendata barang atau obat yang terjual dari toko atau apotek tersebut selama 1 minggu !
3. Setelah itu buatlah tabel dari hasil penjualan tersebut dan tentukan
a. berapa banyak obat yang terjual perhari ?
b. obat apakah yang paling banyak terjual selama 1 minggu tersebut !
c. obat yang tidak terjual atau yang paling sedikit terjual
4. Buatlah kesimpulan dari tugasmu & presentasikan di depan teman-temanmu !
2 minggu.

(ii) Translation

1. Is there a drugstore or pharmacy where you live?
2. Find the place and try to record the goods or drugs sold from the shop or pharmacy for 1 week!
3. After that, make a table of the sales results and determine
   a. How many medicines were sold per day?
   b. Which drug sold the most for 1 week?
   c. Drugs that were not sold or sold the least number of times?
4. Draw a conclusion from your assignment and present it to your friends!

Figure 7. PbNT designed by IM.

The task designed by IM met all the criteria of the frameworks for numeracy and project-based tasks. IM chose the mathematical content of statistical data representation (data tables), which is very likely to use digital tools such as computers or representational tools such as graphs, which is indeed suitable for the school context where IM implemented the teaching practice.



No.	tanggal	Nama obat yang terjual	Jumlah pembelian
1.	20.09.23	Paracetamol	1 tablet
2.	20.09.23	Alpara	1 tablet
3.	20.09.23	Verille acne	1 box

No.	Hari	obat yang banyak terjual	obat yang tdk terjual
1.	ke - 1	Paracetamol 14 tablet	Promag
2.	ke - 2	OBH Combi 7 box	Etrostor

Figure 8. Alternative task solutions by IM.

Among the three participants, the context of the problem promoted by IM is the closest alignment to the frameworks and relates to students' daily life conditions. However, the involvement of context knowledge of medicine or marketing is not used at all in constructing the tasks and compiling project artefacts. A more complete analysis of the suitability of IM tasks with PbNT elements is further explained in Table 6.

Table 6

Element Analysis of PbNT Designed by IM

Element	Description
Context (N,P)	The tasks are authentic (carries a scientific theme about medicine and its sale).
Mathematical content (N,P)	The task encourages students to identify, calculate, and measure (knowing level) statistic/data on drugs sold in pharmacies.
Disposision/student enggagement (N,P)	The command sentence in the task encourages students to look for data in the pharmacy and calculate the maximum/minimum value of the data.
Tools (N)	The use of tools to represent mathematical ideas detected verbally although the task had a great opportunity to involve measuring and representational tools or digital tools.
Critical orientation (N)	The task provides an opportunity for students to make decisions and consider the pharmacies and types of drugs that will be analysed.
Centrality of project (P)	The task is central to the learning activity.
Collaboration (P)	The sentences in the task do not indicate a request for collaboration.
project artefact (P)	The final product is weekly drug sales data.
Construct knowledge (P)	The task constructs knowledge about data, maximum value and minimum value.
Ill-structured (P)	The task is ill-structured. Students take part in defining the type of medicine and how to find the minimum and maximum values from the data.

However, an epistemological obstacle was detected upon further investigation through interviews on IM's knowledge about numeracy and how she implemented the task. The following is the transcript of IM's interview.

R: What do you know about numeracy for students?
IM: According to my understanding, numeracy relates to lessons related to counting.
R: How do you know the meaning of numeracy?
IM: From the word numeracy, according to my understanding, it means related to calculation.
R: What do you know about project task in math learning?
IM: A project task is a task that is done periodically.
R: Can you name the characteristics of project tasks that you know?
IM: It's done periodically. Can't be done all at once. It can be done in groups or individually.
R: How [are] project tasks different from a problem-based tasks?
IM: Problem-based tasks may be completed in one go. Whereas project tasks still require stages so may take longer.
R: Please explain how you implemented your task!
IM: I chose statistics. First, students were divided into groups. Second, students were asked to record the sales of the pharmacy for one week. The data collected was how many sales per day, what drugs sold the most, and which drugs sold the least. The students went to the nearest pharmacy to do the task. This task took 2 weeks as it required preparation and report writing.
R: Where did you get the idea for this task?
IM: I came up with the idea spontaneously without browsing the internet. Since I think numeracy is about counting, the children were asked to collect data and process it. Because in this project, I would want the students to work outside the classroom, looking for data.
R: Why did you choose a pharmacy as your project location?
IM: My goal in creating this task is to make students not only learn arithmetic, but also learn the science of medicine.

R: Why do you want to combine numeracy and science? Is there a connection because it's a project?

IM: I just want students to learn about science besides learning arithmetic.

IM's knowledge of numeracy was based on verbal meaning without any attempt to find out more from other sources. She intuitively connected her numeracy concepts with numeracy knowledge related to statistics. IM did not mention other aspects of projects except for their periodic implementation characteristics either in groups or individually. IM's intuition and knowledge should lead to the emergence of the former education factor caused by lack of prior knowledge of the project and tendency to generalise numeracy concepts. Epistemological obstacles, however, did not appear in the task she designed. In fact, she was the only participant who stated that the task she designed involved multidisciplinary knowledge. Therefore, we conducted further interviews on these matters.

R: Where did you learn about numeracy?

IM: I only became familiar with numeracy recently after the minimum competency assessment at (Indonesian) school or from the math exam questions in the form of numeracy where I teach. From elementary to high school, I also often participated in extracurricular math olympic coaching, where the numeracy tasks were discussed.

R: Where did you learn about the project?

IM: From elementary school to high school, I have had project-based learning, but for subjects other than math, such as art, science, ICT, and history.

R: Please tell me about the learning process.

IM: In Art, teachers explained that we would be working on projects. Teachers explained what materials were needed and how to make them, as well as the assessment. The project was collected after several meetings depending on the difficulty level. In Science, I have participated in project-based learning about melting point. The content was first explained by the teacher. Then, we followed the practicum instructions on the worksheet and presented the results in the form of melting points of various objects.

Just like NR, IM experienced learning with practicum activities and adapted it as project-based learning. However, IM's learning experience in practicum learning in various subjects helped to build the intuition of a cross-curriculum approach to her PbNT. She covered her knowledge deficiencies with practicum-based learning knowledge and Mathematics Olympiad problems from observing her teacher and generalised it into project and numeracy knowledge. Based on the results of the analysis conducted, the epistemological obstacles of three PSMTs in designing PbNT are illustrated in Figure 9.

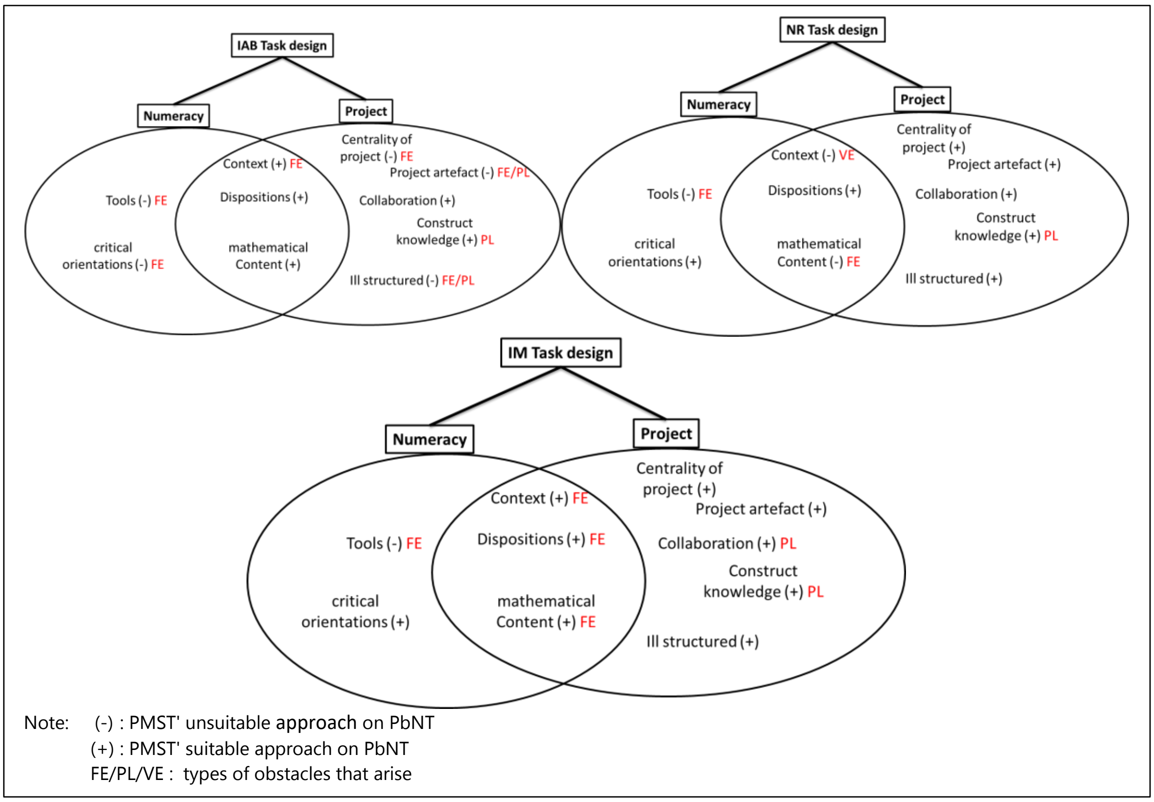


Figure 9. Illustration of the emergence of epistemological obstacles on PbNT of three PSMTs.

Discussion

Findings from the data provide descriptions of how epistemological obstacles emerged in the PSMTs' PbNT design process and affected the tasks they designed. From Figure 9, the epistemological obstacles that emerged in the three PSMTs influenced their approach to the elements of context, tools and knowledge construction. First element of *context*, the PSMTs did not engage knowledge across the curriculum to interpret mathematical solutions and design project artefacts. According to the alternative task solutions created by the PSMTs, they focussed more on forms of mathematical knowledge being taught instead of critical numeracy skills across the curriculum. It is important to have knowledge of mathematical content and contexts outside of mathematics when teaching numeracy (Getenet, 2022). Emerging from this study is that PSMTs' professional capital on numeracy learning will not be developed without appropriate mathematical, contextual and strategic understanding (Callingham et al., 2015), without which they will not be able to make deep connections between learning and the contextual and strategic nature of numeracy.

Second, and in regard to the *tool's* element, the PSMTs had involved tools in PbNT only verbally or implicitly because they had no knowledge of this element. Tools are an important element in the design of rich numeracy tasks because they provide mediation for students to connect with mathematical knowledge through the manipulation of tools and provides opportunities for them to develop higher order thinking skills in mathematical numeracy (Lei & Hu, 2020).

The third element is *knowledge construction*, where the PSMTs tended to directly provide the mathematical concepts needed in doing the numeration project through preaching so that the inquiry is less in-depth and less critical. Justifying the content and context of the task, which consists of reading and questioning what is presented, is a necessary activity in promoting critical thinking (Susandi et al., 2022). The PSMTs' limited ability to design deep inquiry and critical thinking activities on tasks was attributed to their lack of proficiency in higher-order mathematical thinking, motivation to seek more complete references, and experience in designing numeracy tasks that were relatively new to them.

According to the PSMTs' testimony, they find and learn numeracy task design knowledge and skills only through the internet. This process was adopted because the PSMTs interviewed had no desire to explore more knowledge through discussion and feedback from both the website author and their lecturers, regardless of the validity of the information (Watson & Moritz, 2002).

Verbal Expression

The verbal expression factor in epistemological obstacles in Minaidi and Hlapanis (2005) and Murniasih et al. (2020) described as obstacles due to different meanings understood by the PSMTs due to difficulties in interpreting incoming information. However, for the PSMTs who designed PbNT, the obstacles experienced also occurred in the form of distortion of verbal meaning between the tasks intended by PSMT and their written representations. In the case of NR, she had a good understanding of project-based tasks and numeracy. However, the verbal representation of the PbNT written by NR did not bring out what she meant. Therefore, epistemological obstacles in PSMT from verbal factors can arise from verbal input (meaning of incoming information) and/or verbal output (conveying ideas through written representations).

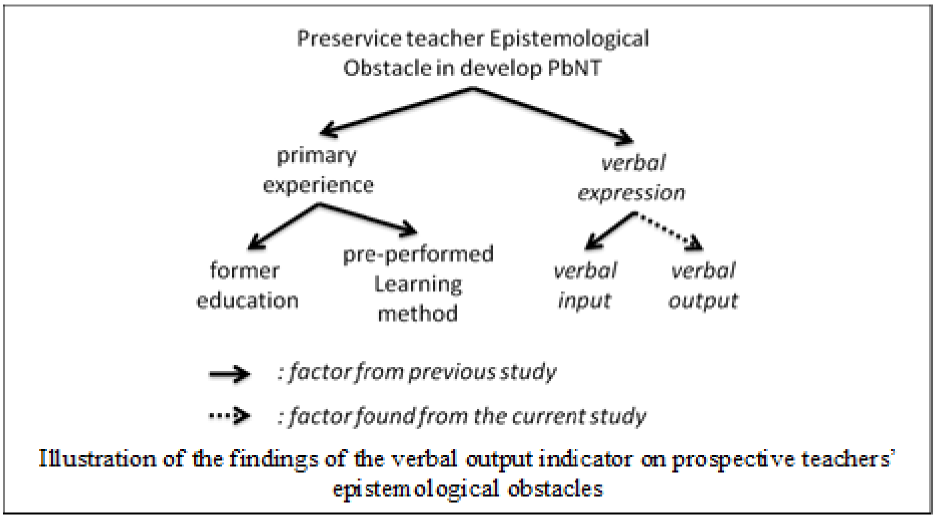


Illustration of the findings of the verbal output indicator on prospective teachers' epistemological obstacles

Figure 3. Diagram of PSMTs' epistemological obstacles in PbNT design based on current study and Minaidi and Hlapanis (2005) and Murniasih et al. (2020).

Former Education

The limited knowledge of PSMT causes them to generalise problem-based task knowledge into aspects of project knowledge because they have many similarities. PSMTs covered their project-based learning knowledge gaps with problem-based learning knowledge. Although problem-based learning and project-based learning have many similarities, project-based learning emphasises more the production of learning artefacts (Grant, 2009). Based on Gronow's (2022) CRIG (Connections, Recognising Patterns, Identifying Similarities and Differences, and Generalising and Reasoning) framework, which is adapted to PSMTs' mathematical pedagogical knowledge, PSMTs who experience obstacles at the stage of identifying similarities and differences tend to generalise project-based tasks with problem-based tasks. The PSMTs focused overly on the mathematical aspects, as in typical mathematical tasks, and paid less attention to the contextual essence of numeracy learning across curriculums. When the PSMTs failed to recognise distinguishing characteristics in the knowledge generalisation, the designed task deviated from the original pedagogical goal due to epistemological obstacles (Murniasih et al., 2020). According to Murtafiah et al. (2020), PSMTs still have some deficits in building ideas, analysing, and evaluating tasks due to limited knowledge. Zamzam et al. (2023) also stated that the difficulties faced by PSMTs in the creative process of designing tasks are deficiencies in understanding material concepts and prior knowledge. In IM's case, her project-based learning and numeracy knowledge deficits were complemented by meaningful learning experiences during school. This finding complements the study results from Murtafiah et al. (2020), which reported that PSMTs do not use their teaching experiences to make decisions about the tasks designed but they may use their learning experience as one aspect of task design. Hence, for PSMTs, the term "experience" pertains primarily to their learning encounters.

Pre-performed Learning Method

Pre-performed learning method factors that occurred in the PSMTs were intuition to use preaching methods in providing the initial concepts needed in the project. The direct preaching method, which has been practised by almost all teachers in Indonesia before the implementation of the new curriculum reform, has been entrenched and has become a solid teaching concept in PSMTs. This is in line with Leavy and Hourigan (2020), who stated that despite having gone through various types of varied learning, the influence of traditional learning is still visible in the tasks designed by PSMTs. In IM's case, the experience of project-based learning in various subjects aside from mathematics and the familiarisation of working with higher order thinking tasks, have formed appropriate intuition for project-based numeracy learning innovations. This is in accordance with Callingham et al. (2015) who determined that in schools that implement project-based learning, teachers have more capacity to recognise numeracy opportunities and increase students' awareness of numeracy. Therefore, meaningful learning in schools is crucial in developing future PSMTs' intuition and innovative thinking.

Limitations

The main limitation of this study is the small number of participants so the generalisation of findings supported by quantitative data analysis cannot be carried out. The findings are based on qualitative data analysis on the tasks designed, epistemological obstacle-based interviews, and field notes. PbNT design process steps performed by PSMTs have not been analysed in depth and structured. For future study, PSMTs' epistemological obstacles can be examined from several frameworks, such as those espoused by Gronow et al. (2022) or Foster and Lee (2021) to see the differences in PSMTs' approaches in connecting mathematical knowledge for teaching and mathematical didactic knowledge in the PbNT design process and not only in the final task designed. Another limitation of this study is that PbNT implementation in the classroom has not been analysed practically because this study was limited to the lesson plans imagined by PSMTs. Further study can be done by analysing from planning to implementation of project-based numeracy learning to explore the epistemological obstacles in each learning process.

Conclusions

This study explored epistemological obstacles experienced by PSMTs when designing PbNT. The approaches chosen by PSMTs who experienced epistemological obstacles in designing PbNT appeared deficient in the involvement of context knowledge outside mathematics (too focused on mathematical aspects), the involvement of tools in the solution process, and the construction of knowledge through critical and inquiry activities. The factors of the prospective teachers' epistemological obstacles in the PbNT design process include 1) verbal expression factor which appears in two forms, namely verbal input, where incoming information are interpreted differently by PSMT; and verbal output, where verbal representations written on PbNT by PSMT do not convey the desired meaning, 2) prior experience factor in the former education occurs because PSMTs' prior knowledge deficit is complemented by generalising other similar knowledge but failing to identify the differences, and 3) the prior experience factor in the pre-performed learning method occur because the same learning method is experienced continuously by PSMTs and becomes embedded view. Thus, PSMTs' lack of experience in designing

PbNT, prior experience/views on numeracy and projects, and embedded views on teaching is likely to influence their approach to task design.

The implementation of this study findings in the form of factors leading to epistemological obstacles in terms of the PSMTs' prior knowledge and learning experience about PbNT can be utilised in the improvement of PSMT education programs in Indonesia. According to Foster and Lee (2021), PSMTs have ranges of experience and prior knowledge that should be taken into account by teacher educators in designing appropriate learning, and use them to develop PSMTs' understandings of significant instructional practices. In a broader context, the approach used by PSMTs in this study can enrich information on how numeracy is interpreted by novice teachers and translated into tasks.

Corresponding author

Cholis Sa'dijah
Mathematics Education, *Universitas Negeri Malang*
Malang, Indonesia
cholis.sadijah.fmipa@um.ac.id

Ethical approval

In accordance with the Universitas Negeri Malang Ethics Research Committee, informed consent of the participants was granted by all participants for their data to be published. Ethical approval was granted by the Universitas Negeri Malang Ethics Research Committee for the data to be reported using the participants first names/initials. The participants gave informed consent for the use of their first names/initials for publication purposes. The authors thanks to Universitas Negeri Malang for supporting this study with Contract No. 4.4.570/UN32.14.1/LT/2024.

Competing interests

The authors declare there are no competing interests.

References

Aldabbus, S. (2018). Project-based learning: Implementation & challenges. *International Journal of Education, Learning and Development*, *6*(3), 71–79. https://eajournals.org/ijeld/vol-6-issue-3-march-2018/project-based-learning-implementation-challenges/

Barak, M., & Assal, M. (2018). Robotics and STEM learning: Students' achievements in assignments according to the P3 task taxonomy: Practice, problem solving, and projects. *International Journal of Technology and Design Education, 28,* 121–144. https://doi.org/10.1007/s10798-016-9385-9

Bishop, J. P., Lamb, L. L., Philipp, R. A., Whitacre, I., Schappelle, B. P., & Lewis, M. L. (2014). Obstacles and affordances for integer reasoning: An analysis of children's thinking and the history of mathematics. *Journal for Research in Mathematics Education*, *45*(1), 19–61. https://doi.org/10.5951/jresematheduc.45.1.0019

Callingham, R., Beswick, K., & Ferme, E. (2015). An initial exploration of teachers' numeracy in the context of professional capital. *ZDM-Mathematics Education*, *47*(4), 549–560. https://doi.org/10.1007/s11858-015-0666-7

Ceolim, A. J., & Caldeira, A. D. (2017). Obstacles and difficulties presented by mathematics teachers recently graduated when adopting mathematical modeling in their classes at elementary education. *Bolema, Rio Claro (SP)*, *31*(58), 760–776. http://dx.doi.org/10.1590/1980-4415v31n58a12

Cesaria, A., & Herman, T. (2019). Learning obstacle in geometry. *Journal of Engineering Science and Technology*, *14*(3), 1271–1280. https://jestec.taylors.edu.my/Vol%2014%20issue%203%20June%202019/14_3_12.pdf

Cheung, K. K. C., & Tai, K. W. H. (2023). The use of intercoder reliability in qualitative interview data analysis in science education. *Research in Science and Technological Education*, *41*(3), 1155–1175. https://doi.org/10.1080/02635143.2021.1993179

Cohen, L., Manion, L., & Morrison, K. (2018). *Research methods in education* (8th ed.). Routledge. https://www.routledge.com/Research-Methods-in-Education/Cohen-Manion-Morrison/p/book/9781138209886

Condliffe, B., Quint, J., Visher, M. G., Bangser, M. R., Drohojowska, S., Saco, L., & Nelson, E. (2017). *Project-based learning a literature review: Working paper.* https://files.eric.ed.gov/fulltext/ED578933.pdf

Craig, T. T., & Marshall, J. (2019). Effect of project-based learning on high school students' state-mandated, standardized math and science exam performance. *Journal of Research in Science Teaching*, *56*(10), 14611–1488. https://doi.org/10.1002/tea.21582

Fisher, M. H., Thomas, J., Schack, E. O., Jong, C., & Tassell, J. (2017). Noticing numeracy now! Examining changes in preservice teachers' noticing, knowledge, and attitudes. *Mathematics Education Research Journal*, *30*(2), 209–232. https://doi.org/10.1007/s13394-017-0228-0

Forgasz, H. J., & Hall, J. (2019). Learning about numeracy: The impact of a compulsory unit on pre-service teachers' understandings and beliefs. *Australian Journal of Teacher Education*, *44*(2), Article 2. http://dx.doi.org/10.14221/ajte.2018v44n2.2

Foster, J. K., & Lee, H. Y. (2021). Prospective teachers' pedagogical considerations of mathematical connections: A framework to motivate attention to and awareness of connections. *Mathematics Teacher Education and Development*, *23*(4), 95–118. https://mted.merga.net.au/index.php/mted/article/view/622/501

Geiger, V. (2018). Generating ideas for numeracy tasks across the curriculum. In J. Hunter, P. Perger, & L. Darragh, (Eds.). *Making waves, opening spaces* (Proceedings of the 41st Annual Conference of the Mathematics Education Research Group of Australasia), Aukland, pp. 314–321. https://files.eric.ed.gov/fulltext/ED592434.pdf

Geiger, V., Goos, M., & Forgasz, H. (2015). A rich interpretation of numeracy for the 21st century: A survey of the state of the field. *ZDM-Mathematics Education*, *47*(4), 531–548. https://doi.org/10.1007/s11858-015-0708-1

Geiger, V., Yasukawa, K., Bennison, A., Wells, J. F., & Sawatzki, C. (2020). Facets of numeracy: Teaching, learning and practices. In J. Way, C. Attard, J. Anderson, J. Bobis, H. McMaster, & K. Cartwright (Eds.), *Research in mathematics education in Australasia 2016–2019* (pp. 59–89). Springer. https://doi.org/10.1007/978-981-15-4269-5_4

Getenet, S. T. (2022). Teachers' knowledge framework for designing numeracy rich tasks across non-mathematics curriculum areas. *International Journal of Education in Mathematics, Science and Technology*, *10*(3), 663–680. https://doi.org/10.46328/ijemst.2137

Goos, M., Geiger, V., Bennison, A., & Robert, J. (2015). *Numeracy teaching across the curriculum in Queensland: Resources for teachers*. https://cdn.qct.edu.au/pdf/Numeracy_Teaching_Across_Curriculum_QLD.pdf

Goos, M., Geiger, V., & Dole, S. (2013). Designing rich numeracy tasks. In C. Margolinas (Ed.), *Task design in mathematics education. Proceedings of ICMI Study 22*, Volume 1, pp. 591–599. http://www.fisme.science.uu.nl/publicaties/literatuur/2013_icmi_study_goos.pdf

Goos, M., Geiger, V., & Dole, S. (2014). Transforming professional practice in numeracy teaching. In Y. Li, E. A. Silver & S. Li (Eds.), *Transforming mathematics instruction: Multiple approaches and practices* (pp. 81–102). Springer. https://doi.org/10.1007/978-3-319-04993-9_6

Gorman, V., O'Keeffe, L., Albrecht, A., & McPhee, J. (2023). Challenges to numeracy across the curriculum: Reflections from a case study. In B. Reid-O'Connor, E. Prieto-Rodriguez, K. Holmes, & A. Hughes (Eds.), *Weaving mathematics education research from all perspectives* (Proceedings of the 45th Annual Conference of the Mathematics Education Research Group of Australasia), Newcastle, pp. 227–234. https://www.merga.net.au/common/Uploaded%20files/Annual%20Conference%20Proceedings/2023%20Annual%20Conference%20Proceedings/Research%20Papers/MERGA45_2023_Gorman_RP293_Final.pdf

Grant, M. M. (2009). Understanding projects in project-based learning: A student's perspective. In *Discipline inquiry: Education research in the circle of knowledge.* Proceedings of the American Educational Reserach Association Annual Meeting. https://www.academia.edu/2845930/Understanding_projects_in_project_based_learning_A_students_perspective

Gronow, M., Mulligan, J., & Cavanagh, M. (2022). Teachers' understanding and use of mathematical structure. *Mathematics Education Research Journal*, *34*(2), 215–240. https://doi.org/10.1007/s13394-020-00342-x

Guo, P., Saab, N., Post, L. S., & Admiraal, W. (2020). A review of project-based learning in higher education: Student outcomes and measures. *International Journal of Educational Research*, *102*, Article 101586.

https://doi.org/10.1016/j.ijer.2020.101586

Haatainen, O., & Aksela, M. (2021). Project-based learning in integrated science education: Active teachers' perceptions and practices. *LUMAT: International Journal on Math, Science and Technology Education*, *9*(1), 149–173. https://doi.org/10.31129/LUMAT.9.1.1392

Hall, J., & Zmood, S. (2019). Australia's literacy and numeracy test for initial teacher education students: Trends in numeracy for low- and high-achieving students. *Australian Journal of Teacher Education*, *44*(10), Article 1. http://dx.doi.org/10.14221/ajte.2019v44n10.1

Hariyani, M., Hermawan, T., Suryadi, D., & Prabawanto, S. (2022). Exploration of student learning obstacles in solving fraction problems in elementary school. *International Journal of Educational Methodology*, *8*(3), 505–515. https://doi.org/10.12973/ijem.8.3.505

Hart, E. W. (2014). Pedagogical content analysis of mathematics as a framework for task design. In C. Margolinas (Ed.), *Task design in mathematics education*. Proceedings of ICMI Study 22, Volume 1 (pp. 337–345). http://dx.doi.org/10.1007/978-3-319-09629-2

Kemendikbud. (2017). *Materi pendukung literasi numerasi*. TIM GLN Kemendikbud. https://repositori.kemdikbud.go.id/11628/1/materi-pendukung-literasi-numerasi-rev.pdf

Kohar, A. W., Rahaju, E. B., & Rohim, A. (2022). Prospective teachers' design of numeracy tasks using a physical distancing context. *Journal on Mathematics Education*, *13*(2), 191–210. http://doi.org/10.22342/jme.v13i2.pp191-210

Leavy, A., & Hourigan, M. (2020). Posing mathematically worthwhile problems: Developing the problem posing skills of prospective teachers. *Journal of Mathematics Teacher Education*, *23*(4), 341–361. https://doi.org/10.1007/s10857-018-09425-w

Lei, H., & Hu, A. (2020). Designing a rich numeracy task in early childhood mathematics education: Teaching addition in a kindergarten in Macao. *Studies in Social Science Research*, *2*(1), Article 1. https://doi.org/10.22158/sssr.v2n1p1

Liljedahl, P. (2015). Numeracy task design: A case of changing mathematics teaching practice. *ZDM-Mathematics Education*, *47*(4), 625–637. https://doi.org/10.1007/s11858-015-0703-6

Machromah, I. U., Utami, N. S., Setyaningsih, R., Mardhiyana, D., & Fatmawati, L. W. S. (2021). Minimum competency assessment: Designing tasks to support students' numeracy. *Turkish Journal of Computer and Mathematics Education*, *12*(14), 3268–3277. https://turcomat.org/index.php/turkbilmat/article/view/10898

Martin, M. O., & Mullis, I. V. (2019). TIMSS assessment frameworks. In *TIMSS & PIRLS International Study Center*. https://timss2019.org/wp-content/uploads/frameworks/T19-Assessment-Frameworks.pdf

Minaidi, A., & Hlapanis, G. H. (2005). Pedagogical obstacles in teacher training in information and communication technology? *Technology, Pedagogy and Education*, *14*(2), 241–254. https://doi.org/10.1080/14759390500200204

Moon, K., Brenner, M. E., Jacob, B., & Okamoto, Y. (2012). Cognitive obstacles and mathematical ideas related to making connections among representations. *Proceedings of the 34th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 645–651). https://files.eric.ed.gov/fulltext/ED585047.pdf

Moru, E. K. (2009). Epistemological obstacles in coming to understand the limit of a function at undergraduate level: S case from the National University of Lesotho. *International Journal of Science and Mathematics Education*, *7*(5), 1057–1059. https://doi.org/10.1007/s10763-009-9168-9

Murniasih, T. R., Sa'dijah, C., Muksar, M., & Susiswo. (2020). Fraction sense: An analysis of preservice mathematics teachers' cognitive obstacles. *Center for Educational Policy Studies Journal*, *10*(2), 27–47. https://doi.org/10.26529/cepsj.742

Murniasih, T. R., Suwanti, V., Syaharuddin, S., Rahaju, R., & Farida, N. (2022). Prospective teachers' perceptions of didactic obstacles in the online mathematics learning. *Jurnal Elemen*, *8*(2), 619–630. https://doi.org/10.29408/jel.v8i2.5740

Murtafiah, W., Sa'dijah, C., Chandra, T. D., Susiswo, & Zayyadi, M. (2020). Novice and experienced mathematics teachers' decision making process in designing math problem. *Journal of Physics: Conference Series*, *1464*(1). https://doi.org/10.1088/1742-6596/1464/1/012030

Mutambara, L. H. N., & Tsakeni, M. (2022). Cognitive obstacles in the learning of complex number concepts: A case study of in-service undergraduate physics student-teachers in Zimbabwe. *Eurasia Journal of Mathematics, Science and Technology Education*, *18*(10), em2158. https://doi.org/10.29333/ejmste/12418

Prabowo, A., Suryadi, D., Dasari, D., Juandi, D., & Junaedi, I. (2022). Learning obstacles in the making of lesson plans by prospective mathematics teacher students. *Education Research International*, *2022,* Article ID 2896860. https://doi.org/10.1155/2022/2896860

Purnomo, H., Sa'dijah, C., Hidayanto, E., Sisworo, Permadi, H., & Anwar, L. (2022). Development of instrument numeracy skills competency assessment (MCA) in Indonesia test of minimum. *International Journal of*

Instruction, *15*(3), 635–648. https://doi.org/10.29333/iji.2022.15335a

Purnomo, Y. W., Kowiyah, Alyani, F., & Assiti, S. S. (2014). Assessing number sense performance of Indonesian elementary school students. *International Education Studies*, *7*(8), 74–84. https://doi.org/10.5539/ies.v7n8p74

Sa'dijah, C., Murtafiah, W., Anwar, L., & Mukhtamilatus Sa'diyah. (2023). Exploring the content knowledge of prospective mathematics teacher students in designing HOTS questions. *The 5th International Conference on Mathematics and Science Education (ICoMSE) 2021 AIP Conference Proceedings, 2569*, Article 040017. https://doi.org/10.1063/5.0113669

Salgado, F. A. (2016). Developing a theoretical framework for classifying levels of context use for mathematical problems. In B. White, M. Chinnappan, & S. Trenholm (Eds.), *Opening up mathematics education research* (Proceedings of the 39th Annual Conference of the Mathematics Education Research Group of Australasia), Adelaide, Volume 1, pp. 110–117. http://files.eric.ed.gov/fulltext/ED572410.pdf

Scheiner, T., Buchholtz, N., & Kaiser, G. (2023). Mathematical knowledge for teaching and mathematics didactic knowledge: A comparative study. *Journal of Mathematics Teacher Education*, *0123456789*. https://doi.org/10.1007/s10857-023-09598-z

Sellings, P., Felstead, K., & Goriss-hunter, A. (2018). Developing pre-service teachers: The impact of an embedded framework in literacy and numeracy. *Australian Journal of Teacher Education*, *43*(4). http://dx.doi.org/10.14221/ajte.2018v43n4.1

Sullivan, P., Knott, L., & Yang, Y. (2021). The relationships between task design, anticipated pedagogies, and student learning. In A. Watson & M. Ohtani (Eds.), *ICMI Study 22: Task Design in Mathematics Education* (pp. 83–114). Springer. https://doi.org/10.1007/978-3-319-09629-2_3

Susandi, A. D., Sa'dijah, C., As'ari, A. R., & Susiswo. (2022). Developing the M6 learning model to improve mathematic critical thinking skills. *Pedagogika*, *145*(1), 182–204. https://doi.org/10.15823/p.2022.145.11

Susiswo, S., Murniasih, T. R., Sa'dijah, C., Muksar, M., & Murtafiah, W. (2021). The development of an instrument on negative fractions to measure the cognitive obstacle based on mental mechanism stages. *TEM Journal*, *10*(3), 1357–1362. https://doi.org/10.18421/TEM103-44

Tamim, S. R., & Grant, M. M. (2013). Definitions and uses: Case study of teachers implementing project-based learning. *Interdisciplinary Journal of Problem-Based Learning*, *7*(2), 5–16. https://doi.org/10.7771/1541-5015.1323

Tegeh, I. M., Astawan, I. G., Sudiana, I. K., & Kristiantari, M. G. R. (2021). Murder learning model assisted by metacognitive scaffolding to improve students' scientific literacy and numeracy skills. *Jurnal Pendidikan IPA Indonesia*, *10*(4), 618–626. https://doi.org/10.15294/jpii.v10i4.32926

Tout, D., Gal, I., van Groenestijn, M., Manly, M., & Schmitt, M. J. (2020). *PIAAC numeracy task complexity schema: Factors that impact on item difficulty*. Australian Council for Educational Research. https://doi.org/10.37517/978-1-74286-609-3

Turineck, L. P., & Cavalcante, A. (2021). Using tasks to elicit mathematics teachers' thinking in financial numeracy. In A. Savard & A. Cavalcante (Eds.), *Mathematics Education in the Digital Era Financial Numeracy in Mathematics Education* (15th ed., pp. 47–53). Springer Nature Switzerland AG. https://doi.org/https://doi.org/10.1007/978-3-030-73588-3

Ubuz, B., & Aydınyer, Y. (2019). Project-based geometry learning: Knowledge and attitude of field-dependent/independent cognitive style students. *The Journal of Educational Research*, *112*(3), 285-300. https://doi.org/10.1080/00220671.2018.1502138

Wake, G. (2015). Preparing for workplace numeracy: A modelling perspective. *ZDM-Mathematics Education*, *47*(4), 675–689. https://doi.org/10.1007/s11858-015-0704-5

Watson, J., & Moritz, J. (2002). Quantitative literacy for pre-service teachers via the internet: Chance and data in the news. *Mathematics Teacher Education and Development*, *4*, 42–55. https://mted.merga.net.au/index.php/mted/article/view/115

Yustitia, V., Siswono, T. Y. E., & Abadi. (2021). Numeracy of prospective elementary school teachers: A case study. *Journal of Physics: Conference Series, 1918*(4), Article 042077. https://doi.org/10.1088/1742-6596/1918/4/042077

Zakiah, N. E., Fatimah, A. T., Sunaryo, Y., & Amam, A. (2020). Collaboration and communication skills of pre-service mathematics teacher in designing project assignments. *Journal of Physics: Conference Series*, *1657*(1). https://doi.org/10.1088/1742-6596/1657/1/012073

Zamzam, K. F., Sa'dijah, C., Subanji, & Rahardi, R. (2023). The creative thinking process of prospective teachers in developing assignments. *Journal of Higher Education Theory and Practice*, *23*(1), 101–108. https://doi.org/https://doi.org/10.33423/jhetp.v23i1.5793

Appendix A: Interview Guideline and Example Task Design

Before Interview:

Explanation of interview purpose: Thank you for being willing to help us with this interview. As you know, we are currently studying the epistemological obstacles of prospective secondary mathematics teachers in the design process of project-based numeracy tasks. Previously, you have designed a project-based numeracy task, please allow us to conduct an interview about it.

Explanation of consent: This interview will be part of our study that will be published in a journal. your name will be initialled to maintain confidentiality. Therefore, before proceeding with the interview, we would like to ask for your consent first. If you agree to this condition, then please sign the following consent form and let's proceed with the interview.

During Interview:

Thank you for agreeing to conduct this interview. Let's look at the project-based numeracy task you designed.

Prior knowledge and experience interview: We would like to explore your knowledge and experience of numeracy and project-based tasks. (The following questions are only initial guides, the questions evolve flexibly according to the interviewee's responses)

Guidelines Questions	Epistemological Obstacle Factor
What do you know about numeracy for students?	Former education (mathematical content knowledge)
What do you know about project-based learning?	Former education (pedagogical content knowledge)
What do you know about project-based numeracy tasks?	Former education (mathematical and pedagogical content knowledge)
Where did you learn about numeracy and projects?	Former education, pre-performed learning method (mathematical and pedagogical content knowledge)
How did you acquire knowledge about the project-based numeracy task? If you self-learned about the project-based numeracy task, please state the sources you used?	Former education, pre-performed learning method, verbal expression (mathematical and pedagogical content knowledge)
Please explain what learning methods you found most effective for learning when you were a student?	Pre-performed learning method (pedagogical content knowledge)
Have you ever experienced project-based learning at school?	Pre-performed learning method (pedagogical content knowledge)
Did you ever learn numeracy at school?	Pre-performed learning method (mathematical content knowledge)

Project based numeracy tasks design process and implementation test-based interview: We want to know how you designed the project-based numeracy task and implemented it according to your imagination. Please read and look again at the project-based numeracy task you designed.

Guidelines Questions	Epistemological Obstacle Factor
Explain the purpose of your project-based numeracy task design?	Former education, pre-performed learning method, verbal expression (mathematical and pedagogical content knowledge)
How did you generate the idea for this project-based numeracy task?	Former education, pre-performed learning method, verbal expression (mathematical and pedagogical content knowledge)
Please explain how you implemented this project-based numeracy task according to your imagination?	Former education, pre-performed learning method, verbal expression (mathematical and pedagogical content knowledge)
Describe the context/mathematical content/tools/student engagement/critical orientation/project centrality/collaboration/artefacts/knowledge construction/structure of your project-based numeracy task? Justify your answer?	Former education, pre-performed learning method (mathematical and pedagogical content knowledge)

PbNT task design example:

Paper production is one of the reasons for forest destruction in Indonesia. Trees are cut down to make the paper we use every day. Let's love the mother earth by starting to recycle used paper.
- Bring unused paper from your house with a minimum area of 609 cm (use a ruler to measure)!
- Together with your group, let's make envelopes and write love letters to our parents!
- How will we make it to minimize the area of wastepaper?