Learner-generated Examples Within a System for Computer-aided Assessment as a Tool for Engaging In-service Teachers with Linear Functions

Siri Ovedal-Hakestad
University of Agder

Received: March 2024 ׀׀ Accepted: September 2024

This paper examines how learner-generated examples (LGE) tasks within a system for computer-aided assessment (CAA) facilitate in-service teachers' (ISTs') engagement with and explorations of the features of linear functions. Employing the concept of example spaces, the study explores how ISTs approach and engage with linear functions through these tasks. Seven ISTs participated in a series of LGE tasks in a CAA system, and their digital responses were collected. Semi-structured interviews were conducted with four participants following task completion. The results reveal common procedures adopted by the ISTs, such as using the "one-unit-right-*a*-up/down" procedure, as well as varying approaches to generation points and plotting them into a coordinate system. ISTs, however, faced challenges in effectively communicating their mathematical explanations. This study highlights the potential of LGE tasks in a CAA system to enhance ISTs' example spaces and improve their understanding of mathematical concepts.

Keywords · linear functions · learner-generated examples · example spaces · computer-aided assessment · in-service teachers

Introduction

Examples play a crucial role in mathematics as they are essential for generalisation, abstraction, and analogical reasoning (Zodik & Zaslavsky, 2008). Traditionally, the reliance has been on examples provided either by the teacher or textbooks. This study, however, shifts focus to be on learner-generated examples (LGE), emphasising the importance of learners creating their own examples of mathematical objects that satisfy given conditions (Watson & Mason, 2006). Such a shift underscores the necessity for teachers not only to carefully select and apply mathematical examples but also to quickly adapt by creating examples that resonate with the individual needs of their students. Despite its significance, this skill is often overlooked in many teacher education programs (Zodik & Zaslavsky, 2008). Thus, incorporating LGE tasks into teacher education could offer a practical approach to this oversight.

This study involves in-service teachers (ISTs) who engaged in a series of LGE tasks within a system for computer-aided assessment (CAA). An example of a series of tasks can be: "(a) Give an example of a number that has four factors. (b) And another, and another. (c) Give an example of a number greater than 500 that has exactly 4 factors" (Sinclair et al., 2011, p. 297). This approach encourages students to explore multiple solutions, fostering learning during the process of making judgments (Breen & O'Shea, 2019), and enhances students' ability to write mathematically (Dinkelman & Cavey, 2015). Further, the approach aims at assembling a repertoire of examples and developing methods of example construction. Recent studies by Sangwin (2019), Brunström et al. (2022), and Kinnear (2024) recommended using systems for CAA when assessing LGE tasks. CAA enables the automatic assessment of higher-order mathematical skills (Fahlgren & Brunström, 2023) and enables opportunity to quickly assess tasks with multiple solutions.

The study involves an analysis of ISTs' responses to a series of LGE tasks in a CAA system complemented by semi-structured interviews. The aim is to examine how LGE tasks within a CAA system facilitate the ISTs' engagement with and explorations of the features of linear functions. Features are perceived as characteristics, specific qualities, or procedures utilised by the ISTs. These features are examined through the analysis of the provided examples, employing the concept of example spaces. Watson and Mason (2006) described example spaces as a collection of examples serving specific functions in mathematical learning. Building on the work of Watson and Mason (2006), Zazkis and Leikin (2007) developed a framework with the intent that example generating tasks could be used as a research tool for describing and analysing participants' knowledge. This is a framework for analysing qualities and structures of example spaces of participant-generated examples that will be utilised in this study (see section "Framework for examples spaces"). By using this framework, the study will focus on the insights gained from the examples provided by the ISTs and reveal not only how these examples demonstrate the features of linear functions they explore but also the potential of LGE tasks to enhance understanding of such mathematical concept. The study is guided by the following research question:

> *How do LGE tasks within a system for CAA facilitate in-service teachers' engagement with and exploration of the features of linear function?*

By investigating the use of LGE tasks in a CAA system, this study uncovers specific characteristics and impacts of these tasks due to their unique nature. The findings will be highlighted throughout this article as a natural extension of the study, exploring its relevance not only to prospective and practising teachers but also to a broader audience, regardless of the mathematical topic being studied.

Theoretical Considerations

This section offers an overview of the theoretical foundations concerning the concept of linear functions, the use of CAA systems for generating examples, the definition and characteristics of what constitutes an "example", and the concept of "example spaces". Additionally, the framework utilised in this study is introduced.

The Concept of Linear Functions

Linear functions hold a fundamental place in mathematics education (Lloyd & Wilson, 1998), and a solid understanding of linear functions is vital in establishing a strong foundation for comprehending algebraic concepts (Pierce et al., 2010). This study delves into the perspective of how LGE tasks within a CAA system facilitate ISTs' engagement with and exploration of the features of linear functions. Investigating the role of ISTs has a dual aspect; the teachers need to understand the concept of linear functions themselves, and in addition, they need to be able to teach this concept to their students.

Moschkovich (1996) stressed that understanding the concept of linear functions includes more than knowing procedures. It involves recognising the relationship between graphical and algebraic representations and identifying the relevant elements within each representation. Similarly, Birgin (2012) suggested that equations, tables, and graphs are the predominant representations of linear functions. Students and teachers should be proficient in comprehending information presented in these diverse formats and perform transitions between them. To foster and facilitate student learning, teachers must possess a robust understanding of functions that includes appropriate representations of functions that together construct a coherent conceptual understanding of the function (Jukic Matic et al., 2022). By integrating these representations, teachers can construct a coherent and comprehensive understanding of functions, enabling them to convey mathematical ideas effectively to their students.

This study, therefore, investigates how LGE tasks within a CAA system facilitate ISTs' engagement with and exploration of the features of linear functions. Specifically, it examines how LGE tasks contributes to ISTs' examination and engagement with the relationship between graphical and algebraic representations, identify relevant elements withing these representations, and the transitioning between different representations. The exploration is conducted through ISTs' generated examples in a CAA system. The following section elaborates on the role of CAA systems in facilitating the exploration of linear functions through ISTs' generated examples.

CAA Systems in Generating Examples

The study delves into the role of CAA in generating examples, with STACK as the primary tool. STACK (System for Teaching and Assessment using a Computer Algebra Kernel) is an open-source CAA system for mathematics developed by Sangwin[1]. CAA has opened the potential to collect a wider range and higher quality of data and to analyse significantly more assessment data than what was achievable with paper-based methods (Yerushalmy et al., 2017). A strength found in the use of computer-aided assessment is the ability to check any answer given in the correct form by applying algebraic equivalence (Sangwin, 2013).

Furthermore, the use of CAA provides the opportunity to simplify the process of assessing, to facilitate teachers' online responses, and to offer immediate and automated feedback (Fahlgren & Brunström, 2023). In STACK, the software aims to determine mathematical properties specified by the teacher. For each question within a quiz, the teacher needs to decide what constitutes a correct answer and establish these properties in the question-making process (Sangwin & Köcher, 2016). Within the scope of this research, the ISTs were provided with immediate feedback after some of the tasks, indicating whether their answer was correct or not. The tasks that included automated feedback were those where the ISTs had to provide an example of two integers with a particular sum. These tasks are detailed in the subsection, *Tasks*. The decision to avoid detailed feedback, like hints for incorrect answers, was made to maintain the tasks' challenge level. Providing hints after a wrong answer in the first task could have exposed the progression in the task series. Additionally, Rønning (2017) noted that offering hints might simplify the original problem, altering the actual challenge faced by the students.

Using CAA systems to facilitate LGE tasks is appropriate because LGE tasks often offer several correct solutions (Breen et al., 2016), all of which can be assessed and evaluated automatically (Yerushalmy et al., 2017). Using STACK for assessing LGE tasks is supported by the recommendations of Sangwin (2019), Brunström et al. (2022), and Kinnear (2024) for the use of computer algebra systems for these purposes.

Examples and Example Spaces

The growing interest in examples within mathematics education highlights their pivotal role in learning, teaching, and in the understanding of mathematical concepts (Zaslavsky, 2019). This paper explores the examples generated by ISTs, drawing from the perspective of Goldenberg and Mason (2008), that an example is deeply embedded in an individual's understanding and engagement with abstract concepts. Iin this study, examples will be restricted to examples of mathematical concepts, and serve as illustrations or instances of mathematical objects, reflecting the views of Alcock and Inglis (2008), Watson and Mason (2006), and Zazkis and Leikin (2008).

Examples are a fundamental part of effective teaching, proven to constitute a fundamental part of a good explanation (Leinhardt, 2001). Despite findings from Iannone et al. (2011), which show no significant difference in performance regarding tasks involving proof production between students who were encouraged to create examples and students who used worked examples, this study emphasises the importance of LGE in understanding and teaching mathematics. The focus on LGE tasks in this study is a step towards engaging ISTs in "exploring, enriching, and extending their appreciation of mathematical structures, concepts, and connections among topics" (Watson & Mason, 2006, p. x).

This study investigates how LGE tasks within a CAA system facilitate ISTs' engagement with the features of linear functions. The features are explored through the concept of example spaces inspired by Watson and Mason (2006). They defined example space as a collection of examples that fulfil a specific function. Example spaces encompass not only the representation of mathematical objects, but also a wide array of associated connections and construction methods (Goldenberg & Mason, 2008). The examples most individuals use are often adaptions of ones they have come across in different contexts, such as in texts or from lecturers (Watson & Mason, 2006). There may be a large potential space of examples arising from past experiences, and what comes to mind in a given situation tends to

[1] https://stack-assessment.org/

be fragments of that potential in each situation. Watson and Mason (2006, p. 76) distinguished between four kinds of examples spaces:

- Situated (local), personal (individual) example spaces that are triggered by current tasks, cues, and environment, as well as by recent experience.
- Personal potential example spaces from which a local space is drawn, that consists of one person's past experience, and that may not be structured in ways that afford easy access.
- Conventional example spaces as generally understood by mathematicians and as displayed in textbooks, into which the teacher hopes to induct his or her students.
- A collective and situated example space, local to a classroom or other group at a particular time, that acts as a local conventional space.

The current study uses collective and personal example spaces to investigate ISTs' generated examples of linear functions. Therefore, a clarification of what is meant by collective example space and personal example space is needed. According to Sinclair et al. (2011), learners are expected to build a collection of examples and strategies for constructing examples based on their experiences, based on their personal use, Watson and Mason (2006) refer to this collection as personal example space. Simultaneously, a shared and collective example space is developed through discussions and negotiations within a group. Individuals may incorporate varying degrees of the negotiated space into their own understanding, resulting in diverse aspects available for future use. Throughout the course of a lesson, a temporary, context-specific, and collective example space also emerges, serving as a resource from which learners can potentially draw. Example spaces can be extended when the learner is active in constructing examples through diverse approaches. This involves restructuring existing knowledge in order to network forgotten experiences and utilise current knowledge in unfamiliar ways, thus expanding and enriching personal example spaces (Watson & Mason, 2006). This study examines the collective example spaces demonstrated by ISTs in their interaction with the CAA system, which may vary from those observed in traditional discussions or classroom environments.

Framework for Example Spaces

Zazkis and Leikin (2007) proposed the concept of using LGE as both a pedagogical tool and as a research instrument. This study adopts the framework proposed by Zazkis and Leikin, focusing on analysing the example spaces of the provided examples related to tasks concerning linear functions. The framework offers opportunities to derive valuable insights regarding the knowledge and understanding from the provided examples and is based on the assumption that "example generating tasks may serve as a research tool in studies that aim to describe and analyse participants' knowledge" (Zazkis & Leikin, p. 19).

The analysis of provided examples of from the ISTs is conceptualised through three categories:

- **Richness:** Encompasses the diversity and variety of examples, and if they are drawn from different contexts.
- **Accessibility and correctness:** Assessing examples based on whether they meet task requirements and ease or difficulty of their generation, focusing on their logical structure and appropriateness.
- **Generality:** Considering the specificity or generality of examples, with an emphasis on broader, general explanations or solutions.

The study utilises this framework to analyse examples provided by ISTs, aiming to explain their example spaces with insights to their knowledge and understanding. Generality, however, is less emphasised in this study due to task design. Richness and accessibility, including the correctness of the provided examples, are the focal points, offering an insight into the personal structure of the example spaces of the participants.

Method

Participants and Data Gathering

This study was conducted within the context of a continuing education course in mathematics for ISTs at a Norwegian university. The course spanned a year, the current study taking place during the second semester. Participants in this course had a reduced amount of regular teaching duties, allowing them to integrate the course as a part of their job. This arrangement meant that, combined with the course, their overall workload was equivalent to a full-time position. A total of 10 teachers enrolled in the course, all of whom were practicing primary school teachers with varied experience in teaching mathematics. Their age ranged between 30 to 55 years and their years in teaching from 5 to 30.

Out of the 10 ISTs, seven voluntarily chose to participate in the STACK-quiz. Following the quiz, four of these ISTs engaged in individual, semi-structured interviews. Given the limited number of volunteers, all were included in the study. This means both groups represented a convenience sample for the study because they were available and willing to participate, allowing a thorough examination of the procedures they used and their engagement while solving the tasks. While the sample size in this study of ISTs is relatively small, this is appropriate given the exploratory nature of the research. As a qualitative study, the small sample size allows for an in-depth exploration of each participant's experience. Although the sample size limits the possibility to generalise broadly, it provides valuable insights into the specific characteristics of LGE tasks in a CAA system and what aspects that contributes to their impact.

Mari, Tina, Lina, and Tara were the four teachers who participated in the interviews. Mari and Lina, both aged between 35 and 40, had around 10 years of teaching experience. Both taught mathematics, Mari in Grades 5–7 (students aged 10–12), Lina in Grades 1–7. Tina, also in the age of 35 to 40, had around 5 years of teaching experience, without mathematics as one of her subjects. Tara, close to 45 years old, had been a primary school teacher for approximately 20 years. She had mostly taught Grades 5–7, with mathematics as one of her subjects.

The four interviews had a duration from 15 to 30 minutes, providing an opportunity for in-depth discussions. They were semi-structured, aiming at uncovering the ISTs' insights into the concept of linear functions by examining the procedures they used and their experiences and thought processes while solving the tasks. This approach allowed for a detailed exploration of the rationale behind their selection of specific examples.

Tasks

Linear functions are mainly taught in the years after primary school; however, it is essential for primary school teachers to gain thorough understanding of linear functions, since "knowing mathematics for teaching obviously requires knowing in detail the topics and ideas that are fundamental to the school curriculum, and beyond" (Ball, 2003, p. 6). Knowledge of linear functions supports the ISTs' ability to teach the topic effectively. A consequence of analysing examples generated is the opportunity to identify characteristics and impact the use of LGE tasks within a CAA system offers. These insights are not only valuable for understanding how ISTs interact with these tasks, but also for informing a broader application of LGE tasks across various educational contexts, extending the benefits to a wide range of students and educators beyond just ISTs.

The tasks underwent several revisions and changes before being selected for this study. Previous research by Ovedal-Hakestad and Larson (2024) introduced similar tasks to a group of pre-service teachers. However, the level of difficulty of those tasks proved to be too high, leading to modifications that were implemented for this study.

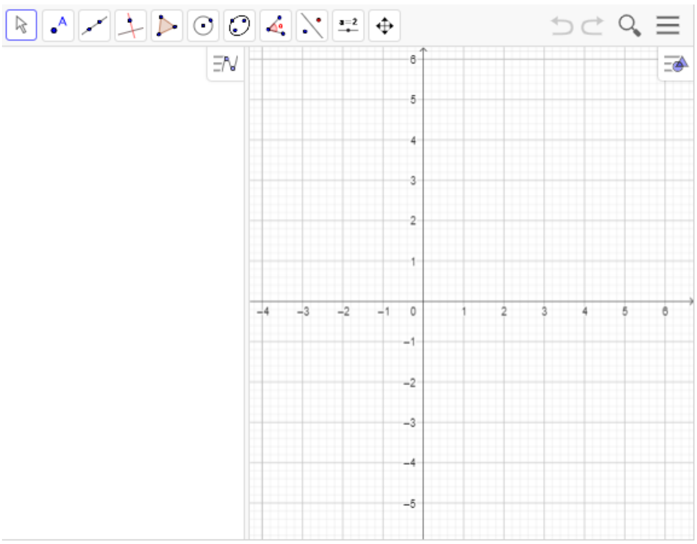
The study involved a series of tasks categorised into four sets, collectively referred to as a "quiz". Set 1 included Tasks 2–7, Set 2 Tasks 8–13, Set 3 Tasks 14–17, and Set 4 Tasks 18 and 19. Task 1 only asked the participants to consent whether their digital responses could be used for research purposes or not. The tasks required ISTs to give examples of two numbers that had the sum of 2 (Set 1), −2 (Set

2), and 5 (Set 3). Thereafter, plot these pairs of numbers as coordinates in a coordinate system given in GeoGebra. Additionally, ISTs were asked to analyse and describe similarities and differences observed among these plotted points. The tasks that concerned generating points were programmed into providing feedback that contained information if the response were correct or not. The open-ended tasks were however assessed manually. STACK provides various options for automating the grading of correct open-ended responses. The commonly known method necessitates the use of predetermined words, with grading depending on their application. This automated process contrasts with the approach in this study, where the tasks were designed to encourage spontaneous and unrestricted expressions from the ISTs, which made a manual assessment more appropriate. Set 1 Included the following tasks:

2. Give an example of two numbers that have a sum of 2, which means that $x + y = 2$.
3. Give another example of two numbers that have a sum of 2, which means that $x + y = 2$.
4. Give another example of two numbers that have a sum of 2, which means that $x + y = 2$.
5. Give another example of two numbers that have a sum of 2, and which you think that no one else in the class will give. Which means that $x + y = 2$.
6. One can let the variables x and y be coordinates for points (x, y) in a coordinate system. Plot at least 4 points with coordinates (x, y) that satisfy $x + y = 2$ in the coordinate system.
7. What similarities do you see between the points (x, y) you plotted in the coordinate system above?

The ISTs were given similar sets of tasks for Set 2, where the sum was equal to -2, and Set 3, where the sum was equal to 5. However, in Set 3 the final two tasks (plotting the points) were excluded. This was because in Set 4 the ISTs should plot all three cases, that is, $2, -2$ and 5. Set 4 comprised Tasks 18 and 19 (Figure 1 & 2).

Task 18 Plot each of the three cases separately, with points having coordinates (*x*, *y*), in the following scenarios: where $x + y = 2, x + y = -2$, and $x + y = 5$.



Tip: It is recommended to plot at least four points from each scenario to effectively represent the cases.

Figure 1. Set 4 Task 18.

Task 19	Describe the similarities and differences between the three cases, namely $x + y = 2$, $x + y =$ -2, and $x + y = 5$.

Figure 2. Set 4 Task 19.

Data Analysis

The data for this study were compiled from digital responses in STACK from all seven participants, of which Tasks 7, 13, and 19 were open-ended responses. Further, the data included interview transcripts from four participants. The analysis was focused on exploring the ISTs' collective and personal example spaces. To examine the collective example spaces, all digital responses from the ISTs were analysed. For investigating the ISTs' personal example spaces, the interview transcripts were specifically utilised, thereby gaining insight into their individual perspectives and understandings. The analysis of the digital responses to the STACK-quiz and the interview transcripts was concentrated on exploring the collective and personal example spaces of the ISTs.

The digital responses obtained from STACK were examined set-wise. This analysis focused on identifying patterns within the provided examples, such as the frequency of specific numbers and the uniqueness of the examples given. Additionally, special attention was given to the variety of ordered pairs generated by the ISTs.

The analysis of the responses to the open-ended Tasks 7, 13, and 19, was guided by Birgin et al.'s (2012) study on students' understanding of and difficulties with linear functions. That study suggested that a comprehensive grasp of linear functions involves recognising the relationships between slope, *x*-intercept, and *y*-intercept. Therefore, terminology related to these concepts was searched for in the ISTs' responses to the open-ended tasks (Tasks 7, 13, and 19). This search revealed subcategories in terminology used by the ISTs, which will be presented in the results section. The subcategories were again used to explore the ISTs' collective understanding of linear functions.

The interviews were transcribed, then the study utilised the framework proposed by Zazkis and Leikin (2007), which offered a structured approach for evaluating the interviews. This framework highlights richness, accessibility, and correctness, as lenses for examining the qualities and structures of the ISTs' example spaces. This approach aligns with Watson and Mason's (2006) notion that the richness of an example space might be an indicator of students' mathematical understanding. Zazkis and Leikin's (2008, pp. 135–136) framework was applied to code each interview into specific subcategories, which are listed Table 2.

Table 2
Framework for Analysing the Qualities and Structures of the ISTs' Example Spaces

Accessibility and Correctness	Richness
Were the answers correct?	Did the examples vary in kind?
Satisfied conditions for the task?	Was there a fluency in any variety?
Were the examples generated with ease or with struggles?	Were the examples routine or non-routine?
Were they pulled out of thin air, constructed using specific procedures, or selected from resources?	How does the personal example space of a participant relate to conventional example space?
Were there any other procedures used for constructing examples or for checking that the conditions of the tasks were satisfied?	How is the personal example space similar to/different from the collective example space?
Were the procedures used for example generation mathematically correct, elegant, or unnecessary?	Were the examples situated in a particular context, such as curriculum or classroom experience?

Here, two examples are given to demonstrate how utterances from the transcripts were coded. The first involves Tara's approach of setting $x = 0$ to determine the value of y, exemplifying a specific procedure for generating examples located under "accessibility and correctness". She explained: "I thought that if I put zero, I could at least find one of them .... So, I put $x = 0$, y is 2, −2, and 5." This statement was categorised as "constructed using a specific procedure to generate examples", because she used the procedure of putting $x = 0$ to generate the value of y.

In the second example, Lina stated that her procedure is to start with the easiest combinations. When asked to elaborate on "easy combinations", she explains: "I am thinking about those you can systematically put in a table like 0 and 1." This statement, where Lina uses her prior knowledge to envisage a table of values for point generation, falls under the "selected from resources" category, a part of "accessibility and correctness". Such coding of the ISTs' utterances facilitated a detailed examination of their approach to generate examples in the context of linear functions.

Results

This section initially presents results concerning features of linear functions through collective example space, followed by an exploration of the findings related to linear functions through personal example spaces.

Collective Example Space

By analysing the digital responses in STACK from the seven participants focusing on collective example spaces, this section focuses on two main aspects: the variety in ordered pairs and the words used in their open-ended answers.

Variety in ordered pairs

The STACK-quiz comprised a total of 19 tasks, where 12 required the ISTs to generate coordinating points, resulting in 84 responses. Notably, 79% of the generated points fell within the range of -10 to 10. One notable observation in the provided pairs is that only one IST provided non-whole integer examples, while the rest of the ISTs only provided whole number pairs. The first two sets of tasks demonstrated greater diversity in examples compared to the final set, indicating a shift in the participants' selection strategies over the course of the tasks.

Tasks 5, 11, and 17 were designed to challenge the ISTs to provide unique examples. The results from these tasks are listed in Table 3. The most common procedure for the participants was to use larger integers when providing these types of examples.

Table 3
An Overview of the ISTs' Responses

Participant	Task 5	Task 11	Task 17
Lina	$(-13, 15)$	$(-6, 4)$	$(-99, 104)$
Tina	$(-11, 13)$	$(-16, 14)$	$(-7, 12)$
Sara	$(-17, 19)$	$(-3, 5)$	$(0, 5)$
Fred	$(-514, 516)$	$(-344, 342)$	$(-755, 780)$
Eric	$(-102, 104)$	$(1004, -1006)$	$(195, -190)$
Mari	$(-100, 102)$	$(-55, 53)$	$(-77, 82)$
Tara	$(\frac{3}{4}, 1\frac{1}{4})$	$(-22, 20)$	$(2, 3)$

Open-ended answers (Task 7, 13, and 19)

The open-ended responses shed light on the collective example spaces of the generated examples among the seven ISTs. The design of the tasks allowed for a natural use of terminology and concepts associated with linear functions. Tasks 7 and 13 asked the ISTs to explain what similarities they saw between the points plotted in the given coordinate system, belonging to the sum of 2 and -2, while Task 19 asked the ISTs to explain the similarities and differences between all three cases (points plotted to the sum of $2, -2$ and 5). Below is a list of the terminology used by the participants, reflecting their grasp of concepts related to linear functions in Tasks 7, 13, and 19.

- Linear function/straight line
- Slope/gradient
- One unit right, one up/down
- Intersection
- Constant term
- Function expression
- Parallel line

In total, five out of seven individual participants used terms like "linear function", "straight line", or related expressions. Four individual participants mentioned the concept "slope/gradient", though only one defined it explicitly. The notion of the "constant term" and its specific value was addressed by four individual participants, where one explicitly pointed to the exact constant term. Intersection was mentioned by four participants, with none stating the x-intercept, while two provided explicit references to the y-intercept.

Personal Example Space

This section presents results related to features of linear functions through the lens of personal example spaces of four ISTs, Tina, Mari, Tara, and Lina, revealed through the interviews. The focus is on the procedures and thought processes these ISTs utilised while engaging in LGE tasks through STACK, with an emphasis on variety, accessibility, and correctness of the provided examples.

Tina

Tina's selection of procedures for the LGE tasks in STACK was based on her prior mathematical knowledge. The first procedure she utilised was the "one-unit-right-a-up/down" in order to explain the gradient of the linear functions. Tina found it challenging to write a mathematical explanation; in particular, she noted difficulty in spotting similarities and differences in the linear lines.

Tina It was like, what are the differences? It was harder to find in the beginning, than the similarities, because there are so many similarities, but it is not that easy to spot the differences. I had to plot more points. I inserted more points, in order to see that they had different starting points.

Tina explained that it was easier to find similarities than differences. To address this, she added more points into the coordinate system for better visualisation and understanding of the differences.

Tina's approach for selecting numbers evolved from an initial random approach to one where she chose numbers closer together.

Tina: On the second one [refers to Set 2] I did not choose a large discrepancy between the numbers.

Tina explained that she first chose sums that did not lie close to each other in the coordinate system. However, this tactic changed, and working on the second set of tasks she chose numbers that were relatively close to each other. This might indicate that she adjusted the chosen points to fit into the GeoGebra-file they were provided with in the plotting task.

Additionally, Tina employed a procedure of selecting more points than necessary for all tasks. For instance, in Tasks 6 and 12, where a minimum of four points in GeoGebra was required, she submitted six and nine points respectively. In Task 18, despite the suggestion to provide at least four points per case, Tina went beyond this, providing 22 points (Figure 3). This can be interpreted as a procedure of verification to convince herself that what she did was correct, in addition to a procedure for locating the linear line.

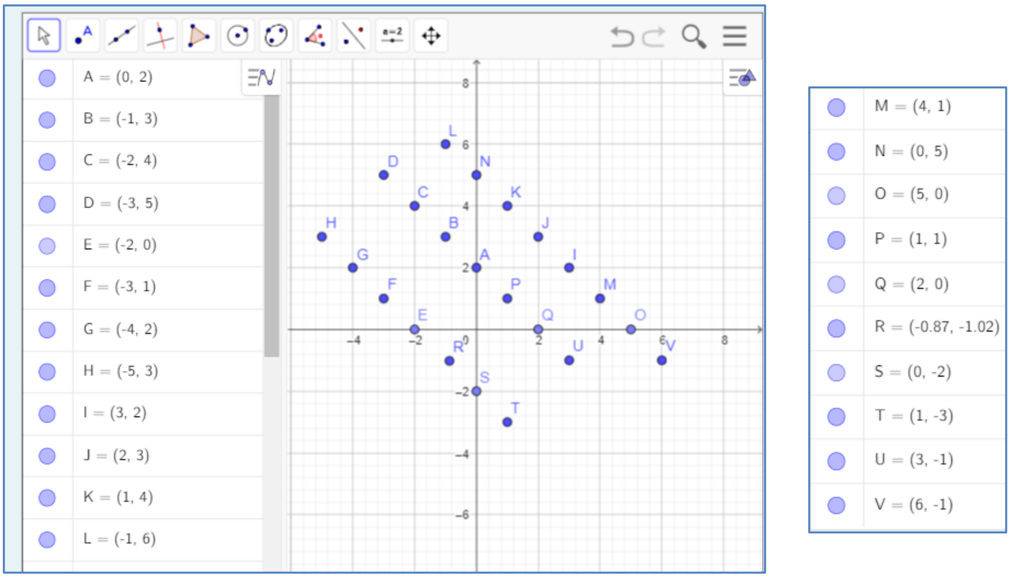


Figure 3. Tina's response to Task 18.

Tina explained:

Tina: I became so unsure about my answers. I started with 4 [points] that went upwards, then I began to think ... "Do they intersect the *x*-axis? The *y*-axis?" So, I had to check this. I had to plot more points to see the differences between the lines.

During the interview, she explained that she became unsure about her own answers. Plotting more points than needed might indicate an approach for self-verification of her solutions.

Mari

Mari also employed the "one-unit-right-up/down" procedure, similar to Tina for gradient explanation.

Mari: When the x-value increases by one, the y-value decreases by one.

Mari explained that she had challenges with connecting the sums to the coordinate system, and stated the following:

Mari: I did it [the tasks] in two parts. What I did with the numbers and I also spent more time finding the function expression on the other part. So I could have spent more time looking at the similarities between the tasks, I see now in retrospect. I did not see that they were linked to the coordinate system. … I was more eager to find the sums there [refers to the first tasks] and then to find the function expression on the other.

Mari's approach to the sets is to focus separately on finding points and then finding the function expression. Reflecting on her own strategy, she observed that more time was spent on the latter, which involved transforming the expression into a standard linear function expression, $y = ax + b$. Then she used this to generate points to be plotted into the coordinate system. This approach indicates Mari's reliance on familiar linear function concepts to validate her answers.

In line with Tina, Mari also explained difficulties regarding mathematical explanations.

Mari: I thought the last one was hard, where I was supposed to argue why the lines were parallel …. I spent a lot of time on that task.

Mari found it difficult to justify why the lines were parallel in the final task, mentioning that she dedicated significant time to understand and explain the connections between the three lines. This highlights a common difficulty faced by participants in expressing their mathematical explanations.

Tara

Tara distinguished herself by being the only IST who used fractions in her examples (Figure 4). Notably, Tara began using fractions by Task 4, even before participants were prompted to come up with a unique example that other classmates might not think about.

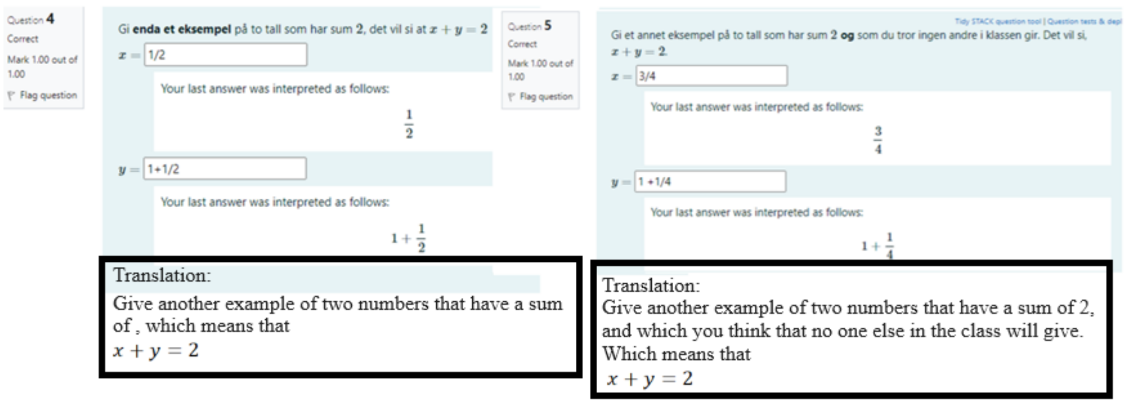


Figure 4. Tara's digital response in STACK to Tasks 4 and 5.

Tara approached the sets by dividing them into two parts, in line with Mari. First, by generating the points, and then plotting different points into GeoGebra. Her initial focus was to find the function expression, in line with Mari's explanation, and use this to generate points. She perceived the number-tasks (finding points/plots) as easy but plotting the points into a coordinate system as more challenging.

Tara took advantage of some previous learnt procedures during this quiz. Her approach involved using her knowledge of algebraic manipulation. She applied the expression $x + y = 2$ and substituted x with 0, leading to the expression $0 + y = 2$, from which she deduced the value of y. Further, she elaborated that she knew that the tasks had to intersect the y-axis in the sums $(2, -2$ and $5)$. She therefore chose y-values close to the intersection $(2, -2,$ and $5)$, such as $y = 1$. She then applied the expression $x + 1 = 2$, and generated the value of x to be 1. She labelled these as the "easy" points and explained that choosing y-values centred around the intersection points would help her draw the linear line.

Lina

Lina also separated the sets into two parts: finding points as the first, then plotting different points into GeoGebra. However, she argued that the tasks did not require plotting the same points provided in the previous tasks. It seems important for Lina to highlight that she did not make any mistakes by making this choice, but she did exactly what the tasks asked for. Lina, as Tina and Mari, faced challenges in writing a mathematical explanation. Lina found it difficult to articulate her understandings in precise terms, even though she knew what the explanation was going to entail.

Lina adopted a systematic approach by manipulating the algebraic expression $x + y = 2$, beginning with substituting x with 0, similar to Tara's procedure. Unlike Tara, Lina visualised this process in a table of values, including a mental image of systematically listing x and y values, and focusing on x-values around 0 to determine corresponding y-values. She described these as the "easiest" combinations of points. When asked to clarify "easy" combinations, Lina immediately compared her work with earlier experiences of working with mathematics and explained how she usually would solve a task. For Lina, adopting a familiar approach and choosing x-values around 0 simplified the tasks, resulting in what she labelled as "easy" points.

Lina, in line with Tina, described adopting a more systematic approach for choosing points throughout the quiz, however not the same as Tina's. Lina implemented the procedure of "adding one and subtracting one" to achieve the same sum, aware that these points would cluster around the origin. Set 3 included the algebraic expression $x + y = 5$, Lina chose to substitute for instance x with 1, that would result in $y = 4$. Further she chose $x = 2$, which would generate the value of y to be 3. She labelled this systematic approach as a way to generate "easy" points.

This detailed examination of the ISTs' varied strategies and terminology is discussed in the following section. The discussion focuses on the ISTs' engagement with the concept of linear functions and LGE tasks through a CAA system.

Discussion

This study investigated how LGE tasks within a CAA system facilitate ISTs' engagement with and exploration of the features of linear functions. Results from digital responses to an LGE-quiz in STACK and insights from in-depth interviews are discussed to address this objective. Analysing the features of the concept of linear functions are done through the notion of example spaces. According to Watson and Mason (2006), an important element when analysing example space is what collection of mathematical objects and construction methods the learner possesses and utilises while engaging with a task. Based on this, the discussion is divided into two parts: first reflecting on the features of linear functions through collective example spaces of the ISTs, second, reflecting on the features of linear functions through their personal example spaces when working with LGE tasks in STACK.

Collective Example Space

The collective example space, unique to a specific classroom context at a particular time (Watson & Mason, 2006), was explored through common procedures adopted by the ISTs as they generated points belonging to a sum, connected the provided points to a coordinate system, and explained with their own words similarities between the points in the coordinate system.

Generating points

The quiz contained three sets of four "give an example" tasks. Fahlgren and Brunström (2023) noted that asking for more than three examples would result in similar examples as earlier provided. However, the number of tasks seemed appropriate in this quiz, as it fostered opportunities for the IST to connect the generated points equal to a sum with a linear function.

These tasks required the ISTs to generate x and y terms in a sum, for example, $x + y = -2$. Of the points provided, 79% consisted of integers in the range of -10 to 10 and predominantly consisted of integers. The last of these "give an example" tasks required the ISTs to provide an example no one else in the class would give, resulting in the procedure of choosing larger integers. This is exemplified by Mari's response to Task 5, where she replied $x = -100$ and $y = 102$. While turning to this procedure was expected by some of the ISTs, the uniformity among the ISTs' approach was surprising. This tendency of choosing larger integers sheds light on a collective lean towards well-known numerical approaches in their example generation, even when asked to generate unique examples. Including a more challenging prompt, such as, "Provide an example no one else in the world would give," might have encouraged the ISTs to vary their responses by providing more complex mathematical examples.

Written explanations

Set 1 and 2 of the provided quiz, asked the ISTs to provide written explanations identifying similarities among points plotted in the coordinate system belonging to the sum of 2 in Set 1, and belonging to the sum of -2 in Set 2. Following, Set 4 involved written explanations where the ISTs were prompted to discuss both similarities and differences in previous plotted points, more specifically related to the sum of $2, -2,$ and 5.

Although this study's tasks did not require in-depth explanation of the linear expression $y = ax + b$, two of the participants included the general function expression of a linear equation in their explanation. The ISTs used this expression to demonstrate that all generated points belonged to the same line, showing their understanding of the underlying mathematical concepts. Moreover, five out of seven participants gave detailed attention to the parameter b, understanding it symbolically as the constant term and graphically as the y-intercept. This focus is noteworthy, especially when contrasted with Pierce et al.'s (2010) observation that the parameter b is often overlooked from students' descriptions in favour of the parameter a, as a more complex parameter. However, the parameter a remained constant -1 in all cases, which might explain why the parameter b received more attention in the given tasks.

The study revealed that interpreting the parameter a graphically as the gradient of the graph was not done by calculating. Some ISTs adopted the "one-unit-right-up/down" procedure or used graph reading to determine the gradient/slope, demonstrating alternative methods to explain the concept of slope without calculating it from two points. The "one-unit-right-up/down" procedure is a technique for identifying the gradient of a linear function that involves starting from an arbitrary point on the graph and then move one unit to the right in a horizontal direction, followed by *a* units vertically in either an upward or downward direction, depending on the positivity or negativity of the gradient (Nilsen, 2013). Phrases such as "moves up on the x-axis" or "moves down on the y-axis", are considered everyday registers (Moschkovich, 1996), and even though a mathematical register might indicate a deeper understanding of steepness, such everyday terms are valuable for schoolteachers.

The outcome of analysing the ISTs responses to the open-ended tasks contrasts with Moschkovich's (1996) findings. In Moschkovich's (1996) work, students had difficulties in explaining components of linear equations, such as x-intercept, y-intercept, the parameter a and b, and their interconnections. This research underscores the challenges ISTs face in understanding and communicating the abstract concepts of linear equations in particular and reveals a higher level of proficiency among ISTs in communicating essential linear function concepts. Furthermore, the integration of tasks where ISTs generate points, plot the points in a coordinate system, and articulate their observations may serve as a promising approach to bridge gaps in mathematical comprehension.

Collective procedures in LGE tasks

The analysis of the ISTs' digital responses in STACK showed a tendency towards common mathematical procedures, such as choosing integers, turning to larger numbers when generating a unique example, labelling the parameter b as constant term and y-intercept, and the "one-unit-right-*a*-up/down" procedure for solving LGE tasks in the provided quiz. The predominant use of integers and standard methods for explaining linear functions, suggests a preference for safe procedures. This uniformity, even when asked to create a unique example, suggests a comfort with established practices. The ISTs' collective approach, possibly influenced by professional experience or coursework, highlights a shared comfort zone with mathematical concepts in the provided quiz.

Although the LGE tasks within a CAA system did not require the ISTs to develop new mathematical knowledge or engage in unfamiliar mathematical concepts, analysing their work through the lens of collective example space provided valuable insights into their preferred strategies and methods for generating examples related to linear functions. Further, the implications of this analysis extend beyond the ISTs themselves. LGE tasks within a CAA system not only encourages students to actively engage in the mathematical content but also offer valuable insights into the tendencies and preferences of a group. For lecturers and teachers, this analysis can inform the design of future lessons and assessments by highlighting patterns of how students engage with these tasks. By gaining insights from collective example space, educators can better tailor their materials to align with student needs and approaches, applicable beyond linear functions.

Personal Example Space

This section of the study focuses on the features of linear functions through the lens of personal example spaces of four ISTs, Tina, Mari, Tara, and Lina, as revealed in the interviews. It delves into exploring the procedures employed while solving LGE tasks in STACK, including how they approached the creation of function expression, choosing specific points for sums, and their systematic approach to listing points.

Procedures for generating points and plotting into a coordinate system

The STACK-quiz was divided into four sets, each comprising tasks for generating points corresponding to a sum and then plotting these points into a coordinate system. Mari and Lina viewed the generating of points and plotting them into a coordinate system as separate tasks, not recognising their interrelation. In contrast, Tina saw the link between those tasks but found it challenging. This difficulty in associating different mathematical representations is consistent with the observations of Birgin et al. (2012) and Schwarz and Dreyfus (1995), who observed that students often see functions, graphs, tables, and formulas as separate entities. These tasks had the intention of connecting pairs of numbers to the concept of linear functions, and therefore an opportunity for the participants to discover these connections. A bit surprisingly, in the first set of tasks, Tina, Mari, and Lina initially plotted points different from their calculated sums. A reason for this can be that they initially did not spot the connection between the generated points and plotting into a coordinate system. As the quiz progressed, they understood the link between the sums they generated and their graphical representation in the coordinate system. This led to a strategic use of their prior mathematical knowledge in generating sums and choosing points to plot into the coordinate system that will be presented below.

In their approach to plotting into the coordinate system, when not using already generated sums, Lina and Tara both used the procedure of starting with $x = 0$ in order to generate the value of y. They explain that, in this way, they knew one of the coordinates ($x = 0$) and could therefore use knowledge as if $x = 0$, then $x + y = 2$, then $0 + 2 = 2$, to find the y-coordinate. This demonstrates an ability to transform unknown elements of the tasks into more manageable mathematical contexts, aligning with their individual approaches with linear functions.

Lina and Tara both favoured generating points near the origin in the given tasks. However, they had different arguments for why these points were referred to as "easy". Lina explained that she visualised a table of values that contained x-values close to the origin and said that this would provide her with the y-value. This might suggest a preference for a traditional method of approaching linear functions and seemed to provide her a sense of security and familiarity when generating points. In contrast, Tara explained that one approach for her was to generate points by choosing y-values centred around the intersection of the y-axis. This procedure would provide her with what she referred to as "easy" points. Lina and Tara's preference for points near $x = 0$, each for their own distinct reasons, reflects a strategy to bring the mathematical tasks into more recognisable forms and thus to make them easier to calculate.

Having this focus, they reveal an adaption of linear functions concepts to fit within their established mathematical comprehension.

Different representations and communication

Mari's approach diverged from the others. She explained that in the initial sets she did not spot the connection between generating sums and plotting points, her primary focus was to find the expression of the linear equation. Then use the function expression to generate points to plot in the coordinate system. This method suggests a mathematical approach to working with linear equations. This contrasts with Tina's approach, which involved selecting more points than necessary when plotting each case into the coordinate system. Tina explained that she had to check several points in order to understand the direction of the line in the coordinate system. This might indicate a procedure to convince herself about the correct linear function and therefore, serve as a self-verification. Tina's approach has similarities with what Antonini (2006) described as the "trial and error" strategy, commonly seen among students' initial approach to LGE tasks. This approach, however, is often quickly abandoned as they gain more experience. The preference for certain types of representations among the ISTs aligns with Birgin et al. (2012), who observed that students often favour specific representations.

This study highlighted that ISTs struggled with effectively communicating mathematical explanations, a key component in mathematics education. Effective communication encompasses more than correct answers; it involves articulating the reasoning behind the answers. Dinkelman and Cavey (2015) suggested that engagement with LGE tasks can improve communication skills over time. However, this study, being a single quiz, could not assess improvement in communication skills, suggesting the need for further, longitudinal research in this domain.

Personal example spaces though LGE tasks

The ISTs provided some distinctive mathematical procedures for solving the LGE tasks in this study. Tara's use of fractions and focus on function expression indicates her preference for mathematical terminology in her example spaces. Mari's approach, focusing on the application of function expressions, reveals a mathematical mindset. Tina, who relied on additional points for verification appears to demonstrate some mathematical uncertainty. Additionally, Lina's examples and remarks show her adherence to the specific rules of the tasks. Viewing the ISTs' solutions to LGE tasks in STACK, combined with the in-depth interviews through the perspective of "richness" and "accessibility and correctness", they demonstrated a diverse range of approaches and logical structures in their task solutions. These different procedures offer insight into the individual understanding and the contexts they utilised while solving the tasks. In addition, the varied approaches demonstrate varied interpretations and applications of features related to linear functions.

Nevertheless, it is important to highlight that the procedures employed by these ISTs while solving the LGE tasks in STACK were only accessible through the interviews. Only examining the digital answers within STACK for these tasks would not provide this level of insight beyond the final written answers. To gain deeper understanding, one must engage in direct communication with the ISTs or consider tasks revisions that could clarify the procedures more transparently.

Limitations

Although the small sample size of seven ISTs limits the ability to generalise the results, there are certain characteristics and aspects of these tasks that may be relevant and valuable to all mathematics students and lecturers. This study focuses on how LGE tasks within a system for CAA facilitate ISTs' engagement with and exploration of the features of linear functions. While the participants of this study were ISTs, the findings suggest that such tasks could also be beneficial for others studying mathematics. Since LGE tasks encourage students to generate examples that meet specific criteria, this could be an effective method for students to apply these concepts in various contexts. This is important for everyone studying mathematics, and in particular mathematics teachers, as this approach has the potential to foster greater creativity and enabling them to apply their knowledge.

Though one objective for practicing LGE tasks is that mathematics teachers should be able to provide examples for their students spontaneously, this study did not investigate whether working with LGE tasks through a CAA system directly results in developing the actual skill. It is worth noting that there has been little focus on practicing this skill in the past, and therefore, using LGE tasks could serve as a practical approach to develop this ability.

The selection of tasks should also be considered. Based on the experience from this study, the difficulty level of the tasks should be carefully designed to reflect the students' abilities. The examples provided should neither be too obvious nor too difficult, as this significantly influences how student approach and engage with the tasks. Although the tasks were well-suited for this group of teachers, incorporating a greater variation of the tasks could allow for understanding the example spaces of different aspects of linear functions. To identify more generalisable patterns, this study could be expanded to include a larger group of teachers and broader range of tasks.

Lastly, employing a system for CAA to implement LGE tasks involve a new context for this group of ISTs. It is possible that the results might have differed if the tasks were presented in a paper format or within in a classroom setting. This is a potential issue concerning the generalisability of the results to other contexts.

Concluding Remarks

This study explored how LGE tasks within a CAA system facilitate ISTs' engagement with and exploration of features of linear functions. This exploration is made possible through the notion of collective and personal example spaces. Through this study, these tasks offer a rich context for exploring characteristics, specific qualities, and procedures utilised by ISTs concerning the concept of linear functions. The study highlights the ISTs' preference for familiar procedures and underscores the importance of clear and effective mathematical communication. By revealing the diverse approaches of ISTs, the study demonstrated the potential of LGE tasks in STACK to enrich understanding of linear functions.

Notably, LGE tasks delivered via a CAA system have demonstrated their potential to offer educators, whether teachers or lecturers, insightful perspectives into the collective mathematical understanding of student groups. The automated corrections feature inherent in CAA systems is particularly valuable, as it allows for the assessment of collective comprehension across groups of any size. This insight could be important in guiding future teaching, allowing for tailored instruction that addresses the collective needs and trends observed within the group of students.

This study indicates that it could be beneficial with a supplement to the tasks in the CAA system to get a better understanding of individual processes when solving the tasks. Relying solely on the examples provided within the STACK system may not always reveal the full spectrum of procedures employed by the ISTs, leaving room for more profound reflections than what is captured in the digital responses.

This paper highlights a pivotal aspect of mathematics education that remains underemphasised in teacher training programs, the skilful creation of mathematical examples. The exploration of ISTs engagement with LGE tasks in a CAA system underscores the potential of such tasks not only to enrich the ISTs' repertoire of examples but also to enhance their understanding of linear functions. By embedding such tasks within teacher education programs have the potential to foster a more practice-oriented training that active involve the ISTs in their learning processes. Such strategic incorporation does not only have the potential to equip ISTs with essential pedagogical tools but also prepare them for their forthcoming roles in schools.

This work has made a valuable contribution to enhancing our comprehension of e-assessment in mathematics, with a specific focus on exploring ISTs' generated examples in a CAA system using the concept of example spaces. It suggests the inclusion of LGE tasks as an integral component of formative digital assessments, offering an insight into valuable information on collective and individual levels of mathematical comprehension.

Corresponding author

Siri Ovedal-Hakestad
University of Agder
Norway
siri.ovedal-hakestad@uia.no

Ethical approval

Ethical approval for the research was granted and informed consent was given by all participants for their data to be published.

Competing interests

The authors declare there are no competing interests.

References

Alcock, L., & Inglis, M. (2008). Doctoral students' use of examples in evaluating and proving conjectures. *Educational Studies in Mathematics*, *69*(2), 111–129. https://doi.org/10.1007/s10649-008-9149-x

Antonini, S. (2006). Graduate students' processes in generating examples of mathematical objects. In J. Novotná, H. Moraová, M. Krátká, & N. Stehlíková, *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 57–64). PME.

Ball, D. L. (2003). What mathematical knowledge is needed for teaching mathematics? *Secretary's Summit on Mathematics, U.S. Department of Education*.

Birgin, O. (2012). Investigation of eighth-grade students' understanding of the slope of the linear function. *Bolema: Boletim de Educação Matemática*, *26*, 139–162. https://doi.org/10.1590/S0103-636X2012000100008

Birgin, O., Gurbuz, R., & Catlıoglu, H. (2012). Determining eighth grade students' understandings and difficulties of linear functions. *Energy Education Science and Technology Part B: Social and Educational Studies*, *4*(3), 1345–1354.

Breen, S., & O'Shea, A. (2019). Designing mathematical thinking tasks. *PRIMUS*, *29*(1), 9–20. https://doi.org/10.1080/10511970.2017.1396567

Breen, S., O'Shea, A., & Pfeiffer, K. (2016). Students' views of example generation tasks. *Teaching Mathematics and Its Applications: International Journal of the IMA*, *35*(1), 27–40. https://doi.org/10.1093/teamat/hrv017

Brunström, M., Fahlgren, M., Vinerean, M., & Wondmagegne, Y. (2022). Designing for a combined use of a dynamic mathematics software environment and a computer-aided assessment system. In J. Hodgen, E. Geraniou, G. Bolondi, & F. E. Ferretti, *Proceedings of the Twelfth Congress of the European Research Society in Mathematics Education* (CERME12).

Dinkelman, M. O., & Cavey, L. O. (2015). Learning about functions through learner-generated examples. *The Mathematics Teacher*, *109*(2), 104–110.

Fahlgren, M., & Brunström, M. (2023). Designing example-generating tasks for a technology-rich mathematical environment. *International Journal of Mathematical Education in Science and Technology*, 1–17. https://doi.org/10.1080/0020739X.2023.2255188

Goldenberg, P., & Mason, J. (2008). Shedding light on and with example spaces. *Educational Studies in Mathematics*, *69*, 183–194. https://doi.org/10.1007/s10649-008-9143-3

Iannone, P., Inglis, M., Mejía-Ramos, J. P., Simpson, A., & Weber, K. (2011). Does generating examples aid proof production? *Educational Studies in Mathematics*, *77*, 1–14. https://doi.org/10.1007/s10649-011-9299-0

Jukic Matic, L., Kehler-Poljak, G., & Rukavina, S. (2022). The influence of curriculum on the concept of function: An empirical study of pre-service teachers. *European Journal of Science and Mathematics Education*, *10*(3), 380–395. https://doi.org/10.30935/scimath/12042

Kinnear, G. (2024). Comparing example generation with classification in the learning of new mathematics concepts. *Research in Mathematics Education, 26*(1), 109–132. https://doi.org/10.1080/14794802.2022.2152086

Kinnear, G., Jones, I., Sangwin, C., Alarfaj, M., Davies, B., Fearn, S., Foster, C., Heck, A., Henderson, K., Hunt, T., Iannone, P., Kontorovich, I., Larson, N., Lowe, T., Meyer, J. C., O'Shea, A., Rowlett, P., Sikurajapathi, I., & Wong, T. (2024). A collaboratively-derived research agenda for e-assessment in undergraduate mathematics. *International Journal of Research in Undergraduate Mathematics Education, 10*(1), 203–231. https://doi.org/10.1007/s40753-022-00189-6

Leinhardt, G. (2001). Instructional explanations: A commonplace for teaching and location for contrast. In V. Richardson (Ed.), *Handbook of research on teaching* (4th ed., pp. 333–357).

Lloyd, G. M., & Wilson, M. (1998). Supporting innovation: The impact of a teacher's conceptions of functions on his implementation of a reform curriculum. *Journal for Research in Mathematics Education*, *29*(3), 248–274. https://doi.org/10.5951/jresematheduc.29.3.0248

Moschkovich, J. N. (1996). Moving up and getting steeper: Negotiating shared descriptions of linear graphs. *The Journal of the Learning Sciences*, *5*(3), 239–277.

Nilsen, H. K. (2013). *Learning and teaching functions and the transition from lower secondary to upper secondary school.* [Doctoral dissertation, University of Agder]. http://hdl.handle.net/11250/139758

Ovedal-Hakestad, S., & Larson, N. (2024). *Pre-service teachers' experiences of working with learner-generated example tasks through a computer-aided assessment system.* Presented at NORMA24: The Tenth Nordic Conference on Mathematics Education.

Pierce, R., Stacey, K., & Bardini, C. (2010). Linear functions: Teaching strategies and students' conceptions associated with $y = mx + c$. *Pedagogies: An International Journal*, *5*(3), 202–215. https://doi.org/10.1080/1554480X.2010.486151

Rønning, F. (2017). Influence of computer-aided assessment on ways of working with mathematics. *Teaching Mathematics and Its Applications: International Journal of the IMA*, *36*(2), 94–107. https://doi.org/10.1093/teamat/hrx001.

Sangwin, C. (2013). *Computer aided assessment of mathematics*. OUP Oxford.

Sangwin, C. (2019). Developing and evaluating an online linear algebra examination for university mathematics. In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education*. Utrecht University and ERME.

Sangwin, C. J., & Köcher, N. (2016). Automation of mathematics examinations. *Computers & Education*, *94*, 215–227. https://doi.org/10.1016/j.compedu.2015.11.014

Schwarz, B., & Dreyfus, T. (1995). New actions upon old objects: A new ontological perspective on functions. *Educational Studies in Mathematics*, *29*(3), 259–291. https://doi.org/10.1007/BF01274094

Sinclair, N., Watson, A., Zazkis, R., & Mason, J. (2011). The structuring of personal example spaces. *The Journal of Mathematical Behavior*, *30*(4), 291–303. https://doi.org/10.1016/j.jmathb.2011.04.001

Watson, A., & Mason, J. (2006). *Mathematics as a constructive activity: Learners generating examples*. Routledge.

Yerushalmy, M., Nagari-Haddif, G., & Olsher, S. (2017). Design of tasks for online assessment that supports understanding of students' conceptions. *ZDM Mathematics Education*, *49*(5), 701–716. https://doi.org/10.1007/s11858-017-0871-7

Zaslavsky, O. (2019). There is more to examples than meets the eye: Thinking with and through mathematical examples in different settings. *The Journal of Mathematical Behavior*, *53*, 245–255. https://doi.org/10.1016/j.jmathb.2017.10.001

Zazkis, R., & Leikin, R. (2007). Generating examples: From pedagogical tool to a research tool. *For the Learning of Mathematics*, *27*(2), 15–21.

Zazkis, R., & Leikin, R. (2008). Exemplifying definitions: A case of a square. *Educational Studies in Mathematics*, *69*, 131–148. https://doi.org/10.1007/s10649-008-9131-7

Zodik, I., & Zaslavsky, O. (2008). Characteristics of teachers' choice of examples in and for the mathematics classroom. *Educational Studies in Mathematics*, *69*(2), 165–182. https://doi.org/10.1007/s10649-008-9140-6