Mathematics Teacher Education and Development

2001, Vol. 3, 28-36

Teachers Content Knowledge, Teacher Education, and their Effects on the Preparation of Elementary Teachers in the United States

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This paper summarises and critiques research on the role of mathematics content knowledge in the preparation and teaching practice of elementary (K-8) teachers in the United States. Research conducted over the last 40 years has given us snapshots of teachers' knowledge at particular points in time, such as during their preservice methods courses. The paper calls for future research to give us longitudinal "videotapes" of teachers' knowledge and how it is developed and used in a variety of contexts.

> Every study or subject thus has two aspects: one for the scientist as a scientist; the other for the teacher as teacher. These two aspects are in no sense opposed or conflicting. But neither are they immediately identical. (Dewey, 1990, p. 200)

In recent years, policymakers in the United States have focused a great deal of time and attention on teacher preparation. In particular, many policy documents are based on the logical assumption that teachers' content knowledge has a significant influence on student learning. For example, the American Council on Education (ACE) recently proclaimed that "A thorough grounding in college-level subject matter and professional competence in professional practice are necessary for good teaching. The data are unequivocal: students learn more mathematics when their teachers report having taken more mathematics " (ACE, 1999, p. 6). The ACE report presents data that suggest that earning a college degree in mathematics, being certified in mathematics, and being mathematically skillful "all contribute to effective teaching of mathematics" (p. 6). Many national policy organisations have created documents specifying the content that teachers should know in order to be effective teachers, (e.g., ACE, 1999; Conference Board of the Mathematical Sciences, 2000; Interstate New Teacher Assessment and Support Consortium, 1993; Leitzel, 1991; National Commission on Teaching & America's Future, 1996; National Board for Professional Teaching Standards, 1991; National Council of Teachers of Mathematics, 1991). At the state level, teacher preparation programs are being influenced by mandates regarding the number and nature of mathematics courses that teachers must complete. It is thus timely to investigate the empirical evidence that might serve as a warrant for these policy decisions.

The issue of what types of knowledge are essential for teaching mathematics in the elementary school has been the subject of numerous conceptual essays and empirical studies for the last 40 years. Research upholds Dewey's claim that knowledge for teaching is different from knowledge for "doing" in a discipline. Merely "knowing" more mathematics does not ensure that one can teach it in ways that enable students to develop the mathematical power and deep conceptual understanding envisioned in current reform documents (e.g., National Council of Teachers of Mathematics, 2000).

Historical Overview

Five major research genres are distinguishable in the literature on teachers' knowledge, and these genres follow a roughly chronological pattern. The earliest studies, conducted in the 1960s and 1970s, were quantitative studies that sought to demonstrate a connection between teachers' knowledge and student achievement (e.g., Begle, 1972, 1979; Eisenberg, 1977). These studies failed to find any statistically significant correlation between measures of teacher knowledge (such as number of mathematics courses taken, major in mathematics, grade point average) and student achievement. Although these studies have been roundly criticised for taking a naïve and simplistic view of teachers' knowledge by using such gross measures as number of courses taken, there has been little effort in the intervening 20 years to develop more appropriate research methods to answer the question about the relationship between teachers' knowledge and students' knowledge.

The 1960s, 1970s, and 1980s saw a flurry of descriptive studies that attempted to characterise the strengths and weaknesses in teachers' knowledge of particular content areas, such as fractions or geometry (e.g., Baturo & Nason, 1996; Graeber, Tirosh, & Glover, 1989; Post, Harel, Behr, & Lesh, 1991; Simon, 1993; Simon & Blume, 1994; Tirosh, Fischbein, Graeber, & Wilson, 1999). Most of these studies were conducted using a combination of quantitative and qualitative research methods. Many studies administered written surveys to large numbers of teachers and conducted follow-up interviews with smaller numbers of teachers. The overwhelming majority of these studies were conducted with preservice teachers. These studies suggest that while elementary teachers generally (although not always) have a command of the facts and algorithms that comprise school mathematics, they lack a conceptual understanding of this mathematics. Their knowledge tends to be compartmentalised and fragmented and, therefore, not easily transferable from one domain to another.

The dismal results of the descriptive studies spawned comparison studies that compared the knowledge of elementary versus secondary teachers, preservice versus inservice teachers, and U.S. teachers versus teachers from other countries (e.g., Ball, 1991, Ball & Wilson, 1990, Fuller, 1997, Ma, 1999). These studies generally employed the same quantitative and qualitative methods as the descriptive studies. The comparison studies showed that while there are some slight differences between various populations, the conceptual knowledge of all populations is uniformly low.

Over time, researchers have come to recognise that the issues surrounding teachers' knowledge, in general, and its implementation in classroom practice, in particular, are multifaceted and complex. Within the last decade, there have been a number of studies that have attempted to capture this complexity by conducting qualitative studies of small numbers of teachers engaged in teaching practice (Borko, et al, 1992; Eisenhart, Borko, Underhill, Brown, Jones, & Agard, 1993; Fernández, 1997; Heaton, 1992, 1995, 2000; Leinhardt & Smith, 1985; Putnam, Heaton, Prawat, & Remillard, 1992, Thompson & Thompson, 1994, 1996). These studies have shown that the relationship between knowledge and teaching practice is anything but straightforward. While a number of elementary teachers with weak content knowledge are predisposed to telling students rules and explaining algorithmic procedures, a number of teachers with strong content knowledge behave similarly.

Teachers' Content Knowledge and Teaching Practice

In most cases, this literature shows that teachers are able to successfully perform computations. However, many teachers are unable to provide conceptual explanations for the procedural tasks they perform. For example, a common finding of these studies is that preservice teachers lack an understanding of quotitive (measurement) division and are prone to rely only on a partitive (sharing) interpretation of division[1] (Ball, 1990; Graeber et al., 1989; Simon, 1993). This becomes particularly problematic in the case of division of fractions where it is almost impossible to make sense of the underlying ideas using a partitive interpretation of division. Many teachers are unable to generate a word problem for a whole number divided by a fraction, often providing a problem that represents a multiplication situation (Borko et al., 1992; Ma, 1999). Teachers tend to rely on their knowledge of whole numbers when working in the domain of rational numbers (Tirosh et al., 1999). This overgeneralisation from one number system to another leads to misconceptions and impoverished ideas about rational numbers (such as the claim that multiplying two numbers results in a product that is larger than either of the two numbers, a claim that is true for whole numbers but false for rational numbers). Further, many teachers do not know the difference between a ratio and a fraction, believing that because they can be represented with the same notation they behave in identical ways (Fuller, 1997; Leinhardt & Smith, 1985).

Another common finding from this literature is that teachers confuse the concepts of area and perimeter (Baturo & Nason, 1996; Fuller, 1997; Heaton, 1992), frequently assuming that there is a constant relationship between area and perimeter. Further, teachers often do not use appropriate units when computing area and perimeter, commonly failing to use square units when reporting measures of area (Baturo & Nason, 1996; Simon & Blume, 1994).

The studies cited above lead to the conclusion that many elementary teachers do in fact lack a conceptual understanding of the mathematics they are expected to teach. However, with few exceptions the literature cited above fails to document that the participants *had the opportunity* to learn mathematics conceptually somewhere in their teacher preparation programs. There are few studies that illuminate the possibilities for enhancing teachers' mathematical knowledge and their teaching practice.

Educators at Michigan State University designed a three course sequence of mathematics content courses for elementary education majors and studied students

[1] See Ball (1990) for a thorough explanation of partitive and quotitive division.

during the classes, during their student teaching, and during their first year of teaching. Schram, Wilcox, Lanier, and Lappan (1988) reported some success in helping preservice teachers expand their conceptual understanding during the mathematics content courses. They found that students developed a conceptual understanding of many of the facts, formulas and rules that they had previously memorised as a result of taking a course that emphasised problem solving, reasoning, discourse, group work, and the use of multiple representations. However, when they followed these teachers into their first year of teaching, they found that some of them struggled to replicate their own learning experiences in their classrooms. Others did not appear to attempt to replicate the type of learning environment they had experienced. The researchers were led to conclude:

> Disciplinary study is necessary to develop in novice teachers a set of intellectual tools and a disposition to engage in mathematical inquiry themselves. But disciplinary study alone may be insufficient…to develop in beginning teachers the knowledge, skills, and beliefs to conceive of teaching as something other than telling or as more than a matter of technical competence. (Wilcox, Lanier, Schram, & Lappan, 1992, p. 23)

This finding lends further credence to the argument that a number of different types of knowledge interact when a teacher makes decisions. Although these teachers possessed some level of desirable knowledge of mathematics, they lacked adequate knowledge of mathematics as a discipline and/or pedagogical content knowledge to enable them to teach mathematics in ways consistent with current reform efforts. Clearly, knowing mathematics for oneself is not the same as knowing how to teach it.

Teacher Education Courses and Teaching Practice

It is tempting to conclude that these studies suggest that prospective elementary teachers need to study more mathematics. However, similar studies have been conducted with prospective secondary teachers, and these studies show that the problem of weak conceptual knowledge of school mathematics is not confined to elementary teachers. For example, Even (1993) found that prospective secondary teachers held an equation concept of functions, expected the graphs of functions to be smooth and continuous, and were unable to provide an explanation of the univalence requirement for functions. The students knew that the vertical line test was a procedural way of determining whether a graph represented a function, but they were unable to provide a conceptual explanation for why univalence is necessary.

Further, studies comparing the mathematical knowledge of prospective elementary and secondary teachers show that secondary teachers' conceptual knowledge of elementary mathematics is not significantly stronger than that of their elementary counterparts. For example, Ball (1990, 1991) compared the mathematical knowledge of preservice elementary education majors and preservice secondary mathematics education majors on the topics of division (including division of fractions, division by zero, and division in algebraic expressions) and place value in the multiplication of large numbers. The secondary majors were more successful at obtaining correct answers than the elementary majors, but they were not adept at explaining the reasons behind the rules they invoked and their knowledge was not connected across various contexts. Thus, Ball concluded that although the secondary mathematics majors had successfully completed a number of advanced mathematics courses, this academic preparation did not provide them with "the opportunity to revisit or extend their understandings of arithmetic, algebra, or geometry, the subjects they will teaching" (p. 24). She further noted that simply requiring more mathematics of prospective teachers will not increase their understanding of school mathematics. Rather, a *different kind of mathematics* is needed.

Alternative certification programs (for those already holding a bachelors degree in a content area) have gained popularity in recent years as the need for more teachers rises. Ball and Wilson (1990) compared the mathematical content knowledge of students in traditional teacher education programs and alternative route certification programs at both the entry and exit points of the programs. The mathematics content of the study dealt with the relationship between perimeter and area, proof by example, division by zero, and division of fractions. Upon entry to the teacher education programs, neither group was able to explain the mathematics underlying the problems presented, and there were no significant differences between the groups. At the conclusion of the teacher education programs, both groups showed increased evidence of mathematical understanding, but again there were no significant differences between the groups. Ball and Wilson concluded that neither group had "opportunities to unpack mathematical ideas or to make connections" (p. 7) and that neither group was prepared to teach mathematics for understanding. Their findings support Ball's (1990, 1991) claim that requiring teachers to study more traditional mathematics is not the answer as students who have pursued this course of study are not substantially better prepared to teach school mathematics.

It is striking to read the comments from prospective teachers as they are asked to solve mathematical problems or as they engage in reflecting on their teaching. In many cases, these teachers are fully aware that they lack a conceptual understanding of mathematics. For example, one student teacher noted, "I don't just like saying 'Well, this is *pi*. Remember it,' … [but] where does *pi* come from? Well, I don't know." (Eisenhart et al., 1993, p. 18). Another preservice teacher noted, "I am really worried about teaching something to kids I may not know. Like long division–I can *do* it–but I don't know if I could really *teach* it because I don't know if I really *know* it or know how to *word* it" (Ball, 1990, p. 104). It is to their credit that these future teachers are aware of, and concerned about, their mathematical competence and its potential impact on their teaching.

Critique of Existing Research

Three weaknesses in the research on the nature of teachers' mathematical knowledge are worthy of note. First, these research studies have addressed a fairly narrow range of mathematics content areas. The topics of place value, division, rational numbers (more specifically, fractions, with considerably less attention to decimals and ratios), and geometry (focusing almost exclusively on area and perimeter) have been addressed by numerous researchers. Perhaps it has been taken for granted that teachers understand addition and subtraction of whole numbers, patterns, and counting–fundamental topics in the kindergarten and first grade curricula. A number of more contemporary mathematical topics (such as probability, data analysis, functions, transformational geometry, number theory) have been addressed by only a few researchers. Given the recent emphasis on elementary mathematics as more than arithmetic, it seems necessary to know more about teachers' knowledge in domains other than number.

For example, an area of mathematical understanding that seems to be crucial to enabling teachers to enact current reform visions, but that has received limited attention in the research literature, is preservice elementary teachers' understanding of mathematical justification. If teachers are to orchestrate discourse in their classrooms and encourage students to share their emerging mathematical ideas, teachers must have a sense of what constitutes a valid mathematical argument (Ball, 1994). Studies suggest that preservice teachers are prone to accept inductive evidence, such as a series of empirical examples or a pattern, as a sufficient proof (Martin & Harel, 1989; Simon & Blume, 1996). Simon and Blume (1996) paint a vivid picture of the challenges and opportunities of engaging a class of preservice teachers in mathematical arguments. Studies such as this one illuminate the nature of preservice teachers' thinking and demonstrate how their thinking impacts, and can be impacted by, instruction.

Second, these studies generally present "snapshots" of teachers' knowledge at a particular point in time. Few studies provide a longitudinal "videotape" of teachers' knowledge and how it changes over time. Mathematics teaching is an intricate and complicated endeavor, and we need data that captures how, when and where teachers' mathematical knowledge is accessed, applied, and changed. Research that shows us how teachers' knowledge changes as a result of coursework and teaching experience can help us strengthen opportunities for learning. For example, the two volumes edited by Schifter (1996a, 1996b) contain rich detail about teachers' experiences in professional development programs and in their classrooms and provide insight into the elements that were forces for change. Similarly, Heaton's book (2000) about her experiences in both the classroom and teacher education provide details about her struggles with mathematics and how the mathematics she knew applied to the mathematics of the school curriculum. Studies of this nature are necessarily small in scope in terms of numbers of participants, but if we gather enough evidence from such studies, a summary analysis will be possible.

Third, research conducted in this genre has generally failed to provide us with rich data about teachers who *do* possess strong conceptual knowledge of mathematics. We have very few examples of the reasoning of teachers who are able to think through problems and provide suitable explanations. Most of these studies report that 50% or fewer of the teachers studied lacked conceptual understanding of mathematics. However, we rarely read any data about the other 50% of the teachers who did possess some conceptual understanding of the mathematics. It would be enlightening to see examples of teachers with strong mathematics content knowledge and an analysis of what mathematics and what reasoning processes these teachers use to solve novel problems. Further, it would be useful to know how, when, and where these teachers developed this conceptual understanding. Studies that show us what teachers who can do and use mathematics look like can suggest avenues for change and areas in which to invest resources.

Directions for Future Research

There is no clearly defined body of knowledge that informs teaching. Rather, teachers need multiple types of knowledge, each of which is rather ill-defined and amorphous. Because of the enormous complexity of teaching and learning, a new approach to research on teacher knowledge is needed. An approach that combines a variety of perspectives - those of mathematics educators, mathematicians, sociologists, psychologists, anthropologists and others - is needed in order to bring a richness to the research that begins to parallel the richness that exists in teachers' knowledge.

It seems clear that we need more in-depth studies of teachers in action in various contexts as learners of mathematics and as teachers of mathematics. Studies that are longitudinal in nature, that provide us with "videotapes" rather than just snapshots of teachers' knowledge, are needed to enhance our understanding. However, as we begin to accumulate a substantial collection of studies that investigate in detail the knowledge and practice of individual teachers, we must guard against viewing these studies as simply a collection of stories. Researchers must return to these stories and conduct cross-case analyses in order to begin to develop a theory about teachers' knowledge, teachers' practice, and student learning. As Cooney (1994) noted, "if we are to move beyond collecting interesting stories, theoretical perspectives need to be developed that allow us to see how those stories begin to tell a larger story" (p. 627).

The study of teaching and teachers' knowledge is as important to educational reform today as it was 40 years ago. As Shulman (1983) noted, teachers are the key ingredient in our educational system.

> ...the teacher must remain the key. The literature on effective schools is meaningless, debates over educational policy are moot, if the primary agents of instruction are incapable of performing their functions well. No microcomputer will replace them, no television system will clone and distribute them, no scripted lessons will direct and control them, no voucher system will bypass them. (p. 504)

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