Bridging the Research-Practice Gap: Developing a Pedagogical Framework that Promotes Mathematical Thinking and Understanding

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This paper documents the findings of a three-phase research project that led to the development of a teaching/learning framework for promoting thinking and understanding in mathematics classrooms. The framework characterises students and teachers as co-learners in the classroom environment, and is the result of a school/university partnership. The partnership explored pedagogical practices using "accounts of practice" that are based on teachers' perceived reality, grounded in classroom contexts and viewed through the conceptual lenses of a researcher. The pedagogical framework contributes to a growing body of empirically based knowledge that seeks to bridge the gap between theory and practice in mathematics education.

Issues associated with the transfer and application of research findings to school-based practice have been acknowledged in recent literature related to research in mathematics education (Malone, 2000, Richardson, 1997). The need for the collection of authentic data from classroom contexts, combined with the interpretation of that data from a theoretical and a practical perspective have been flagged by many mathematics educators (Fraivillig, Murphy & Fuson, 1999; Schoenfeld, 1999; Simon & Tzur, 1999; Steffe & D'Ambrosio, 1995). These "flags" are linked with the need to translate these interpretations into useful images of pedagogical practice that help to bridge the gap between research and practice. In turn, these images of practice can be used to guide current and pre-service teachers towards constructing their own models of reality in relation to mathematics teaching and learning.

The research reported in this paper seeks to address the research-practice gap in two ways. Firstly, the practical, classroom-based nature of this inquiry sought to provide authentic and grounded descriptions of pedagogical practices that were based on the lived experiences of teachers. This was made possible by the formation of a collaborative school/university partnership (Bobis, 1998), and the use of teachers' accounts of their perceived reality viewed through the conceptual lenses of a researcher and described as "accounts of practice" (Simon & Tzur, 1999). This methodology was chosen to "explore theoretical issues in contexts that really matter" (Schoenfeld, 1999, p. 14) to increase the authenticity of the findings. Secondly, these accounts of practice were used to develop a formal knowledge base of what mathematics classrooms might look like when teachers seek to promote mathematical thinking and understanding. The recurrent themes that emerged from the data collected in this study led to the formation of a pedagogical framework that was grounded in classroom practice. The framework provides a rich description of the instructional strategies and assessment practices evident in classrooms that reflect social constructivist principles of learning and teaching. The framework aims

to meet the needs of practising and pre-service teachers by providing images of teaching that promote mathematical thinking.

Theoretical Perspectives

The assumptions in this paper reflect a social constructivist perspective of teaching and learning, and a corresponding belief that assessment should be an integral and constructive part of the learning process. This perspective acknowledges that:

- learning involves the active construction of knowledge through personal experience and is influenced by prior knowledge as well as student and teacher attitudes and approaches towards learning (Ernest, 1994; Middleton & Spanais, 1999);
- learning does not occur in isolation and is not fixed, but rather it is socially negotiated and expressed through language that focuses on explanation and clarification (Ernest, 1994; Yackel, Cobb & Wood, 1992);
- learning is enhanced through collaboration with more knowledgeable others through a scaffolding process where learners progress from assisted to independent performance (Brown, Ellery & Campione, 1998; Hogan, 1997; Vygotsky, 1978); and
- assessment is an integral part of the learning process and should be consistent with learning principles (Bransford, Brown & Cocking, 1999; Clarke, 1997; De Lange, 1995; Yackel, et al. 1992).

However, the principles of social constructivism do not dictate specific teaching methods (Simon, 1995). Therefore, it could be argued that classroom and pre-service teachers seeking to merge their beliefs about learning with their pedagogical practices could benefit from the abstraction of shared practices that reflect the principles of social constructivist learning and teaching. The essence of these principles suggests that students need to make sense of the information they are constructing through socially interactive processes so that understanding becomes the goal of learning.

Teaching for Understanding

Mathematics needs to be seen as a way of ordering and explaining our everyday lives and the problems with which we are confronted. Over the last three decades there has been a continued belief that the mathematical experiences we plan for students at school should be relevant and meaningful to their everyday lives. Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier and Human (1997), in their synthesis of research on teaching for understanding, indicated that we understand something if we see how it is related or connected to other things we know. They go on to suggest that students make connections to other things they know through reflection and communication. If communication and reflection foster the development of connections, then it stands to reason that classrooms that provide for these experiences will facilitate understanding (Fraivillig, Murphy & Fuson, 1999; Hiebert, et al. 1997; Silver & Smith, 1997). Similarly, the teacher's role in such classrooms can become more defined with these goals in mind.

The focus of this study was to describe the types of instructional and assessment strategies to emerge from the collected data that presented opportunities for student reflection and communication. Identifying the strategies that promoted mathematical thinking and understanding led to the formation of a pedagogical framework. The goal of such a framework is not to dictate teaching practice, but to provide a guide that builds on, and is consistent with identified theoretical constructs.

Background

Currently in New South Wales (NSW) primary schools, and other educational systems across Australia, teachers are required to incorporate Working Mathematically outcomes into the teaching and learning of mathematics. These new outcomes provide another strand within the syllabus that highlights the importance of student-centred learning and thinking mathematically, in other words, students making sense of mathematics. In effect, these outcomes are providing a framework for pedagogical practice in mathematics classrooms that reflects current constructivist principles of learning.

The outcomes are designed to highlight students' thinking and learning processes and include a focus on student questioning, problem solving, communicating, verifying, reflecting and using technology (Board of Studies, 1998). The study did not seek to test any predetermined hypotheses. Instead it sought to describe teachers' perceived reality as they attempted to integrate the new outcomes and experienced the process of change in their teaching and assessment practice.

Research Design and Analysis

In keeping with the goal of documenting teachers' accounts of practice, a qualitative design that focused on description and interpretation was deemed appropriate. A variety of techniques for data collection and analysis were selected. A collaborative action research approach was complemented by the use of classroom observations and pre- and post-lesson interviews (Kemmis & McTaggart, 1988; Wolcott, 1992), and case study research (Gall, Borg & Gall, 1996).

The study evolved over an eighteen-month period and comprised three phases that produced three sets of data. The first two phases sought to identify the pedagogical practices that promoted students' mathematical thinking, and provided evidence of addressing the Working Mathematically outcomes. Data were gathered from a total of six classrooms during the first two phases of the study. The third phase looked at the effectiveness of current and changed assessment practices in identifying the quality of students' mathematical thinking, and how these assessment practices were linked to the teaching and learning of mathematics. This phase focused on four of the previous participants (see Smith, 2000a for an extensive account of the findings from each phase).

The methods of analysis in this study have strong parallels with methods described by Simon and Tzur (1999). Teacher perceptions were analysed using line-by-line analysis and annotated transcripts to identify emerging themes. The focus on using teachers' perceptions of their practice and describing them from a

researcher's perspective in relation to current knowledge in the field "contribute(s) to the growth of empirically based knowledge in the field" (p. 254). Simon and Tzur suggested that this method of accounting for the teacher's perspective from the researcher's perspective used particular conceptual lenses that may not occur to teachers in the field, but are of theoretical importance to the communities of mathematics education researchers and teacher educators. This method highlights the benefits of nurturing a school/university partnership that will allow for practical inquiries that are embedded in real contexts and lead to descriptions of what social constructivist classrooms might look like. The teachers in this study saw themselves as life-long learners who were constantly striving to improve their classroom practice so that their beliefs and practices continued to merge together. As such, their descriptions may benefit other practising and pre-service teachers as well as mathematics educators.

Phase One

The first phase documented the action research undertaken by the current author as a teacher-researcher (Bobis, 1998) who was teaching Year 6 in a small rural central school in NSW over a six-month period. During this phase of the project the teacher-researcher developed a descriptive account of her pedagogical practices using a reflective journal, student work samples and programming documents for data collection and analysis. The analyses produced some important insights for classroom teachers seeking to promote thinking and understanding in their classrooms. The key elements to emerge were:

- Classroom instructional strategies: these strategies involved: making connections with other concepts and prior knowledge using class concept maps; focusing on open-ended tasks that catered for all learning styles; the use of student and teacher modelling of solutions using the "think-aloud" strategy; and explicit teaching of thinking and reflective practices.
- Assessment strategies that made thinking more visible: these strategies were integrated with instruction and aimed to promote thinking, reasoning, and communication, as well as make thinking more visible. They included: observation grids; students' written procedures and verification of solutions; revisiting work samples to allow for student self-corrections and providing feedback for both the students and the teacher. Student concept maps were also used to "bring thinking out" and establish a shared vocabulary that could assist students to put their thinking on paper. These maps also allowed for assessment of prior knowledge and misconceptions, and often provided direction for instruction.

A vital component of this approach was sharing the purpose and criteria for assessment activities with the students so they had clear goals for learning. These assessment activities were naturally derived from the teaching/learning process and were not a separate "assessment event" (Clarke, 1997).

Phase Two

The second phase also focused on descriptive accounts of the pedagogical practices of five classroom teachers from the same rural central school setting over

a further six months. The teachers, Toni, Kim, Anna and Andrea were relatively new to teaching (less than five years experience) and Michelle had been teaching for ten years. They were teaching the full range of classes Kindergarten to Year 6. Ethnographic methods of classroom observations and pre- and post-lesson teacher interviews were used to gain perceptions of practice. In addition, the vetting of analysis by the participants was undertaken in order to support the triangulation process.

This phase involved a school/university partnership (Bobis, 1998) between the school and the previous teacher-researcher who had since taken a lecturer's position at the local university. The partnership allowed a collaborative approach for data to be gathered and provided an opportunity to extend the initial phase and establish any common emerging themes. The data collected during this phase highlighted the importance of appropriate teacher questioning that required higher levels of thinking, as well as teaching strategies that fostered "thinking-centred" classroom discourse. Both of these components promoted the active involvement of students through:

- Finding out about students' prior knowledge at the beginning of a unit and/or lesson: this was achieved through the use of class concept maps to establish a shared understanding of language, and to "find out what language is in their heads and what ideas they have about a concept." (Kim and Anna)
- Student and teacher modelling of solutions and strategies (think aloud strategy): "They have to teach me, not me teach them." (Toni) "Come out and show us how you got your answer…who did it a different way?" (Anna)
- Creating a supportive learning environment where risk taking was encouraged: "I'm big on giving them a chance to answer. I'm trying to get the kids to be patient with each other." (Toni) "I often give them a clue to get them going." (Kim)
- Promoting student language through the use of "wait time" and class, group and paired discussions: "I use partner discussions to get everyone involved" (Kim) and "I'm very conscious of how long I wait for an answer because I used to jump in a lot." (Andrea)

Central to both sets of data was the notion of the teachers as reflective practitioners who saw themselves as co-learners with their students and who consistently sought to share in the joy of learning.

Phase Three

The third phase of the study was a natural extension of the second phase. Having gathered data related to current classroom pedagogy in the first two phases, the participant teachers and the co-researcher were able to use these data to collaboratively develop a focus for the third phase of the study. Action research was chosen as a model for this phase to ensure a community of co-researchers was established. The "thematic concern" (Kemmis & McTaggart, 1988) that was identified by the participants was a need for developing authentic assessment tasks that would provide evidence of mathematical thinking and understanding. Case

studies were also used to report the divergent interests of the participating teachers, which allowed each teacher to develop individually and still remain in a shared learning community (Gall, Borg & Gall, 1996; Lampert, 1998). This thematic concern recognised that the change towards a "thinking curriculum" forces us to focus on "thinking assessment" as well (De Lange, 1995). There were four common elements of pedagogical practice that emerged from this phase of the data. Each of the participating teachers:

- Made connections with prior knowledge and revisited concepts to introduce new ones by using concept maps to "find out what language is already in their heads" (Anna and Kim); using open-ended tasks at the beginning and the end of a unit to "help bring out misconceptions a lot earlier" (Anna), "find out what they know and get a snapshot of where they are, compared to the end" (Michelle), "cater for the differences much better" (Anna) as well as "be more challenging and interesting" (Kim).
- Shared with students the purpose and criteria for assessment and instruction "to give them something to strive for and know which direction they need to improve on" (Kim) and "If it isn't obvious to them that I'm assessing, even during class discussions then I need to be more explicit" (Michelle).
- Planned opportunities for students to verbalise, clarify and record thinking through explanation or verification of solutions, and written definitions of mathematical terms on students work samples (Anna and Andrea), portfolios that contained work samples of open-ended tasks and written procedures that were used for three-way interviews (Kim, Anna, Michelle), as well as written procedures for explaining a mathematical process (Michelle and Kim).
- Linked assessment with instruction at the programming and planning stage because 'It gives purpose and direction to instruction.' (Anna); "When I'm programming, I think of which open-ended tasks will be assessment tasks." (Kim); and "I plan it (assessment) to occur more as you go through because it provides better feedback and information about what direction to take." (Anna).

These findings further developed and confirmed the components in the first two phases and validated the emerging pedagogical framework.

Results

Some common themes emerged out of the research shedding light on pedagogical practices that, on the one hand, are practical and achievable in classrooms and, on the other hand, produce evidence of students' mathematical thinking and achievement. This study demonstrates that we can link practical and achievable classroom instruction to the production of work samples that provide evidence of mathematical thinking, in other words, instruction that is directly linked to assessment.

A Framework for Promoting Thinking and Understanding

The data collected in each of the three phases was synthesised to form a framework for promoting thinking and understanding in mathematics classrooms. This framework seeks to highlight the rich data that was collected through "accounts of practice". The pedagogical practices that promoted student thinking and sense making have been combined with the opportunities for assessment that were identified by the co-researchers in this study. These opportunities, which were essentially learner centred and embedded in instruction arguably lead to assessment that is learner centred. Six key concepts emerged to subsume the findings from each phase of the study. They reflected the "conceptual lenses" used by the researcher to account for teachers' observed and perceived practices. The six emerging concepts became:

1. Guided thinking in a supportive classroom environment;
2. Verbalising thinking;
3. Clarifying thinking;
4. Inking thinking;
5. Learner centred experiences; and
6. Learner centred assessment.

Figure 1 represents the "framework for promoting thinking and understanding in mathematics classrooms". Each concept is explained further in the following section.

Guided thinking in a supportive classroom environment. The first concept relates to the teacher's role as a facilitator of student learning where the emphasis is on guided discovery within a community of learners (Brown, et al., 1998). It views the teacher as a model and "fellow player" in the learning process (Clarke, 1997) and provides a role description for teachers who are seeking to increase learner centred experiences that lead to learner centred assessment. Using content specific open-ended tasks (Sullivan, 1999) that require students to think, reflect and communicate mathematically in a way that makes sense to them became the focus of the classroom design.

This notion of sense making can only occur when opportunities for cognitive and metacognitive activity are explicitly designed by the teacher to occur during classroom instruction. Guided thinking in a supportive classroom environment acknowledges the affective domain of learning, and highlights the importance of positive attitudes, risk taking, sharing the purpose of learning experiences, motivation, and viewing mistakes as opportunities for learning (Middleton & Spanais, 1999).

PROMOTING THINKING AND UNDERSTANDING IN MATHEMATICS CLASSROOMS
Requires:

1. GUIDED THINKING IN A SUPPORTIVE CLASSROOM ENVIRONMENT

- Using content specific open-ended tasks
- Teacher modelling of thinking and reflective practices (teachers as learners)
- Teacher questioning that promotes thinking
- Sharing criteria and purpose for assessment and instruction
- Planning for assessment during programming
- Positive attitude towards learning mathematics

Which lead to opportunities for reflection and communication through:

2. VERBALISING THINKING

- Class discussions that develop topic concept maps
- Teacher questioning (open-ended) that elicits thinking and establishes prior knowledge
- Student predictions
- Verbal explanations that lead to a shared vocabulary

These opportunities for thinking characterise:

5. LEARNER

Which create assessment opportunities for:

- Finding out about the learner's prior knowledge and misconceptions
- Guiding instruction
- Establishing natural language

These opportunities characterise:

6. LEARNER

3. CLARIFYING THINKING

- Student concept maps that make connections to prior knowledge and new knowledge
- Peer tutoring
- Revisiting related topics
- Student/Teacher modelling of solutions
- Think/pair/share solutions

These opportunities for thinking characterise:

CENTRED

Which create assessment opportunities for:

- Teacher and student monitoring of progress
- Feedback on teaching/learning to guide further learning
- Observing/listening to students' explanations

These opportunities characterise:

CENTRED

4. INKING THINKING

Work samples that show evidence of thinking:

- Student concept maps
- Written procedures
- Explanation of mathematical terms
- Labelled diagrams and pictures
- Presentation of solutions
- Justification of solutions

These opportunities for thinking characterise:

EXPERIENCES

Which create assessment opportunities for:

- Developing portfolios
- Three-way interviews
- Student self-assessment
- Comparison of pre- and post-lesson work samples
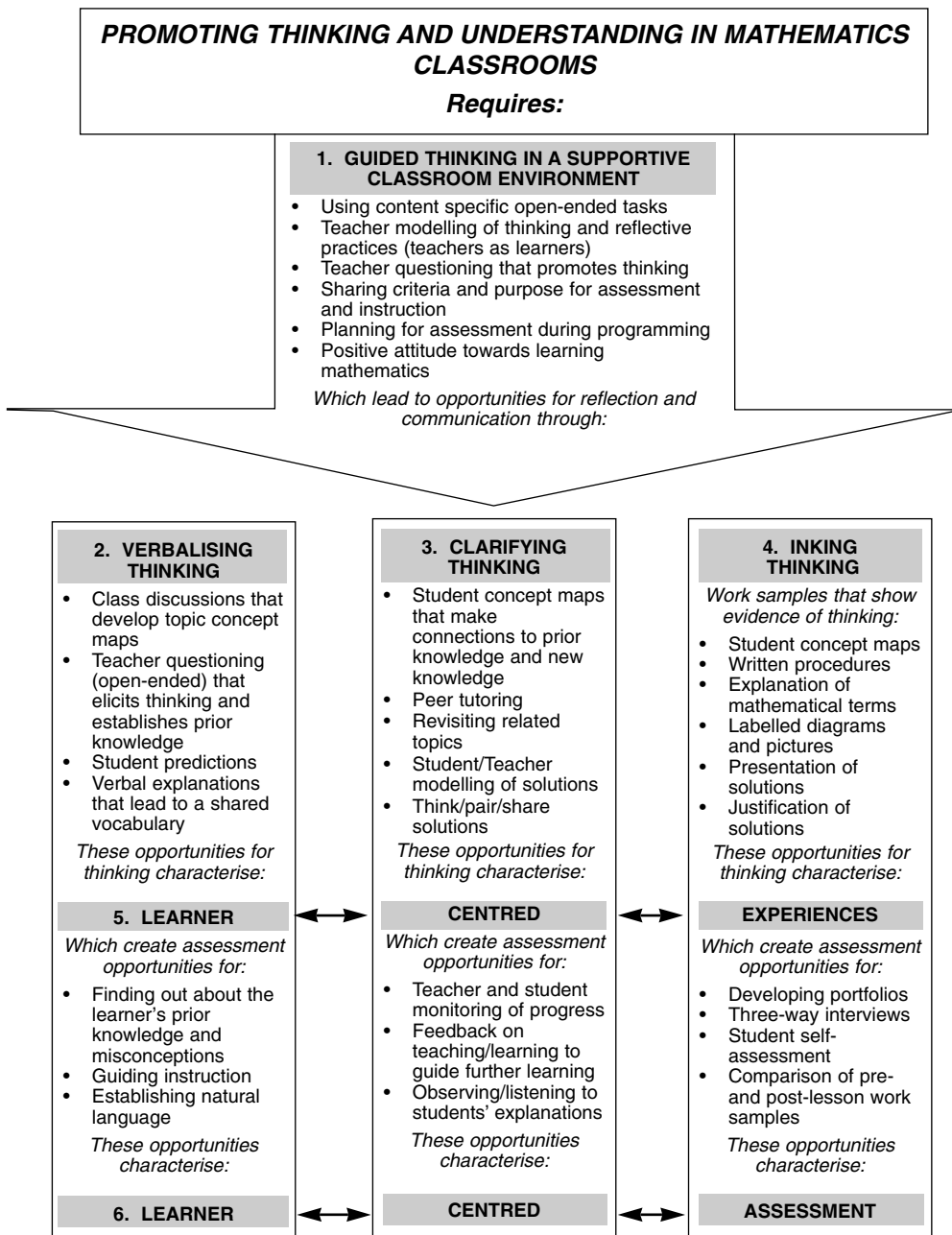
These opportunities characterise:

ASSESSMENT

Figure 1. A framework for promoting thinking and understanding in mathematics classrooms.

Verbalising thinking. The second concept incorporates all aspects of classroom practice that promote student language and "bringing thinking out" where the emphasis is on communicating meaning verbally in a variety of classroom settings. It subsumes the elements of oral communication as a whole class using strategies such as concept maps to establish a shared vocabulary, teacher questioning to elicit thinking, and opportunities for active engagement in class discussions. The verbalisation process is crucial to the framework because it focuses on eliciting and modelling the natural language used by students to explain concepts. This promotes rehearsal and practice of language that in turn can be used to assist them to "ink their thinking".

Clarifying thinking. The third concept focuses on more metacognitive processes such as students reflecting on, and monitoring their progress and includes experiences that are collaborative and social in nature. Instructional strategies such as think/pair/share, student concept maps, as well as teacher and student modelling of "think aloud" strategies and solutions were identified in the study as effective ways to clarify student thinking and reinforce communication and reflection. Teachers in the project also planned opportunities for students to revisit their work samples so they could clarify and reflect on their progress. All of these strategies provided opportunities for self-assessment during the clarifying process. By articulating their thinking in collaborative groups, students often self-corrected themselves as they explained and monitored their solutions.

Inking thinking. The fourth concept has been named "Inking thinking" to highlight the importance of students' written representations of thinking, and is borrowed from Fogarty (1995). This element refers to the need to collect evidence of students' thinking through authentic work samples that are naturally derived from learning experiences. These work samples were the result of students recording their solutions and responses to content specific open-ended tasks that further developed the metacognitive processes of reflection and verification of solutions.

The teacher's role here was to make explicit the purpose of the tasks and what was expected of students as they completed the tasks. It characterised the notion of shared and negotiated goals for learning. An example can be illustrated through an excerpt from the teacher-researcher's descriptive account of her practice during her unit on Volume in Phase One of the study. The students were set the open-ended task of designing a new three-litre container for milk that would fit easily inside the fridge. This task was set at the end of the unit to assess transfer of knowledge to a novel situation. She stated that:

> Assessment criteria were shared with the students so that they knew I was looking for evidence of the relationship between volume and capacity as well as documentation that was presented in an organised way to show their thinking and reasoning. (Anecdotal notes written in the unit of work on volume, May 1998)

This practice of sharing criteria allows for students to self-assess their work as they progress through the tasks and leads to opportunities for setting personal goals for learning. The concept of "inking their thinking" during open-ended tasks lead to the use of Progress Books (Portfolios) to show evidence of students' improvement over time. The open-ended tasks produced solutions that were "more practical, because before I was only putting in worksheets and they were boring and didn't

show their thinking and how they went about doing a task" (Kim). Students' work samples were used for teacher assessment, student self-assessment, and as part of the three-way reporting system between the student, the classroom teacher and the parents or caregivers.

Learner centred experiences. There is an underlying assumption embedded within the concepts identified in this framework. The terminology "learner centred experiences" and "learner centred assessment" have been deliberately chosen to incorporate one of the emerging themes from the data. The notion that teachers are co-learners and models for learning was evidenced throughout the three phases of the research project. Deliberately phrasing the terminology towards "learner centred" rather than "student centred", allows the teacher to be placed as a co-learner in the teaching/learning process. Likewise, this terminology allows the student to become the teacher, and the teacher to become the learner during student and teacher modelling of solutions and strategies that were observed in the project classrooms. Perhaps the best descriptions came from two of the participating teachers when they stated:

> What we are actually doing as teachers is exactly what we are asking the kids to do. You know, reflect on what we've done, think about your thinking…verbalise your thinking! When you have this situation where you are working as a team you can get very excited about things. It's so much more interesting to do it this way and work together. (Michelle, post-lesson interview transcript, 1999);

> My big thing in every lesson is to ask questions that get them to tell me things – not me telling them. They have to teach me, not me teaching them. I told them today that they were the teachers as well as me…I try to leave it free to see what happens and where things go. This might sound awful, but some of my best lessons have been when I just go with the flow. (Toni, post-lesson interview transcript, 1999)

Clearly, one of the central themes that emerged from this data to characterise a social constructivist learning environment was the notion of a shared and active role in the teaching and learning process. The concept of "learner centred experiences" sees both the teacher and the students as "scaffolders of learning" (Hogan, 1997), and highlights assessment opportunities that are embedded in learning experiences.

Learner centred assessment. Evidence of the integral nature of assessment and instruction was a feature of each participant's classroom. The identified "thinking concepts" (verbalising, clarifying, and inking) provide opportunities for assessment, which are documented underneath the thinking components in the framework. All the features in this emerging framework are taken from data collected within the study, which validates its practical, classroom-based nature, and reflects a focus on thinking that integrates assessment and instruction and fosters metacognitive processes. It seeks to show how experiences that are learner centred can be symbiotic to assessment that is learner centred (Smith, 2000b).

Discussion

While each concept has been described separately, they are interdependent. The six concepts within the framework combine to create an image of a thinking centred mathematics curriculum. There is a resounding message for educators that a

thinking curriculum where students are encouraged to make sense of mathematics requires assessment practices that provide evidence of thinking and sense making. Resnick and Resnick (1992) suggested that higher levels of thinking are involved in the most basic competencies and that thinking should pervade the whole curriculum. Similarly, Bransford et al. (1999) maintained that "assessment should reflect the quality of students' thinking as well as what specific content they have learned" (p. 232), and call for assessment frameworks that integrate cognition and context.

Clarke (1997) reminded us that it is through our assessment that we communicate most clearly to students which activities and learning outcomes we value. The findings represented in this paper highlight the nexus between assessment and instruction in a pragmatic framework that focuses on assessing students' thinking and prior knowledge related to mathematical concepts in order to direct instruction. Likewise, Cooney, Badger and Wilson (1993), after spending a year observing teachers' assessment practices in classrooms, stated that:

> It is imperative that assessment be seen as an integral part of instruction. It provides a window to students' thinking and a compass for instruction. Equally important, what gets assessed – and how it gets assessed – sends clear signals to students about what teachers think is important. (p. 239)

Planning for assessment to be learner centred is a creditable and achievable goal for all classroom teachers. It has been recognised by the co-researchers in this study that the likelihood of this happening is enhanced if assessment is planned during the process of programming learning experiences (see Phase Three comments). If we value communicating ideas and thinking about why an answer makes sense, and if we truly believe that affective factors such as persistence, risk taking and monitoring progress are important characteristics for students to possess, then our assessment and instruction must reflect these beliefs. The framework presented in this paper highlights pedagogical practices that promote mathematical thinking and understanding that are validated by relevant theoretical principles and similar research studies.

Conclusion

The methodology used in this study sought to understand practice from the point of view of practice (Lampert, 1998). Extensive use of participant voices increased the authenticity of the framework components, as did the multiple sources of data taken primarily from the classroom context. The teachers' accounts of their practices have been viewed through the conceptual lenses of a researcher to produce images of what classrooms that promote mathematical thinking and understanding might look like.

The framework is an attempt to bridge the gap between theory and practice so that pre-service and practising teachers can be guided by the descriptive images of instructional strategies and the learner centred nature of the roles of teachers and students. Shared understandings derived from collaborative research have been illuminated so that other researchers and teachers may be guided towards further investigation and elaboration of such a framework.

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