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Mathematics Education for Early Childhood: A Partnership of Two Curriculums

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This paper describes the mathematics education component of one early childhood teacher education program at the Auckland College of Education. It explores how the school mathematics curriculum can provide a meaningful framework for the mathematical learning of very young children. It identifies how the interweaving of the mathematics curriculum document with the New Zealand early childhood curriculum can support the early childhood teacher to optimise the provision of mathematically-rich environments. It examines a variety of issues pertaining to the early emergence and development of mathematical concepts.

Early Childhood Education in New Zealand

Curriculum for Practice

Why all the fuss about curriculum? Curriculum as a philosophical statement, as a tool for accountability, or as a foundation for the total learning experience, curriculum documents are here to stay. Historically educational practice has always been driven by the philosophy or the underlying "theme of the times" (Heaslip, 1997). In early childhood education in particular, beliefs of recent times included:

- 1960s: a period of *compensatory* learning, providing for what was perceived to be lacking in the home
- 1970s: an emphasis on *language* and the implications on early learning of the stages of language development
- 1980s/90s: a need for *curriculum-based* learning and the use of curricula as a framework for the processes of learning.

The early childhood curriculum, *Te Whaariki* (Ministry of Education, 1996), is the first national curriculum statement for the early childhood sector in New Zealand and as such has provided the beginning of an exciting era for early childhood education. The key feature of *Te Whaariki* is that it espouses a child-centred approach, sensitive to the holistic development and learning of the child.

Teacher Education at ACE

Since 1996, Auckland College of Education programs in teacher education for early childhood have included a stronger emphasis on subject knowledge than has been the tradition in New Zealand, through a delivery structure based on the *New Zealand Curriculum Framework* (Ministry of Education, 1993). The framework contains seven essential learning areas: Language and Languages, Mathematics, Science, Technology, Social Sciences, The Arts and Health and Physical Well-being. It is designed to cater for learners from five to eighteen years of age (Years 0 to 13 in

the New Zealand school structure). A key feature of the framework is that it links children's learning with the world outside school, through the additional component of eight essential skills: communication, numeracy, information, problem solving, self-management and competitive, social and co-operative, physical, and work and study.

The decision to provide curriculum knowledge and experiences through the Centres for Learning, based on the *New Zealand* Curriculum Framework (e.g. Centre for Technology Education, Centre for Mathematics Education) was made, however, with the intention that the early childhood pathway remain committed to the philosophy of *Te Whaariki.* The intention was that providing student teachers with a broad base of subject-knowledge and experience would allow them to examine the holistic nature of a child's play from a broad and confident knowledge base in terms of the national curriculum. The decision was enabled by the recognition that not only are the essential skills of the framework apparent in children before school, but so too is the close relation between the learning areas and the everyday emergent curriculum of young children as they journey from birth to school.

The structured use of both curriculums is referred to throughout the paper as the *dual-curricular* approach to teacher education as used in the Bachelor of Education (Teaching). It is an expectation of this program that early childhood student teachers are empowered to reflect for themselves on the interface of the two curriculums: *Te Whaariki* and each specific curriculum/subject area of the *New Zealand Curriculum Framework.* This paper focuses in particular on the partnership of *Te Whaariki and Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) as they support each other in the mathematics education modules of the Bachelor of Education (Teaching). It examines how the acquisition of knowledge of the school curriculum document can enhance the ability of early childhood teachers to provide for mathematics learning through play.

The Supporting Curriculum Documents

Mathematics in the New Zealand Curriculum

The underlying theme of the national mathematics curriculum is a problem-solving approach to the learning and teaching of mathematics. For the strands of the document see the Appendix A. Recommendations from within the curriculum are:

- [Concept learning] should be taught in such a way that [children] develop the ability to think mathematically.
- [Children] learn mathematical thinking most effectively through applying concepts and skills in interesting and realistic contexts which are personally meaningful to them.
- Rather than remembering the single correct method, problem solving requires [children] to search the information for clues and to make connections to the various pieces of mathematics and other knowledge and skills which they have learned.
- Critical reflection may be developed by encouraging [children] to share ideas, to use their own words to explain their ideas, and to record their thinking in a variety of ways.

- As new experiences cause [children] to refine their existing knowledge and ideas, so they construct new knowledge. (Ministry of Education, 1992, p. 11)

This would suggest that the mathematics curriculum has constructivism as its underlying philosophy and raises questions such as: what does this mean in terms of mathematical learning? what is mathematics knowledge? how is the learner's contribution valued? and what are the implications for the teacher?

Te Whaariki

The principles and strands of *Te Whaariki* (see Appendix B) form the basis for worthwhile learning and at the core is the recognition of the "importance of the social context within which children are cared for and learning takes place" (Ministry of Education, 1996, p. 7)

- It is about the individual child.
- Its starting point is the learner and [the] knowledge, skills, and attitudes that the child brings to their experiences.
- [It encourages children to] grow up as competent and confident learners and communicators ... secure in their sense of belonging and in the knowledge that they make a valued contribution to society.
- [It advocates that children] learn strategies for active exploration, thinking and reasoning.
- [It upholds that children] discover and develop different ways to be creative and expressive.
- [It allows for children to] develop [their own] working theories for making sense of the natural, social, physical, and material worlds.

(Ministry of Education, 1996, pp. 9,16)

This would suggest that the early childhood curriculum also has constructivism as its underlying theoretical framework and again raises questions such as: how is children's knowledge valued? how do different aspects of play contribute to this knowledge? and again, what are the implications for the teacher?

Both the mathematics curriculum document and *Te Whaariki* highlight the importance of empowerment as a key factor in the growth of the learner. *Growth* is defined by the acquisition of knowledge and the ownership of that knowledge and *empowerment*, through the self esteem which develops from the confidence of holding that knowledge and from the process in which the knowledge was gained. Why then might the dual-curricular approach to curriculum knowledge at the Auckland College of Education be viewed as a source of tension?

Source of Tension

A possible source of tension for this approach to early childhood pre-service teacher education arises due to the emphasis in *Te Whaariki* of "a model of learning that weaves together intricate patterns of linked experience and meaning rather than emphasising the acquisition of discrete skills." (Ministry of Education, 1996, p. 41). Further to this, writing about the implementation of *Te Whaariki*, concern has been expressed by Carr and May (1996) that focusing on individual subject areas

could inhibit this holistic approach for pre-service teachers. While Cullen is in agreement that adults who work with young children do need to be sensitive to children's interests and everyday experiences, she argues that they also need to "have the teaching strategies and subject knowledge which allows them to extend children's foundational knowledge" (1996, p. 119).

In a recent report on the use of *Te Whaariki* in early childhood centres, the Education Review Office (ERO) (1998) are of the opinion that the broad view of curriculum, as adopted in *Te Whaariki*, and its deliberate aim of being sufficiently open to be inclusive of the wide range of centres within New Zealand, carries risks in terms of children's cognitive and intellectual development. The report (1998, p. 12) states

> few programs being implemented by early childhood education centres were linking *Te Whaariki* to the essential skills and learning areas of the national curriculum statement for schools. This may be because the links between the two documents are somewhat tenuous …

This raises the question 'What are realistic expectations of the early childhood teacher in terms of the cognitive development of children?' Previously there was little opportunity in New Zealand, for the current early childhood teachers to participate in teacher education programs embracing in depth the principles of both early childhood and subject-based pedagogies. A wealth of subject-knowledge is required to work confidently across the whole framework of the national curriculum while remaining focused overall on the wholeness of the play contexts. ERO appears to omit this historical influence and channel its criticisms, in terms of poor curriculum links, at the statements in *Te Whaariki* itself.

The Education Review Office (1998, p. 12) continues

> In failing to identify a positive relationship between early childhood education and school education *Te Whaariki* creates the impression that early childhood education exists in a vacuum, is complete in itself and has no relationship with further learning.

Carr and May (1996) meanwhile, reporting on the establishment of *Te Whaariki*, stress that one of the underlying thrusts in the need for an early childhood curriculum was to prevent the national curriculum for schools starting a downward move into the early childhood years. Carr reiterates this strongly in her recent assessment project:

> The community must decide to what extent the school curriculum should further push down into the early childhood years, and particularly if in doing so there is any trade off against the early childhood curriculum principles and goals. (Ministry of Education, 1998, p. 3)

It seems there needs to be a further perspective to this dilemma and Auckland College of Education, while remaining totally committed to the value of play and to the sound holistic principles of *Te Whaariki*, has risen to the challenge of providing for the development of individual subject knowledge within a holistic framework. As part of their program, student teachers experience three compulsory modules in mathematics education, with the option of further modules in their final year. The modules are designed to encourage an increase in personal mathematical understanding to enable student teachers to recognise and enhance the

mathematical knowledge displayed by infants, toddlers and young children within a holistic play environment.

Mathematical Knowledge in the Early Years

Developing Mathematical Understanding

Current theories on knowledge construction, and mathematical knowledge in particular, remain embedded in an overall belief in constructivist learning. However as debate continues to embrace the broad spectrum of constructivism, in terms of compatibility or otherwise of a variety of constructivist principles (Begg, 1999), interest has developed in exploring further the 'learner as expert', focusing more explicitly on what is the happening for the learner as the key participant in a constructivist learning environment.

In 1956, elaborating on real understanding of a mathematical proposition as opposed to merely 'knowing the rules', Wittgenstein (cited in Kieran, 1994) wrote "Acting with understanding, it seems, involves being able to see the significance and consequences of the proposition, to see alternatives to it and alternative approaches in producing it and to be able to situate it or interweave it with one's more informal intuitive knowledge of that area of mathematics."

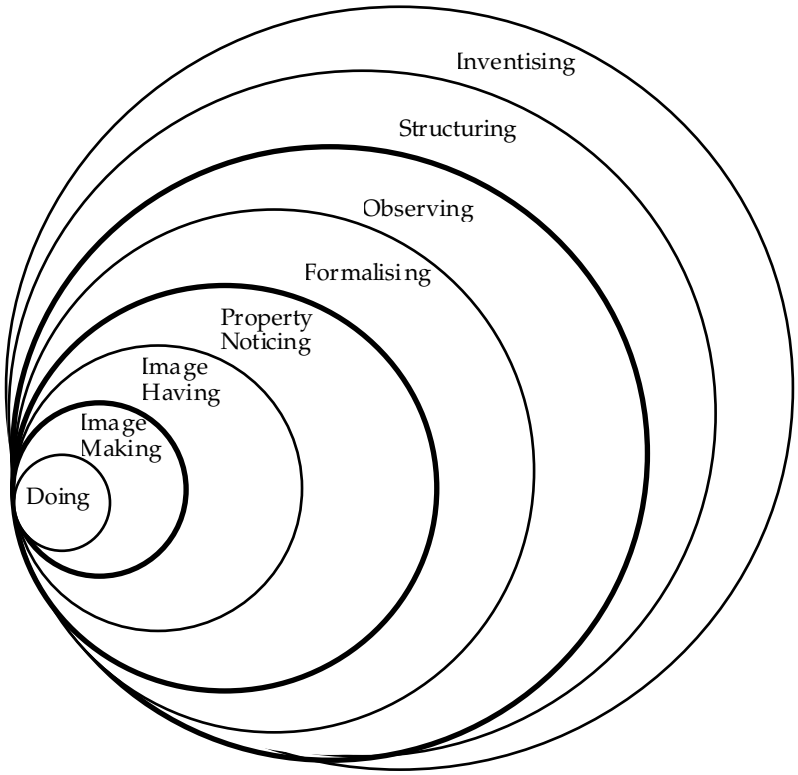

Figure 1. Model of a dynamical theory of the growth of mathematical understanding.

Responding to definitions such as this on the meaning of mathematical understanding Pirie and Kieran (1989) present a model to demonstrate the growth of mathematical understanding which theorises the process of attaining mathematical understanding as a sequence of phases having at its core the action of "doing" mathematics (see Figure 1).

This "doing" phase of the process they later named "primitive knowing" (Pirie & Kieran, 1994) and emphasise their view of seeing the process as recursive: that "primitive knowing" implies a starting place for growth of mathematical understanding in any one mathematical situation, revisited as part of the cycle of developing formalised understanding. "Primitive knowing" remains relevant for the very young child, or even infant, and should be recognised in terms of the intuitive knowledge that sparks the interest the child brings to an activity. As such very young children should also have the opportunity to develop their understanding through phases of the model. As well as supporting the belief in the learner as key participant in the construction of knowledge, as espoused in the constructivist-based mathematics curriculum, this is in keeping with the child-centred philosophical base underlying *Te Whaariki.*

Pound (1999) like Wittgenstein defines mathematical learning as the ability to make connections while Merttens (1997) draws attention to the importance of making connections not only for oneself but with those of one's peers. The importance of responding to the mathematical thinking of others emphasises the importance of interaction within a socially constructed learning environment (Elliot, 1993). The effectiveness of working with or beside others is apparent in both the mathematics curriculum and *Te Whaariki.*

The Mathematical Thinking Underlying the Understanding

How does mathematical thinking manifest itself in holistic play situations? Ways of 'showing' mathematical thinking include problem solving, representation of ideas and apparent disposition.

Bell (1996, p. 184) although writing about algebraic thinking in school-age children suggests that "generic problems can provide authentic algebraic experiences that not only cover the strategies for problem-solving ... but also develop the key algebraic abilities of writing, reading and manipulating algebraic expressions". The power of generalising is a recognised indicator of algebraic thinking (Lee, 1996; Mason, 1996) and at a very young age children are generalising (Curcio & Schwartz, 1997) as they discuss relationships evident in a variety of situations involving proportional reasoning.

Morris (1998) writes about "visible thinking" in terms of painting and other graphics. Through their art work children demonstrate a variety of ideas which can indicate their mathematical thoughts. Looking beyond graphic representation visible thinking can encompass involvement in drama and music. Because disposition can be a barrier to mathematical development at any stage of learning it is important that children learn to enjoy mathematical thinking at a very young age (Pound, 1999).

Support for Teachers through the Curriculum Documents

Linking through Constructivism

Constructivism, as identified through the expectations of both the mathematics curriculum and *Te Whaariki* recognises the learner as the key person in the learning experience and the teacher as the facilitator of tasks which allow the learner to construct their own knowledge. In early childhood environments the role of the teacher includes the preparation and provision of planned play and requires the recognition of the power of spontaneous play (Jones & Nimmo, 1994). Crucial to both types of play, in terms of providing a constructivist learning environment, is the style of interaction and facilitation of the children's activity in order to turn them into rich learning situations, and also the content knowledge to be confident in recognising and developing mathematical understanding further (Corneille, 1997). A working knowledge of *Te Whaariki* together with both the Mathematical Processes strand and the Mathematical Content strands of the mathematics curriculum document (see Appendix A), as gained through the dual-curricular approach, is desirable.

Mathematical Processes and Te Whaariki

The links between *Te Whaariki* and the mathematics curriculum are most obvious through the learning outcomes of *Te Whaariki* and the achievement objectives of the Mathematical Processes. In early childhood settings in New Zealand children are encouraged to create as they wish, to explore and enjoy, and to find their own ways of getting out of difficulty. At school level the current New Zealand mathematics curriculum purports an approach more aligned to the activities encouraged in early childhood programs and these are identified particularly in the under-pinning strand, the Mathematical Processes, as experiences through which understanding of mathematical content can be acquired. It is stated in this section of the document that learners should be allowed opportunities to be creative, to have a go, to discard unsatisfactory attempts and try another way, to plan for themselves ways around perceived difficulties or stumbling blocks.

The examples in Table 1 illustrate a similarity in approaches to learning and teaching in both documents, and emphasise the constructivist beliefs which form the theoretical frameworks of each.

A theoretical base and curriculum knowledge about the 'how' of teaching enables teachers to explain what they see and to take appropriate action (Orton, 1992), following observation of children's play but theories about facilitating play are not sufficient. To optimise the potential for the development of mathematical concepts emerging through play, teachers also need sound content knowledge of the concepts themselves in order to address the 'what' in teaching.

Table 1
The Mathematical Processes and Te Whaariki

Mathematical Processes		*Te Whaariki*	
Problem solving	Students should be seeking solutions through trial and error	Exploration	Young children are encouraged to feel comfortable about saying "I don't know"
Developing logic and reasoning	Students should be recognising and working with patterns in a variety of forms and contexts	Belonging	For infants a regular but flexible pattern is established for the the day, for example, going in the pushchair for a walk or going outside
Communicating mathematical ideas	Students should be working cooperatively as part of a group by listening attentively, generating ideas, and participating in reflective discussion	Contribution	Toddlers are encouraged to contribute to small-group happenings, for example, joining in dance, or bringing chairs around the table for snack time

Mathematical Thinking and Mathematical Content

The indicators of mathematical thinking emerge through the play situations and everyday experiences of the child and planning for these experiences needs to address the notion that any underlying mathematical concept formation can be interesting and enjoyable (Hughes, Desforges & Mitchell, 1996). The thinking that underlies mathematical understanding in play situations aligns itself with the emergence of concepts as presented in the content strands of the mathematics curriculum. As children play, and think mathematically, they are engaged in thinking across all content strands. Using the structure of the mathematics curriculum as a framework this early mathematical thinking can always be described using the words of the content strands. Some examples of early mathematical thinking are:

- *geometric thinking* as children learn about space and themselves within space; as they explore shapes through all their senses and discover which shapes fit where, and more importantly which don't
- *algebraic thinking* in recognising pattern in manipulative play and through music; through making connections, and generalising in a variety of problem-solving situations

- *statistical thinking* as they sort and categorise their collections, and talk about their decision-making
- *numerical thinking* through contextual situations such as birthdays, counting as they climb the steps of a ladder or as they clap hands in a song
- *measurement thinking* when they discuss the growth of a plant, or decide on the size of a block building

For the early childhood teacher then it is important to have the confidence in their own knowledge of mathematical content, to value the conceptual thinking emerging through children's play, to recognise its potential for higher level thinking and to have the ability to take action accordingly.

Implications for Teacher Education

The early childhood teacher is required to recognise conceptual development through the variety of skills used by the child and by making appropriate assessment decisions in order to plan for further mathematical learning based on the child's interests. Integrated assessment and planning need to address the use of appropriate language and the provision of resources which have the potential to enrich mathematical development. Informed insight into the mathematical thinking of infants through to young children can lead to the creation of environments that are meaningful in terms of a child's personal context.

The Dual-Curricular Approach and the Student Teacher

Cullen (1999) writes that for programs in teacher education to achieve authenticity there needs to be provision for subject knowledge to be embedded soundly in domain knowledge. In order to maintain the philosophical belief in holistic learning in early childhood this is particularly important. At the Auckland College of Education study of the mathematics curriculum and identification of how its philosophy is embedded in the principles of the early childhood curriculum equips student teachers with the knowledge and confidence to provide rich mathematics contexts for the children. Haynes (in press) reports on a study which explores the perceptions of student teachers on their dual-curricular program across a variety of learning areas, including mathematics. Responses indicate a belief in the professional strength they possess through their dual-curricular program and suggest a knowledgeable approach to children's learning across the national curriculum. This would seem to indicate a positive anticipation to addressing the concerns of the Education Review Office (1998).

However in order to utilise the links between *Te Whaariki* (Ministry of Education, 1996) and Mathematics in the New Zealand Curriculum (Ministry of Education, 1992) it is essential that teachers enjoy mathematical activity for themselves and feel confident to plan effectively for children through the provision of appropriate materials and thoughtful dialogue. It is the personal knowledge and disposition of the teachers which enables them to take a "national curriculum and turn it into a child's curriculum" (Malaty, 1996).

The intention of the Bachelor of Education (Teaching), is that subject-based module delivery through the Centres for Learning will enable early childhood student teachers to achieve a level of empowerment which enables them to view the

challenges of a dual-curricular approach positively, rather than as a source of tension. With the mathematics curriculum and *Te Whaariki* documents in partnership as a framework for assessment and planning, then "the incidence in early childhood centres of both ad hoc structured maths teaching and the loose 'maths is everywhere' approach will cease and early childhood mathematics will gain purpose and meaning" (Hill, 1995). Kleinberg and Menmuir (1997) agree that too broad a focus on curriculum in early childhood can mean that valuable mathematical learning opportunities can be overlooked. They emphasise a focus on "learning to be mathematical" and recommend a content element to curriculum decision-making. As stated by a group of current student teachers (Benseman, Bent, Healy & Ikimau, 1998) the strength of this curriculum partnership should enable teachers to feel confident in providing children in early childhood with a productive start to their mathematics education.

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Appendix A: Mathematics in the New Zealand Curriculum

Mathematical Processes

- Problem Solving
- Developing Logic and Reasoning
- Communicating Mathematical Ideas

Mathematical Content Strands

- Number
- Measurement
- Geometry
- Algebra
- Statistics

Appendix B: *Te Whaariki*

The Principles

- Empowerment
- Holistic development
- Family and Community
- Relationships

The Strands

- Well-being
- Belonging
- Contribution
- Communication
- Exploration