Mathematics Teacher Education and Development 1999, Vol. 1, 4-21

Scaffolds for Learning: A Sociocultural Approach to Reforming Mathematics Teaching and Teacher Education

Merrilyn Goos
University of Queensland

Most research interest stimulated by the mathematics education reform movement has been centred on schools, and on supporting teachers in implementing inquiry based teaching approaches in mathematics classrooms. This paper argues that similar research efforts need to be invested in pre-service teacher education if genuine and lasting reform of mathematics teaching is to be achieved. Parallels are drawn between two separate studies which applied sociocultural theories of learning to explore the nature of expert scaffolding in structuring learners' thinking. The first study, set in a senior secondary school classroom, shows how the teacher's scaffolding helped students to engage critically and independently with mathematical tasks. However, for many student teachers the supervised practicum continues to offer more traditional models of teaching. A way of resolving this dilemma is suggested by the second study, which investigated the actions of a mentor in scaffolding student teachers' post-lesson reflections. It is proposed that such a mentor may play a useful role in the professional development of supervising teachers.

In recent years educational policy makers, practitioners and researchers have called for significant changes to the way mathematics is taught in schools. In both the United States and Australia, influential curriculum documents have been published which articulate new goals for students' mathematics learning, and promise to bring about a radical re-thinking of current mathematics teaching practices. These moves for curriculum reform are supported by current research in mathematics education which has re-conceptualised mathematics teaching and learning as a social and communicative activity that requires the formation of a classroom community of learners, where the ways of thinking, modes of inquiry, communicative conventions, values, and beliefs characteristic of the wider mathematical community can be progressively enacted and appropriated.

These curriculum proposals and theoretical formulations clearly have profound implications for teacher development and change, and their translation into practice by both in-service and pre-service mathematics teachers remains problematic. The aim of this paper, therefore, is to consider the challenges to be faced by mathematics teachers in developing new ways of working with students in their classrooms, and by teacher educators in preparing pre-service students to critically reflect on the dilemmas of practice.

The paper argues that a sociocultural perspective on learning can provide a theoretical rationale for classroom reform, by demonstrating how the central concept of the zone of proximal development (ZPD) can be applied to a community of learners. Brief episodes from a secondary school classroom are presented to illustrate how such a classroom community can be constituted, with specific reference to teacher scaffolding of students' learning. The second part of the paper

examines implications of the sociocultural perspective for pre-service teacher education, and analyses the role of a mentor in scaffolding student teachers' post-lesson reflections focusing on conflicts between teaching approaches promoted by the university and practicum schools.

Towards Reform of Mathematics Teaching

The last decade has witnessed the emergence of an international reform movement in mathematics education that has promoted goals and practices which stand in contrast to those of traditional instruction. In the United States, for example, the National Council of Teachers of Mathematics (NCTM) has recognised that goals for students' learning need to extend beyond mastery of a predetermined body of knowledge and procedures to include mathematical reasoning and problem solving, communication, and connecting mathematical ideas and applications (National Council of Teachers of Mathematics, 1989). So that these goals may be achieved, the NCTM has recommended moving away from over-reliance on teaching practices such as exposition and individual seatwork, towards activities which promote students' involvement in constructing, applying, and evaluating mathematical ideas (National Council of Teachers of Mathematics, 1991). A similar shift in priorities has occurred in Australia, where the intent of the NCTM Standards is echoed in the *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991).

While these curriculum policy documents can suggest what a "reformist" mathematics classroom might look like, changes to classroom practice are unlikely to occur unless they can be justified on theoretical grounds and implemented in ways considered feasible by teachers. From a theoretical perspective, goals for school mathematics are derived from a conception of what mathematics is, and what it means to understand mathematics. There is growing agreement within the mathematics education research community that learning to think mathematically involves acquiring not only skills, strategies and knowledge, but also habits and dispositions of interpretation and meaning construction (Schoenfeld, 1994), that is, a *mathematical point of view.*

If seeing the world in the way that mathematicians do is a fundamental element of mathematical thinking, then mathematics education can be interpreted as a process of enculturation into the practices of the discipline; most importantly, students' understanding of what the discipline is about is shaped by their participation in the classroom mathematical community. Within the last decade an emerging body of literature (e.g., Collins, Brown, & Newman, 1989; Lampert, 1990; Lave, Smith, & Butler, 1989; Schoenfeld, 1989) has begun to argue that mathematics is an inherently social and collaborative activity, and that mathematics classrooms should therefore engage students in these authentic practices of the wider mathematical community. If students are to develop mathematically powerful forms of thinking, as well as appropriate epistemological values, then mathematics classrooms must support a culture of sense-making in which students learn by immersion in the authentic practices of the discipline.

The principles outlined above are consistent with a sociocultural perspective on learning, which emphasises the socially and culturally situated nature of

mathematical activity, and views learning as a process of appropriating the cultural tools—language structures, symbol systems, forms of representation, and structures of reasoning—recognised by a community of practice (Resnick, Pontecorvo, & Säljö, 1997). How this appropriation occurs can be understood by considering a key sociocultural concept—Vygotsky's notion of the zone of proximal development (ZPD) (Lave & Wenger, 1991; Renshaw, 1996; Saxe, 1991).

The most widely known definition of the ZPD is the distance between what a learner can achieve alone and with the assistance of a more advanced partner, such as a teacher or peer tutor. This form of guidance is frequently referred to as scaffolding, defined as assistance which enables learners to accomplish a task unable to be completed individually, and intended to move them closer to a level of competence which will allow them eventually to complete such a task on their own (Mercer & Fisher, 1992).

While the notion of the ZPD has previously been applied only to individual learners, it has recently been extended to include whole classes of students (Brown & Campione, 1995). Here, the class is considered a *community of learners* where children are inducted into more disciplined and rigorous modes of thinking that involve exploration, speculation, conjecture, gathering evidence, and providing proof. Whether conceptualised in terms of scaffolding or participation in community of practice, the ZPD represents a challenge for students to move beyond their established levels of competence and adopt the conventions of language and reasoning of the discipline of mathematics.

The metaphor of "community" is attracting increasing interest from researchers seeking to provide an educational perspective on recent efforts to reform mathematics teaching and learning (e.g., Forman, 1996; Lampert, 1990; Renshaw & Brown, 1997). The next section describes one attempt to create such a classroom in an Australian secondary school.

Establishing a Community of Inquiry in a Secondary Mathematics Classroom

The aim of this study was to investigate the teacher's role in establishing a community of mathematical inquiry (Goos, Galbraith, & Renshaw, 1999). The study was conducted over a two year period in eight senior secondary school mathematics classrooms, and made use of ethnographic and case study techniques such as participant observation, semi-structured interviews, survey instruments, and analysis of video and audio tapes and student work samples.

Evidence from these sources indicated that one classroom, more than others, exemplified the theoretical principles outlined in the previous section and, hence, approximated a community of inquiry. While a detailed analysis of the teacher's actions in establishing this culture is beyond the scope of this paper (see Goos, Galbraith, & Renshaw, 1999, for further details), the following episodes from the Year 11 classroom have been selected to illustrate the teacher's role in scaffolding the processes of mathematical inquiry.

The Teacher's Role in Scaffolding Students' Thinking

Early in the school year the teacher placed explicit emphasis on modelling the processes of mathematical inquiry and scaffolding students' mathematical thinking, as the following example from two consecutive lessons on matrices early in Term 2 illustrates. The aim of the lessons was to have the students discover for themselves the algorithm for finding the inverse of a 2 x 2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. The teacher first chose a matrix A with a determinant of 1 and asked the students to find the inverse A^{-1} by using their existing knowledge of simultaneous equations to solve the matrix equation $AA^{-1} = I$. He then elicited students' conjectures about the general form of the inverse matrix, based on the specific case they had examined. Since the nature of the example ensured that students would offer $\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ as the inverse, the teacher was able to provide a realistic context for students to test this initial conjecture. A counter-example, whose inverse was found to have the form $n\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$, allowed the students to find a formula for n, which only then was labelled by the teacher as the determinant.

In this example, the teacher scaffolded students' thinking by asking questions that prompted them to clarify, justify, and critique their conjectures and assertions. Thus the processes of inquiry were structured through teacher questions such as "Can you check via matrix multiplication that you do get the identity matrix?", "How could we verify this?", and "How is this [the form of the inverse for the counter-example] related to your conjecture?".

As the year progressed, the teacher gradually withdrew the structured support illustrated above to move students towards independent performance. This change is noticeable in a lesson on vectors which occurred about one third of the way through the school year. In this lesson the teacher asked students to develop a method for finding the angle between two vectors $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 1 \end{pmatrix}$, based on their knowledge of the formula for the dot product, $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \vartheta$, encountered for the first time only the previous day. Now the students were expected to advance their thinking without the teacher's scaffolding, and most spontaneously formed small groups and pairs to work on the task without his assistance for over ten minutes.

Some students were still coming to terms with the teacher's insistence that they explain the reasoning that led to their answers. For example, when one calculated that the angle between the vectors was 22.4° and announced that he had "the answer", his friend pointed out that the teacher "doesn't want the answer, he wants *how* you work out the angle". This point was reinforced by the

teacher himself when he reconvened the class and nominated a student (Alex) to come to the blackboard to present his solution. As Alex began to calculate the value of $\begin{pmatrix}3\\2\end{pmatrix}\cdot\begin{pmatrix}5\\1\end{pmatrix}$, the teacher reminded him that he wanted a general equation first before any numerical substitution. Other students then began offering Alex suggestions and hints as to how to proceed:

Adam: Rearrange it, Alex.

Aaron: Yeah, rearrange it.

Alex: Using ...?

Aaron: Using, like, symbols.

Adam: Look up on the board! (i.e., at the formula $\mathbf{a}\cdot\mathbf{b}=|\mathbf{a}||\mathbf{b}|\cos\vartheta$) Just write down the equation.

Alex: So you work out **a** dot **b** using this method— (starts to substitute numbers again)

Teacher: I don't want to see anything to do with those numbers at all!

Aaron: Alex, rearrange that equation so you get theta by itself. (Alex begins to do so.)

Teacher: How's he going? Is he right? (Chorus from class, "Yes". Alex finishes rearranging formula to give $\vartheta=\cos^{-1}\left(\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}\right)$.) Alex, that's great, that's spot on!

Here Adam and Aaron appear to have appropriated teacher-like scaffolding strategies in moving a peer's thinking forwards, and they bypassed the teacher completely in directing their comments to the student at the blackboard. Note that the purpose of their actions was not to publicly display their own knowledge for the teacher's evaluation and approval – often the only reason why students are permitted to speak in traditional classrooms – but to assist in the construction of a jointly owned solution.

This classroom provided ample evidence of the teacher's role in scaffolding students' learning. While the teacher explicitly modelled the processes of mathematical inquiry, he did not do so in a way that reduced these processes to a series of mechanical steps for students to follow. Instead, the teacher provided a consistent structure for inquiry which elicited the necessary mathematical substance from the students, such as conjectures that were subsequently tested, refined, and validated.

Towards Reform of Pre-Service Teacher Education?

Studies such as that reported above show that a sociocultural perspective on learning can offer a way to understand how current goals for curriculum and pedagogical reform may be accomplished in mathematics classrooms. However,

few studies have applied sociocultural theories to *teacher* learning in general, and pre-service teacher education in particular (see Frykholm, 1998; Samaras & Gismondi, 1998). There is merit, therefore, in asking how such a perspective could shed light on the process of mathematics teacher preparation in the current climate of educational reform.

First, we should remember that student teachers themselves may not have experienced as learners the kind of teaching promoted by the mathematics education reform movement, and exemplified in the classroom described in the first part of this paper. The literature on teacher socialisation confirms that prospective teachers come to pre-service programs with well established beliefs about teaching, which arise largely from their experience as students in classrooms and all too frequently are reinforced by their practicum observations and initial teaching experiences (Brown & Borko, 1992; Thompson, 1992). It is vital for the pre-service program to model alternative approaches which immerse student teachers in the same kind of mathematical experiences we hope they will create for their own students. In other words, the notion of a community of inquiry which embraces challenge as a means of growth is as relevant to the teacher education classroom as it is to the school mathematics classroom.

Just as learning mathematics within a classroom community has been described as a process of enculturation into the practices of the discipline, it is helpful to think of learning to teach as enculturation into a professional community characterised by particular values, beliefs, and practices. However, as foreshadowed above, the problem then arises as to *whose practice* students are to adopt. This question is often analysed in terms of the perceived discontinuity between the learning experiences offered by the university program and the supervised practicum—the gap between theory and practice. In addition, continuing to structure the practicum along the lines of a craft apprenticeship (Zeichner, 1996) tends to exacerbate the difficulties experienced by student teachers of mathematics in reconciling the theory they learn at university with the practices they observe in schools, since the apprenticeship model assumes that supervising teachers function as experts who pass on their accumulated craft knowledge rather than adaptive learners who are continually inquiring into their own practice.

As Frykholm (1998) points out, "as long as the traditional apprenticeship model is maintained, teacher educators run the risk of having their students 'apprenticed' to the very practices that the math education community has endeavoured to change" (p. 318). Even if the teacher education program is successful in reproducing the kind of mathematics classroom culture illustrated in the school based example above, we cannot assume that all our pre-service students will have the opportunity to work with similarly reform minded supervising teachers. A lack of connection between course work and supervised school experience thus becomes critical at a time when practising teachers themselves are being challenged to re-examine their own assumptions about mathematics learning and teaching (Frykholm, 1999).

The problems of practice, and particularly any disparities between the philosophies and expectations of the school and the university, clearly deserve systematic analysis by student teachers within the context of their own practicum experience—not only on their return to the university, but while they are actively engaged in teaching. Although teacher education programs commonly emphasise *reflective practice* (Cooney, 1994; Schon, 1987), it seems that student teachers have few

structured opportunities for detailed reflection on their own teaching, and on issues related to this experience (e.g., Borko & Mayfield, 1995).

The notion of learning through reflection is consistent with the sociocultural perspective on learning, as demonstrated by recent studies which have begun to apply sociocultural theories to teacher education. For example, Samaras and Gismondi (1998) describe a reflective teacher education program for early childhood and elementary settings, in which both coursework and fieldwork were based on Vygotskian principles incorporating collaborative partnerships between student teachers in a variety of formats. In the field of secondary school mathematics, Frykholm (1998) has piloted a new model of supervision which aimed to form a collaborative and self-critical community comprising student teachers, their supervising teachers, and university based graduate students who acted as mentors for the practicum. Both studies identified benefits of reflecting with a partner whose role was to act as a sounding board for ideas and to stimulate discussion of specific lessons and more general issues related to mathematics teaching.

However, it is rarely acknowledged that reflection of this kind is a skill which needs to be *learned*. In addition, studies of reflective learning in teacher education seldom examine in detail the actions of the mentor or partner in *facilitating* reflection. The next section describes one such study, in which pre-service students were offered structured opportunities for post-lesson reflection under the guidance of a mentor. The study highlights the mentor's scaffolding role in assisting one student teacher to analyse and resolve some of the conflicts between his practicum and university experiences.

Reflection and Learning to Teach

This study aimed to map relationships between the process of reflection and changes in the cognitions and practices of student teachers during the practicum component of a one year Postgraduate Diploma in Education course. A total of 58 students, who were preparing to teach in either mathematics, science, or the social sciences, participated in the study over a period of two years. A full description of the research design and data gathering methods is provided in Evans, Galbraith, and Goos (1993). For the purposes of this paper, interest centres on the use of semi-structured interviews that guided students' reflective analysis of their teaching.

Students were interviewed immediately after a lesson they taught during the last two weeks of each teaching practice block (of five weeks and six weeks respectively). These Reflective Interviews sought a self-evaluation and also prompted the student teachers to elaborate on goals and teaching methods, changes in knowledge and beliefs as a result of the lesson, and goals for future teaching. The interviewer (the author) acted as a mentor who drew attention to and asked questions about lesson events and the student teacher's interpretations in a non-judgmental way.

Students also participated in more active Reflective Interventions for two lessons during the second practicum session. These interviews used a prompt in the form of a Reflection Card, shown in Figure 1. The rows correspond to important lesson features for the students in the classroom: involvement or engagement,

STUDENT LEARNING	Teaching Approaches		Opportunities for Feedback	Indicators
	Expectations	Actions		
Engagement Involvement • attitude to learning				
Learning Process • how students learn				
Progress • how well students learn				
Social context • environment in which students learn				

Figure 1. Reflection Card.

learning processes, progress made during the lesson, and the social context in which they learned. The columns refer to major lesson features for the teacher: expectations and actions concerned with teaching approaches, the opportunities the student teacher created to obtain feedback on the progress of the lesson, and the indicators or cues during the lesson which the teacher actually used as feedback. The interviewer/mentor sought reflections on each of the sixteen cells, while the student teacher used the discussion to make written notes in each cell.

In the following section, results of qualitative analyses of interview transcripts are used to explain the mentor's role in promoting and scaffolding reflection.

The Mentor's Role in Scaffolding Reflection

The theory behind the mentoring process stemmed from the scaffolding interpretation of the zone of proximal development, which for student teachers encompasses aspects of their emerging teaching skills which have not yet developed fully, but which are taking shape under the guidance of other people. The mentor's role in scaffolding reflection involved modelling strategies for analysing the lesson, offering feedback on the students' analysis, questioning to elicit reflections that the students would not produce alone, and providing a consistent structure to help students organise and explain their experience (Tharp & Gallimore, 1988).

By comparing students' responses to the first and second Reflective Interviews it was possible to explore to what extent they had benefited from the mentor's assistance. This was done by constructing maps from the transcripts of the interviews. Reflection Maps centred on the students' goals and methods for the lesson, and the factors involved in their formation. These factors included the

student teacher's beliefs about teaching, ideas from the university program, and contextual features such as the particular group of students, constraints seen as being set by the supervising teacher, resources, time available, and curriculum requirements.

The interviews assisted student teachers to identify and analyse tensions arising from conflicts between their own beliefs about teaching, the theoretical perspectives offered by the university program, and the teaching approaches modelled by their practicum supervisors. The effectiveness of the mentor's scaffolding in promoting this kind of reflection is demonstrated through the following case study of Damien, a student teacher whose main teaching area was in mathematics.

Damien

One of the mathematics classes taught by Damien was a Year 10 group with a reputation for disruptive classroom behaviour, lack of motivation to learn, and low achievement. Damien's supervising teacher preferred to set these students tasks that they could readily achieve so that they might experience success, and he was firmly wedded to a teaching approach limited to whole class exposition and questioning. He encouraged Damien to model his own teaching on this approach, and was not responsive to Damien's tentative requests to try the more inquiry based, student centred methods promoted by the university program.

First Reflective Interview. By the time of the lesson observed for the first Reflective Interview, Damien was experiencing increasing discipline problems with the students, who were more accustomed to their regular teacher's firm management than to Damien's gentler approach. His Reflection Map for this interview is shown in Figure 2. The map shows his concern with the students' experiencing success as a first step towards changing their negative attitudes towards mathematics, and his corresponding approach of setting very easy tasks. This policy was reinforced by his low expectations of the students' abilities and behaviours, and by his supervising teacher. While Damien wished to help individual students understand the subject matter, he did so through whole class teaching and seat work. These methods became the context within which he tried to apply his beliefs about student involvement and encouragement.

Damien's interview responses also portrayed three major aspects of his personal context. He was unwilling to try methods he had not seen practised, and as a school student he had only ever *experienced* whole class expository teaching; he did not wish to become an authoritarian teacher, despite his classroom management problems; and he recognised conflict between his supervising teacher's methods and those endorsed by the university course. In theoretical terms, then, the assistance offered by Damien's supervising teacher was not well matched with the ZPD which defined the direction in which Damien hoped his teaching and management skills would develop. After four weeks of practice teaching, Damien was surviving, but was not really comfortable with what he was doing—in other words, he was experiencing at first hand the theory-practice dilemma referred to earlier, and the question of *whose practice* to adopt had become a significant issue.

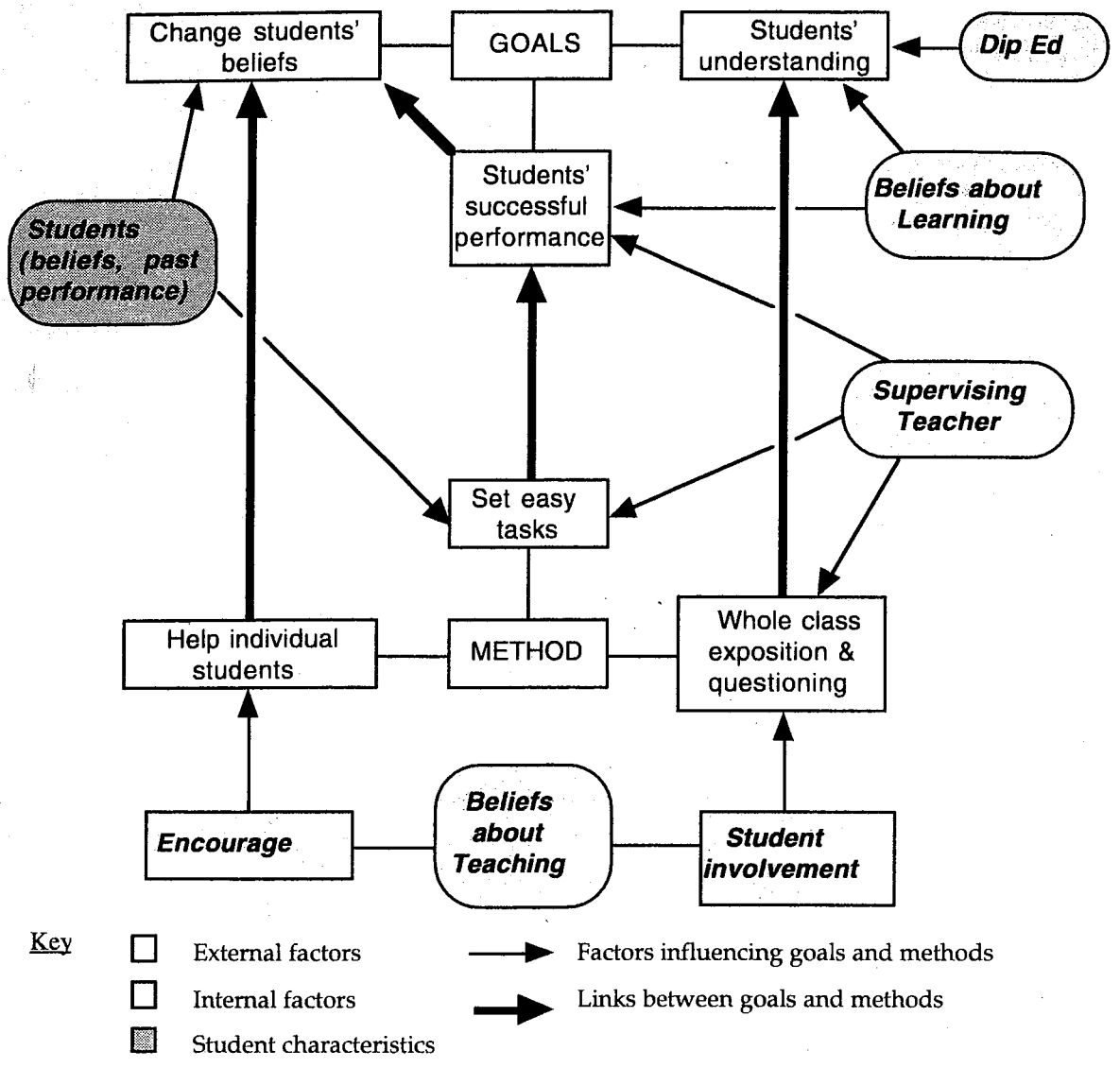

Figure 2. Damien's first Reflection Map.

First Reflective Intervention. The first lesson observed during the second teaching practice session brought Damien's problems to a head. Afterwards, when analysing the lesson with the aid of the Reflection Card, he realised that none of his expectations regarding student involvement, learning or behaviour had been achieved (see Figure 3). The analysis helped Damien to identify four problems, and to propose solutions to two of them.

STUDENT LEARNING	Teaching Approaches		Opportunities for Feedback	Indicators
	Expectations	Actions		
Engagement Involvement • attitude to learning	Attention, willingness to work. Enjoyment, confidence.	Named disruptive Ss, called for quiet. Gave positive feedback for correct answers.	General disruptive behaviour made it difficult to give individual attention.	Ss used delaying tactics, calling out, moving around room. Rude to each other and to me.
Learning Process • how students learn	Listen to teacher, copy example from board. Practise on similar tasks.	Whole class, work example on board. Asked questions. Worksheet for individual practice.	Verbal questioning, checked progress on worksheet.	Few Ss responded to questions. Many confused.
Progress • how well students learn	Low ability class – low expectations. Performance before understanding.	Chose simple example & exercises.	Toured to check progress, asked questions.	Few Ss finished worksheet. Few worked unless urged to do so.
Social context • environment in which students learn	Respect, courtesy. Discussion, cooperation.	Insisted on quiet when I speak. Threatened to remove one S.	Observed and listened to Ss.	Ss off task, restless, rude. Unwilling to help each other.

Figure 3. Damien's Reflection Card for first intervention.

First, Damien recognised that his students were responding to his very *modest lesson goals* by refusing to do the work he had prepared, and he resolved to raise his expectations of progress. Second, he acknowledged that the students' behaviour, and his own mild personality, made it difficult for him to gain their *attention and respect* by using the stern management techniques exemplified by his supervisor. While acknowledging that he could not change his personality, Damien decided to try a firmer classroom manner in future. Damien's perception of the students' behaviour, and his own belief in individual satisfaction as a means of creating *enjoyment and confidence*, made him unwilling to try *small group discussion* or other activities in which he feared losing control of the classroom. Activities of this kind, which give students some control over the pace and direction of the lesson, might also help overcome the discipline problems which prevented him from giving individual attention, feedback and encouragement to those students who wanted to work.

Damien was not aware of a fifth problem, concerning his expectations of students' learning processes. Although he believed that students learn better if they *think for themselves*, the teacher-led, whole class approach approved by his

supervisor provided few opportunities for them to engage with and make sense of the mathematics they were doing.

Second Reflective Intervention. The second intervention coincided with a visit by the university liaison tutor. In contrast to his usual teacher-centred approach, Damien's supervising teacher encouraged him to try a practical activity that allowed the students to discover for themselves the angle properties of isosceles and equilateral triangles. Students were to use rulers and compasses to construct triangles of given side lengths, measure the angles, tabulate their results, and draw the relevant conclusions. This change represented a high risk venture for Damien, but it paid a handsome dividend. Not only did he carry out his stated intentions to set higher expectations of his students and to take a firmer stand on discipline, but he was also able to try out a teaching strategy which allowed his students to "work things out for themselves", created enjoyment and confidence, and made it possible for him to give much more individual help and encouragement. It also had the unplanned effect of stimulating discussion between the students, whereas previously they had been expected to work individually and in silence (see Figure 4).

STUDENT LEARNING	Teaching Approaches		Opportunities for Feedback	Indicators
	Expectations	Actions		
Engagement Involvement • attitude to learning	Willingness to work, confidence. Wanted full attention today.	Positive "can-do" attitude. Encouraged individuals.	Able to observe & question Ss as they worked on task.	Ss asked me questions, wanted to show me their work. On task talk.
Learning Process • how students learn	Work things out for themselves. Learn by doing – "hands on" activity.	Prepared investigation task, provided necessary resources.	As above	S's comments to each other – sharing results, asking each other questions.
Progress • how well students learn	Raised my expectations, wanted Ss to understand.	Asked questions that pushed for understanding. Design of activity.	Toured room, observed, listened to Ss. (More time for this as I wasn't "out the front")	Ss showed me their work.
Social context • environment in which students learn	Respect, courtesy. Discussion.	Shifted talker at first sign of trouble. Extra firmness at start of lesson.	Had time to tour & listen.	Ss explaining to each other, cooperating. All participating (including "shifted" S).

Figure 4. Damien's Reflection Card for second intervention.

This temporary removal of the constraints which had limited Damien's development provided the mentor with a valuable opportunity to make the event into an occasion for learning. Her scaffolding of the reflective conversation helped

Damien to identify the reasons for the lesson's success, as the following excerpts illustrate.

The mentor (M) typically offered minimal assistance to students answering interview questions. However, the following example shows how a simple prompt for additional information **(PA)** elicited more than an answer to the question, as Damien (D) reflected on the benefits of group discussion and compared this with his own experience of school:

M: Any other signs that they were involved on task? **(PA)**

D: Oh yes they were talking amongst themselves about the work, a lot of them. I mean *it's much better to have them working together on the work than even by themselves, I think, because they are able to talk and express themselves mathematically, which is important. Something which I never had the opportunity to do myself which is a real downfall I think.*

Probing to explore an idea further **(PE)** was another scaffolding strategy employed by the mentor. The excerpt below shows how she attempted to extend the boundaries of Damien's ZPD by inviting him to re-consider his expectations of students' behaviour.

D: ...Today I particularly ... before the lesson I said to myself, "I'm going to make sure I do my transition between the five things in the beginning and to the next thing properly." I sort of had that expectation to do that.

M: So that's, ... it seemed like you really wanted their full attention.

D: Yes.

M: And what was the reason for that particularly today? **(PE)**

D: Well obviously because (liaison tutor) was there. Isn't that terrible? *If I had that expectation all the time I'm sure it would make things a lot better.* (...) The other thing I decided before the lesson I was going to ... if anyone misbehaved or something like that I would make sure that they sat down the front. I'd do that early in the piece, which actually worked quite well.

M: So do you think that's because you made a stand early in the lesson? **(PE)**

D: I think so. Like that's not to say ... *I should have done it a couple of other times as well I think.*

These excerpts, although brief, give some indication of how the mentor's questions not only drew attention to significant aspects of the lesson, but also stimulated further reflection by the student teacher (highlighted in italics above).

Second Reflective Interview. This lesson was with a different Year 10 mathematics class, but with the same supervising teacher—who continued to model and recommend transmissive teaching approaches. However, Damien's Reflection Map for the lesson (Figure 5) shows that he had internalised the reflections elicited in the second intervention, and adopted a slightly more liberal approach consistent with his own beliefs and the practices endorsed by the university's pre-service program. In fact, his belief structure remained intact, but his goals were now a little more specific, and "changing students' beliefs" was replaced by aiming to have students work things out for themselves as a means of understanding. He only partly retained whole class teaching as a method and now elaborated this method with specific statements about the nature of the mathematical examples to be used. He

made use of an activity worksheet (a modest advance, to be sure, but one which was significant in the light of his previous experience), which became the vehicle for his helping individual students. The way in which he related his beliefs to these goals and methods became detailed, explicit, and theoretically grounded (e.g., note his references to the benefits of student discussion and explaining). In short, Damien had found a way of reconciling his goals, methods, and beliefs, with the expectations of his supervising teacher.

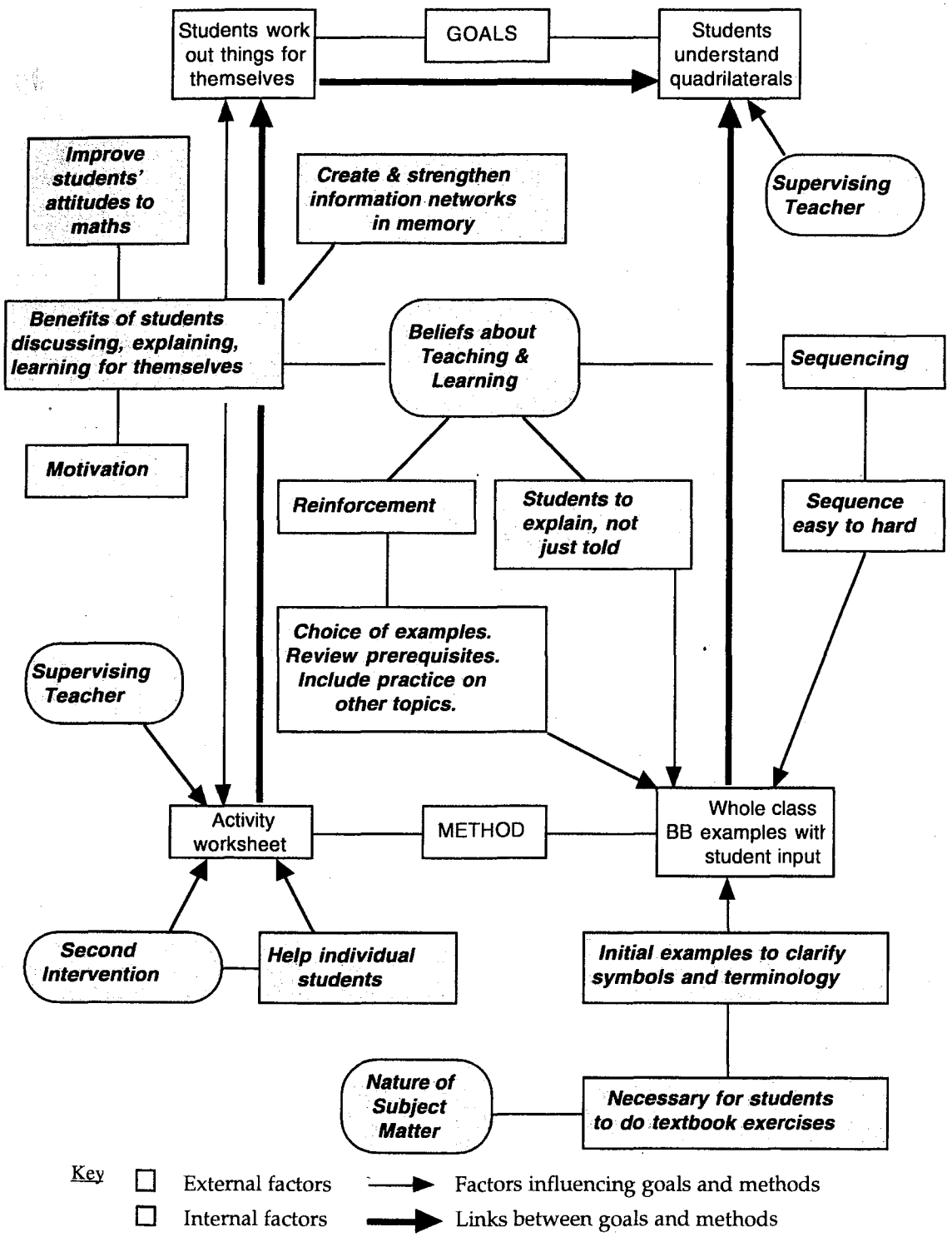

Figure 5. Damien's second Reflection Map.

Discussion

Possibilities for new approaches to mathematics teaching are suggested by the concept of the classroom as a community of inquiry, within which students learn to think mathematically by participating in the intellectual and social practices that characterise the wider mathematical communities outside the classroom. The first part of this paper examined one teacher's actions in establishing such a community in a senior secondary school classroom, and paid particular attention to the nature of expert scaffolding in creating zones of proximal development that extended students' mathematical thinking.

Despite the success of this teacher in socialising his students into mathematical practice, the widespread adoption of the teaching approaches documented in the first part of the paper remains problematic. For example, school structures and philosophies may make it difficult for teachers to adopt new roles and move out of their traditional position as the dispenser of knowledge. It must also be acknowledged that participation in a community of inquiry makes unfamiliar demands on students as well as teachers, and it is unreasonable to expect students to quickly embrace changes that challenge their ideas about what mathematics is, and how it is best learned.

Similarly, teacher beliefs raise a further barrier to change. It is widely recognised that teacher beliefs about the nature of mathematics and how it is learned influence the features of the classroom environment they create (Fennema & Loef-Franke, 1992; Thompson, 1992). This close connection between epistemology and pedagogy has significant implications for teachers' ability to translate into practice the changing goals of mathematics education. Because their ideas about mathematics were formed as a result of their own school experience, many teachers may not have learned to think mathematically themselves and are thus ill-equipped to model cognitive processes such as conjecture and generalisation (Schifter, 1993). As beliefs appear to be formed as a consequence of teachers' own experiences of schooling, and are often reinforced by practice teaching during their pre-service preparation, it is difficult to see how the cycle of (teacher beliefs $\rightarrow$ student beliefs $\rightarrow$ teacher beliefs) can be broken.

This problem was highlighted in the second part of the paper, which considered implications of sociocultural theories of learning for pre-service education of mathematics teachers. Again, expert scaffolding was the focus, in the form of semi-structured interviews that elicited student teachers' post-lesson reflections. There are clear parallels between the actions of the mentor and those of the teacher who featured in the first part of the paper, in that both used probing questions as a means of structuring the processes of inquiry they wished to develop in their students. In the case of the student teachers, the row and column headings of the Reflection Card also provided a visible reminder of the substance of such inquiry. However, as the card and associated interview were not subject specific, it may be beneficial to revise the intervention so as to target issues in mathematics education.

Since only four lessons taught by student teachers were observed and analysed in this way, it is impossible to determine whether the interviews produced lasting effects in terms of their ability to learn through reflection on their own practice. Nevertheless, the evidence presented here suggests that the mentor's scaffolding was successful in creating a zone of proximal development that challenged students

to analyse practical and theoretical dilemmas. Particularly important was the way in which the Reflective Interviews and Interventions sought information on contextual details such as student characteristics, supervising teacher requirements, and other material constraints, and invited the student teachers to explain how these features influenced lesson planning and delivery. Such an approach, in providing opportunities to articulate and defend goals, assumptions and beliefs, reveals how students define their own situation, and acknowledges the problematic nature of teaching.

Supporting reflective learning through scaffolding is one of many possible ways in which sociocultural theories could contribute to the re-vitalisation of pre-service teacher education. As argued earlier, reflection is crucial if student teachers are to analyse contradictions between the teaching approaches they are exposed to during the practicum and the reformist vision of mathematics teaching typically offered by university programs. The connection between reform and reflection is also evident in the NCTM's *Professional Standards for Teaching Mathematics* (National Council of Teachers of Mathematics, 1991), which identifies the ability to analyse one's own teaching and students' learning as one of the goals of reform oriented pedagogy.

The notion of learning in a community of practice also deserves further investigation within pre-service teacher education, in order to address the issue of *whose practice* student teachers come to adopt. Brown and Borko (1992) have argued that a common culture of practice could be created if universities worked more closely with schools, supervising teachers, and mentors for beginning teachers to support good mathematics teaching as described by current curriculum documents and classroom based research. Such an arrangement suggests a greater role for university based teacher educators in the professional development of supervising teachers (Borko & Mayfield, 1995). For example, modelling the mentoring approach to post-lesson conferences documented in the second part of this paper may lead to improvements in the quality of supervisory conferences—often criticised for their lack of depth and avoidance of controversial issues (Zeichner, 1996).

Conclusion

The two studies reported here were conceived and conducted independently of each other, and, indeed, the mentoring research project was not specifically directed at mathematics teaching. Nevertheless, juxtaposing the studies in this way has suggested possibilities for future research in mathematics teacher education. In particular, mentoring of student teachers via the interview procedures described in this paper appears to be beneficial in helping them to analyse not only the particular lessons they had taught, but also relationships between their beliefs about teaching and learning and the contextual constraints of practice teaching. Further research is needed to orient the interventions towards current issues in mathematics education, and to extend the mentoring relationship to include supervising teachers as teacher educators.

References

Australian Education Council. (1991). *A national statement on mathematics for Australian schools.* Carlton, Vic: Australian Education Council and Curriculum Corporation.

Borko, H. & Mayfield, V. (1995). The roles of the cooperating teacher and university supervisor in learning to teach. *Teaching and Teacher Education, 11,* 501-518.

Brown, A. L. & Campione, J. C. (1995). Guided discovery in a community of learners. In K. McGilly (Ed.), *Classroom lessons: Integrating cognitive theory and classroom practice* (pp. 229-270). Cambridge, Ma: Massachusetts Institute of Technology Press.

Brown, C. A. & Borko, H. (1992). Becoming a mathematics teacher. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 209-239). New York: Macmillan.

Collins, A., Brown, J. S., & Newman, S. E. (1989). Cognitive apprenticeship: Teaching students the craft of reading, writing, and mathematics. In L. B. Resnick (Ed.), *Knowing, learning, and instruction: Essays in honor of Robert Glaser* (pp. 453-494). Hillsdale, NJ: Erlbaum.

Cooney, T. J. (1994). Research and teacher education: In search of common ground. *Journal for Research in Mathematics Education, 25,* 608-636.

Evans, G., Galbraith, P., & Goos, M. (1993). Reflection in the development of teaching skills. (*Proceedings of the annual conference of the AustralianAssociation for Research in Education*) [Online]. Available: http://www.swin.edu.au/ aare/conf93/evang93.059.

Goos, M., Galbraith, P., & Renshaw, P. (1999). Establishing a community of practice in a secondary mathematics classroom. In L. Burton (Ed.), *Learning mathematics: From hierarchies to networks* (pp. 36-61). London: Falmer Press.

Fennema, E. & Loef-Franke, M. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). New York: Macmillan.

Forman, E. A. (1996). Learning mathematics as participation in classroom practice: Implications of sociocultural theory for educational reform. In L. Steffe, P. Nesher, P. Cobb, G. Goldin, & B. Greer (Eds.), *Theories of mathematical learning* (pp. 115-130). Mahwah, NJ: Erlbaum.

Frykholm, J. A. (1998). Beyond supervision: Learning to teach mathematics in community. *Teaching and Teacher Education, 14,* 305-322.

Frykholm, J. A. (1999). The impact of reform: Challenges for mathematics teacher preparation. *Journal of Mathematics Teacher Education, 2,* 79-105.

Lampert, M. (1990). Connecting inventions with conventions. In L. P. Steffe & T. Wood (Eds.), *Transforming children's mathematics education: International perspectives* (pp. 253-265). Hillsdale, NJ: Erlbaum.

Lave, J., Smith, S., & Butler, M. (1989). Problem solving as everyday practice. In R. I. Charles & E. A. Silver (Eds.), *The teaching and assessing of mathematical problem solving* (pp. 61-81). Reston, Va: NCTM.

Lave, J. & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation.* Cambridge: Cambridge University Press.

Mercer, N. & Fisher, E. (1992). How do teachers help children to learn? An analysis of teachers' interventions in computer-based activities. *Learning and Instruction, 2,* 339-355.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, Va: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics.* Reston, Va: National Council of Teachers of Mathematics.

Renshaw, P. (1996). A sociocultural view of the mathematics education of young children. In H. Mansfield, N. Pateman, & N. Bednarz (Eds.), *Mathematics for tomorrow's young children* (pp. 59-78). Dordrecht: Kluwer Academic Publishers.

Renshaw, P. D. & Brown, R. A. J. (1997). Learning partnerships: The role of teachers in a community of learners. In L. Logan & J. Sachs (Eds.), *Meeting the challenges of primary schools* (pp. 200-211). London: Routledge.

Resnick, L. B., Pontecorvo, C., & Säljö, R. (1997). Discourse, tools, and reasoning. In L. B. Resnick, R. Säljö, C. Pontecorvo, & B. Burge (Eds.), *Discourse, tools, and reasoning: Essays on situated cognition* (pp. 1-20). Berlin: Springer-Verlag.

Samaras, A. P. & Gismondi, S. (1998). Scaffolds in the field: Vygotskian interpretation in a teacher education program. *Teaching and Teacher Education, 14*, 715-733.

Saxe, G. B. (1991). *Culture and cognitive development: Studies in mathematical understanding*. Hillsdale, NJ: Erlbaum.

Schifter, D. (1993). Mathematics process as mathematics content: A course for teachers. *Journal of Mathematical Behavior, 12*, 271-283.

Schoenfeld, A. H. (1989). Problem solving in context(s). In R. I. Charles & E. A. Silver (Eds.), *The teaching and assessing of mathematical problem solving* (pp. 82-92). Reston, VA: NCTM.

Schoenfeld, A. H. (1994). Reflections on doing and teaching mathematics. In A. H. Schoenfeld (Ed.), *Mathematical thinking and problem solving* (pp. 53-70). Hillsdale, NJ: Erlbaum.

Schon, D. A. (1987). *Educating the reflective practitioner*. San Francisco: Jossey-Bass Publishers.

Tharp, R. G. & Gallimore, R. (1988). *Rousing minds to life: Teaching, learning, and schooling in social context*. New York: Cambridge University Press.

Thompson, A. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: Macmillan.

Zeichner, K. (1996). Designing educative practicum experiences for prospective teachers. In K. Zeichner, S. Melnick, & M. L. Gomez (Eds.), *Currents of reform in preservice teacher education* (pp. 215-234). New York: Teachers College Press.

Author

Merrilyn Goos, Graduate School of Education, The University of Queensland QLD 4072.
E-mail: <M.Goos@mailbox.uq.edu.au>