

Enactivism as a Theory and Methodology for Mathematics Teacher Educators' Learning as Teachers and Researchers

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This article proposes a methodology for becoming mathematics teacher educators (MTEs), in addressing the question, "How is enactivist learning done?" This methodology draws on enactivism, tying together methodological and theorising work done 25 years apart by two of the authors. The methodology has five dimensions—Using dissonance; Staying with the detail; Finding conviction; Making it real; Going meta—and operates as a methodology for MTE teaching, learning and researching. We offer three illustrations of these methodological dimensions, showing how they cross the three aspects of MTE practice. Towards the end of the article, we share the outcomes of a four-way discussion, using narrative interviewing protocols, consistent with the methodology dimensions. We exemplify how new awarenesses linked to action, enactivist learning, arise for one member of the group when being interviewed by the others. We show how the methodological dimensions are relevant to levels of MTE teaching, MTE learning, and MTE researching practice.

Keywords • mathematics teacher education research • enactivism • reciprocal narrative interviewing • structural coupling • observer dependence

Introduction

The four authors of this article are mathematics teacher educators (MTEs), currently working within two contexts, the University of Bristol in the United Kingdom and the University of Agder in Norway. Laurinda Brown and David Reid met in 1995; and Tracy Helliwell and Alf Coles had Laurinda Brown as one of their doctoral supervisors. This group of authors (referred to hereinafter by their first names only) forms a community of practice using a theory of cognition, enactivism, that posits "the enactment of a world and a mind on the basis of a history of the variety of actions that a being in the world performs" (Varela et al., 1991, p. 9). Enactivism, as a theory of learning, also provides a methodology that supports learning as MTEs, teaching as MTEs (Helliwell, 2021) and researching (Reid, 1996). As a group, and in various combinations of authors, we have described our practices as MTEs (Brown, 2015; Brown & Reid, 2006; Coles & Brown, 2001; Coles & Helliwell, 2025), developing theory over time using enactivist ideas. This focus contributes to a growing body of research concerned with MTE expertise as it develops through professional practice (Helliwell et al., 2026b; Potari, 2026). This article addresses the question posed in the call for manuscripts for this special issue, "How are methodologies used, adapted, or developed for the study of MTE work?" Hence the title of this article, "Enactivism as a Theory and Methodology for Mathematics Teacher Educators' Learning as Teachers and Researchers."



Methodology

We approach enactivism not only as a theoretical perspective but also as a methodology for examining our own learning as mathematics teacher educators and researchers. This approach follows Reid's (1996) proposition that, because research is itself a form of learning, enactivism can function simultaneously as a theory of learning and a methodology for researching learning. From an enactivist perspective, the observer is not independent of the phenomenon observed; rather, observations are necessarily observer-dependent and arise through the observer's own history and interactions (Maturana & Varela, 1987; Reid, 1996; Reid & Mgombelo, 2015). Thus, our own participation in the research was treated as what came to be noticed and understood.

Methodologically, we are attending to our interactions with one another, our experiences, and our engagement with enactivist theory. We view our learning as emerging through recurrent interactions rather than as the acquisition of knowledge that exists independently of those interactions (Reid & Mgombelo, 2015). Our analysis therefore involves moving recursively between experiencing, dialoguing, engaging with theory, and writing (Reid, 1996; Brown, 2015). This is consistent with developments of enactivist methodology in mathematics education, which emphasise the observer's participation (Simmt & Kieren, 2015). In this sense, methodology is not a predetermined procedure applied to the research; it is enacted through our ongoing participation in the process of doing, reflecting, and learning.

We begin by introducing two methodological frameworks for enactivism, developed at different methodological levels written 25 years apart: namely, research principles and teacher education purposes. The first, which also serves in this article as an introduction to enactivist theory, is based on the influential ideas from a conference paper written by David in 1996 called "Enactivism as a Methodology" (Reid, 1996), where he developed a set of *methodological principles for researching in mathematics education*. Tracy then shows how, in her 2021 doctoral study, she drew on some of David's methodological principles (Reid, 1996) to articulate *a methodology for mathematics teacher education* (Helliwell, 2021). Through studying her developing practice as a new mathematics teacher educator, having transitioned from teaching in schools, Tracy used enactivist theory to support her teaching, learning and researching. We illustrate Tracy's methodology for mathematics teacher education, which was formulated at the level of purposes (Brown & Reid, 2006; Helliwell et al, 2026a) or actions, through discussion of three cases, to give insight into how these purposes are realised and developed. Finally, we exemplify our reflexive use of Tracy's methodology for our continuing process of learning by presenting and reflecting on parts of a collaborative discussion. The following two methodological discussions are written in the first person by David and Tracy, respectively.

Methodological Discussion 1: Enactivism as a Research Methodology: David

In 1996, I presented the way in which I had, with colleagues, employed the ideas of enactivism as the methodology of my doctoral study (see Reid, 1996). This has remained, explicitly or implicitly, my way of working ever since, and some of the ideas I presented have been elaborated and expanded by others. A selection of this work can be found in the *ZDM-Mathematics Education* issue on enactivism as a methodology (Reid et al., 2015).

Enactivism is a biological theory of cognition in which cognition is understood as embodied action. The basic principles are outlined by Maturana and Varela (Maturana & Varela, 1992; Varela et al., 1991). Methodologically, a key point is captured in the words of Maturana, "An observer has no operational basis to make any statement or claim about objects, entities or relations as if they existed independently of what he or she does" (1988b, p. 30). This quotation succinctly sums up the key idea of observer dependence, which, along with structural coupling, provides the basis for enactivism as a research methodology.



Observer Dependence

Maturana was a biologist who developed his ideas out of his work on the vision systems of frogs and pigeons. He began by making the usual assumption that he could observe what is truly going on in the way animals perceive the world. He concluded that frogs and pigeons do not represent an external world, they construct one:

In 1965 when I was studying color vision in pigeons, I realized that I could no longer pretend that one saw the colors as features of an external world, and that I had to abandon the question, "How do I see that color?" and ask instead, "What happens in me when I say that I see such a color?" (Maturana, 2002, pp. 5–6)

This is a fundamental principle of enactivism as a methodology. An enactivist researcher does not ask, "What do I observe?" but rather, "What happens in me when I say I observe something?" Maturana recognised through his work on perception, that perception is an active process. Without a feedback loop of action and perception, perception does not occur: "Cognitive structures emerge from the recurrent sensorimotor patterns that enable action to be perceptually guided" (Varela et al., 1991, p. 173). Thus, cognition is fundamentally an active process. The phrase "knowing is doing" captures this insight.

Structural Coupling

Maturana described living systems, including mathematics education researchers, as "structure determined systems":

If I have a living system ... then this living system is in a medium with which it interacts. Its dynamics of state result in interactions with the medium, and the dynamics of state within the medium result in interactions with the living system. What happens in interaction? Since this is a structure determined system ... the medium triggers a change of state in the system, and the system triggers a change of state in the medium. What change of state? One of those which is permitted by the structure of the system. (Maturana, 1987, p. 75)

Structural coupling tends to be self-reinforcing, either because of the structures of the entities involved, or because they form part of an entity whose structure requires maintaining structural coupling among its parts: "We speak of structural coupling whenever there is a history of recurrent interactions leading to the structural congruence between two (or more) systems." (Maturana & Varela, 1992, p. 75).

Most aspects of the structure of a living system cannot be observed directly, but its actions. Structural coupling of two systems is therefore visible in the coordination of their actions. Actions can be coordinated at different levels. Two tango dancers, for example, coordinate where and when they step in a relatively simple coordination of actions. But they also act in ways that modify how they act. When the follower does not follow the leader's lead, for example, the follower signals that the leader should take a different action. This disrupts the existing coordination of actions and triggers a different coordination of actions. In this case, the coordination of actions serves to coordinate other actions: they are *coordinations of coordinations of actions*. This is what Maturana described as "communication", or to use the verbal noun preferred by some enactivists, "languaging".

In his writings in the 1970s and 1980s, Maturana used the phrase "consensual domain" to describe the domain of recurrent coordinations of actions between two structurally coupled living systems. He described this as "a consensual domain of interactions, because it arises as a particular manner of living together contingent upon the unique history of recurrent interactions of the participants" (Maturana, 1988a, p. 35).

In my 1996 paper, I wrote about how I see structural coupling in research:

In all research we establish a relationship, a structural coupling, with the milieu which is to be our topic of study. We interact with the people, objects, chemicals, and ideas we find there. By so doing we modify the milieu for each of its inhabitants, and the ... entities in the milieu adapt in ways determined by their structures but triggered by our presence. At the same time, we engage in the process we are



there for, adapting in response to the triggers offered by the milieu. Note that an important part of our structures are our theories, beliefs, and biases. The changes which can be triggered in us, that is, what we can learn about the research context, are determined by our theories, beliefs and biases. What we learn is determined by what we know. (Reid, 1996, pp. 205–206)

I described the research process as involving structural coupling with the data, as well as structural coupling with other researchers:

As a community, researchers in a field form the context in which their research occurs. I have to learn in ways which allow me to remain in interrelation with the participants and other aspects of my own research, and simultaneously in ways which allow me to remain a member of this research community. This establishes constraints. There are constraints offered by research data and constraints offered by the research community. (p. 206)

I also provided an example of how observer dependence, combined with the structural coupling of researchers with their colleagues, could produce a research method that captured “multiple consensual contradictory perspectives” that say different things about the data while remaining mutually intelligible. This can be achieved in several ways. For instance, through “the participation of a number of researchers, each with her or his own agenda, theories, and background” (p. 207), or by a single researcher returning to the data sufficient time has passed for their evolving structure to enable them to find something new. A researcher can also consciously adapt a perspective, different from one used previously. In each case, the goal is to see the data in multiple ways and to use observer dependence to explore the possibilities the data afford.

Methodological Discussion 2: Developing a Narrative-Enactivist Methodology for Becoming a Mathematics Teacher Educator: Tracy

In my doctoral thesis, “*Developing a narrative-enactivist methodology for becoming a mathematics teacher educator*” (Helliwell, 2021), I presented a multi-stage, multi-dimensional methodology for becoming a mathematics teacher educator, drawing on ideas from narrative inquiry and enactivist philosophy. The motivation for this work was both personal and professional, arising from my need to learn a new way of being with mathematics teachers as a novice mathematics teacher educator.

In developing this methodology, I articulated three interconnected methodological levels in becoming a mathematics teacher educator (Helliwell, 2021):

- a methodology for researching my process of becoming a mathematics teacher educator (i.e., a *research methodology*);
- a methodology for learning to teach mathematics and mathematics teachers (i.e., a methodology for *mathematics teacher educator learning*); and
- a methodology as a way of working with mathematics teachers (i.e., a methodology for *mathematics teacher education*).

Enactivism as a Research Methodology

At the level of *research methodology* (the same level as David articulated in his 1996 conference paper), I developed a set of eight principles to guide my process of research (see Helliwell, 2021, for a detailed elaboration of all eight principles). The three of these principles (engaging in all four stages of narrative inquiry; utilising multiple perspectives in multiple ways; and enabling new ways of making distinctions) which connect most closely to those articulated by David are described here as a basis for a narrative-enactivist methodology for mathematics teacher education.



Engaging in all four stages of narrative inquiry

In terms of narrative as a process of inquiry, Clandinin and Connelly (2000) commented, “In the construction of narratives of experience, there is a reflexive relationship between living a life story, telling a life story, retelling a life story and reliving a life story” (p. 71). Retelling is not mere repetition; it is a generative act that requires us to move beyond viewing stories as fixed objects or entities, allowing “possibilities [to] emerge for reliving in more thoughtful and responsive ways in the future” (Clendenin et al., 2011, p. 34). For David, observer dependence and structural coupling are important principles underpinning enactivism as a methodology. Observer dependence and structural coupling are both concepts connected to this process of narrative inquiry in the sense that we cannot go beyond what is observable yet through engaging with the four stages of narrative inquiry—living, telling, retelling, and reliving of life stories—we are bringing with us our histories of structural coupling whilst expanding the space of what is possible to observe.

Utilising multiple perspectives in multiple ways

In the previous methodological discussion, David described having multiple people viewing data from different perspectives or an individual looking differently over time. Since my doctoral research was a form of self-based study, the principle of multiple perspectives involved me reporting on my shifting perspectives through telling and retelling stories that aimed to capture and express my process of becoming a mathematics teacher educator whilst actively shaping the process. By drawing on “creative analytic practices” (Bochner & Ellis, 2016; Richardson & St. Pierre, 2018) that combine the process of telling stories with analytical techniques more associated with traditional data analysis, I was able to analyse data from two complementary perspectives—a creative arts perspective and a formal analytical perspective.

Enabling new ways of making distinctions

From an enactivist perspective, learning (and therefore, researching) involves changing how we make distinctions, that is, changing how we see and engage with the world (which includes data we may have collected). According to Maturana (1988a) the “basic operation [that we perform] in the praxis of living is the operation of distinction” (p. 12). What we observe, however, is shaped by our current structures; thus, many stimuli that could potentially trigger a response, go unnoticed. By expanding the range and differentiation of our interactions, those that trigger responses can also expand, and in return new ways of seeing are made possible. Dreyfus and Dreyfus (1991) described this as a process of becoming expert through developing increasingly refined discriminations, enabling ever more nuanced and intelligent responses. In both methodologies, learning in and from experience is important. For David, learning in ways interrelating with other researchers in his group with different views and about his own focus (Reid, 1996). In my study, researching as an active process of perceptually guided action, was a process of making new and increasingly refined distinctions (Helliwell, 2021).

Enactivism as a Methodology for Mathematics Teacher Education

The methodological principles outlined above informed my approach to analysing a range of conversations between myself and a group of in-service mathematics teachers over the course of a year-long professional development programme. Through enacting these research principles, a methodology for mathematics teacher education emerged, which I ultimately expressed as five methodological dimensions, specifically: Using dissonance; Staying with the detail; Finding conviction; Making it real; and, Going meta. For each dimension, I developed a range of actions that shaped my learning and informed my practice as a mathematics teacher educator. The five dimensions, and examples of associated actions, are presented in Table 1.

Table 1
Methodological Dimensions for Becoming a Mathematics Teacher Educator

Methodological dimension	Description	Examples of associated mathematics teacher educator actions
Using dissonance	Developing an open mind. Trapping judgements and a false sense of accomplishment. Learning about oneself and others. Triggering a range of considered and thoughtful responses in the moment of interactions	• Staying alert to instances of dissonance, in the moment, provoking a more thoughtful response, rather than reacting emotionally or judgementally. • Prompting others in retrospectively re-entering moments of dissonance as a way of identifying issues in relation to practice. • Encouraging others to locate their own moments of dissonance in the future.
Staying with the detail	Developing (new and different kinds of) insights. Letting go of assumptions and preconceptions. Being open to newness and difference. Releasing self from judgement and becoming curious. Listening like everything being said is true (or listening openly).	• Becoming attuned to the different modes of talk and prompting for clarity when ambiguous labels/terms/phrases are used. • Noticing when a slide from one mode of talk to another might be taking place and responding by resisting the slide or encouraging it. • Identifying opportunities for doing mathematics before considering implications on practice.
Finding conviction	Nurturing a sense of agency and fostering a sense of ownership both in self and in others.	• Fostering a sense of ownership, through supporting others in establishing their own teaching issues and avoiding the urge to impose one's own teaching issues on others. • Nurturing a sense of agency through organising the gathering of potential teaching strategies and associated actions, supporting the emergence of new categories. Flagging modes of speaking that act to limit possibilities.
Making it real	Expanding the space of the possible by realising possibilities. Committing to not going beyond what is observable, observing one's own actions in relation to the actions of others. Constantly and critically reviewing practice.	• Creating the conditions for mathematics teachers to learn for themselves. Expanding what is possible for teachers by developing a repertoire of possible actions that may be triggered in a range of situations. • Using mechanisms that specifically support the move to enacting new practices, such as prompting ideas to be considered in relation to practice; making and sharing commitments; immediately putting ideas into practice; conducting small-scale research projects; using writing as a mode of inquiry; imagining the playing out of hypothetical situations.
Going meta	Supporting learning and supporting the observing of learning (in self and in others).	• Attending to the process of talking about teaching and learning mathematics as well as the content. Noticing both the content and form of comments made and commenting on them at a meta-level. • Supporting the seeing of process and form in others and remarking when this seeing is shared. Commenting on what is the same and what is different and inviting others to do the same. • Commenting on instances of observing learning (either in teachers or in teachers' descriptions of their students).



Three Illustrations

In this section, the focus is on the five methodological dimensions developed by Tracy in her doctoral work (Helliwell, 2021) and summarised in Table 1. David, Laurinda and Alf draw on Tracy's five methodological dimensions to talk about three illustrations from their experiences of being mathematics teachers and teacher educators. The aim is to explore the question, "How is enactivist learning done?" Note that these illustrations come from teaching that occurred before Tracy developed her methodological dimensions, but they are nonetheless visible there, reflecting the common enactivist perspective we share. The key contribution of this article is this methodological approach to becoming an MTE, which we end by using in a reflexive manner via narrative interviews. This methodology is relevant to researching, learning and teaching as an MTE and the illustrations cross these levels. The following illustrations are written in the first person from the perspective of the author(s) named in each subheading.

Illustration 1: David

I am a teacher educator, and a researcher, but I do not (usually) research teacher education. As a teacher educator, I base what I do on the principle that becoming a teacher is a matter of knowing, not knowledge, and that knowing is doing. Concretely, I teach a course on teaching algebra. This course falls into two parts. In the first part, the preservice teachers solve problems together, and we reflect on those experiences using a set of distinctions drawn from literature: generative/transformational/meta-level (Kieran, 2004); arbitrary/necessary (Hewitt, 1999); and specific/generic/general (Mason & Pimm, 1984). In the second part, the preservice teachers teach each other familiar mathematics from school but using unfamiliar methods and materials. This is referred to as the Teacher-of-the-Week (TotW) lessons. In 2023, a doctoral student, Karoline Kongshavn, observed my class and interviewed the preservice teachers as part of her research, giving me a new perspective on my practice. She has written several articles about her observations (e.g., Kongshavn, 2023), and this has given me the opportunity to see my work as a teacher educator from a different perspective.

The problems I have chosen are such that they are genuine problems. The preservice teachers have not seen them before and do not have ready-made methods to solve them, but they can be solved in the timeframe of a 90-minute class session. When some solutions have been found I select groups to present their solutions to the class. At first, the preservice teachers are reluctant to question their peers' solutions, and my role is to ask clarifying questions about the assumptions being made and the notations and representations being used (Staying with the detail). As time goes on the preservice teachers become more comfortable doing this themselves. The presentations of solutions also provide opportunities to make connections with the literature (Going meta). For example, when generic examples are used in an argument, or a solution involves a transition from generative to meta-level algebraic activity.

Before the preservice teachers deliver their TotW lessons, I teach two model lessons. In these I emphasise that those in a student role should not pretend to be in school. They should be themselves. Although the topics taught are all in the school curriculum, they should learn something genuinely new about these topics in the TotW lessons. This is often the logical underpinnings of algebraic methods they have been taught but can also be seeing concepts in a new light via new representations of them. In any case, the experience for the TotW teachers should be one of teaching something the students do not already know (making it real).

Kongshavn's research (e.g., Kongshavn, 2023, 2025; Kongshavn & Knipping, 2025) has given me insights into how the preservice teachers experience the TotW lessons, as she interviewed them before and after they taught their lessons. The following comment, made by a preservice teacher, Agnes, before she taught her TotW lesson, shows that she is thinking of her lesson in what I would call a "teacherly" way, by anticipating the students' difficulties. Agnes (pseudonyms used to identify



participants in Kongshavn's studies) is also showing meta-awareness: the point of teaching a TotW lesson is to experience uncertainty (using dissonance, staying with the detail, making it real):

Agnes: I think it might be challenging to formulate questions so that they understand what they are going to do. Because that is at least something that I think has been a bit challenging when we have worked with the tiles in the past. Like, oh, what should I do now, what is it that I am supposed to find out? Where should I begin? But I suppose that's also part of the process. (Kongshavn, 2023, p. 3230)

The next observation, made after teaching a TotW lesson, indicates that the preservice teachers face some of the real dilemmas familiar to all teachers:

Aurora: I saw something that was quite interesting, at the very back – these boys. They had made – it was like – these pieces. Then there was a positive and a negative, and then they had one that was unspecified. So, they made a general formula with sort of the one that was unspecified. And it was those smart guys in our class, so they were completely certain that what they did was right. And I was like – it might be correct, but I just can't seem to see it at this point. And I was just like, "What is this?" It was probably correct, but ... (Kongshavn, 2023, p. 3231)

Facing these real dilemmas in the controlled context of a university class allows us to reflect on them, and to use such experiences of dissonance to discuss possible teacher actions.

The practice of staying with the detail when listening to students' solutions of problems is one that at least some of the preservice teachers also recognise as a guiding principle for teaching, related to expecting explanations. The following observation from a post-lesson interview illustrates this:

Barbara: I feel like, often—and one can notice this both today, and also in David's lessons—that whenever someone comes up and explains they're like; "and then you do like this, and then you do like that" [imitating students], and that many of them do not explain it step by step, because, like, "but you guys also know that" [imitating students]. And then it's like, well, yes, I do know that. But if you challenge them, like; "but why?" "I don't know" [imitating students]. (Kongshavn & Knipping, 2025, p. 13)

After each TotW lessons (and my model lessons) the preservice teachers reflect on the lesson, both in writing online and in a face-to-face discussion. The first prompt in the discussion is to mention things they liked about the lesson, and the ideas that come up become guiding teaching principles. To me it is important that these principles arise from the preservice teachers' own experiences and comments, rather than being given by me. I feel it is important that they are the preservice teachers' observations (Finding conviction).

Here are two comments from online reflections on lessons:

PST4: I think the tasks had a nice connection so that we could use what we had arrived at in further explanations. [...] Also, I liked that you asked us to explain our solutions during group work.

PST5: The lesson was built in such a way so that the ways we learned to use the algebra tiles in one task could be used further on in the next task, especially the use of zero pairs. (Kongshavn, 2025, p. 51)

Principles such as asking for explanations and building on immediately prior learning are ones that I expect to emerge, but each year is a little bit different and to me it is more important that the preservice teachers arrive at these principles themselves than that they arrive at some "optimal" set of principles.

This comment shows how "Going meta", using concepts from the readings, also inform the reflections after the lessons:

PST7: In this lesson, I think there was a good transition between how we explored the square numbers in the times table, to using the tiles to express the difference between a specific square number and the numbers if you went one or more notches to the side, and finally to expressing the square sentences when using the tiles. Feel it showed a good way to work from generic examples and then develop it further into something general. (Kongshavn, 2025, p. 51)

I have not previously described my approach to teacher education through Tracy's five methodological dimensions (Helliwell, 2021), though evidently, it is possible. This reflects their relevance to the enactivist perspective I use to describe what I do. I see what I am doing as an MTE as



fostering a “reformulation of their praxis of living,” to use the words of Maturana (1988b). What is occurring is a change in the kinds of descriptions and reflections the preservice teachers offer, a change in how they reformulate their praxis of living. As an observer I might describe these as more “teacherly”, but I do not see the preservice teachers as appropriating principles or norms. Rather, they are engaging in activities with their peers in which their actions (including their reformulating) become coordinated, and such that the community they form acts in a more teacherly way. How exactly this occurs I do not know. I ask them to act like teachers (in their TotW lessons) and to reflect on what they have done. In most cases doing what teachers do also means that they know in a way more like the way teachers know, becoming more like teachers than students.

Illustration 2: Alf and Laurinda Co-teaching and Co-researching in Mathematics Classrooms and Working with Prospective Teachers

We met in 1995, when Laurinda was a mathematics teacher educator and Alf was a mathematics teacher. We became co-teachers and co-researchers being able to swap roles when Laurinda visited Alf’s school at most once a week. We would plan the teaching together the evening before and reflect afterwards. Laurinda had begun to work with enactivist ideas focusing on multiple views, working collaboratively. At the start of each year, the teachers in Alf’s department discussed what being a mathematician meant to them. Their ideas formed a poster displayed in the teachers’ rooms. All views were collected and there was no sense of there being wrong answers. As a head of a school department, Alf had elements of an MTE role and the practice of gathering views on being a mathematician contained an element of working collaboratively on finding conviction, in relation to teaching mathematics. Laurinda observed that Alf, early in the year, teaching a class new to him, made comments that pointed to behaviours that students were involved in that supported their learning, such as “this group are getting organised.” These meta-comments (Going meta) were apparent less frequently as the year progressed and some of the meta-comments became the children’s ways of learning mathematics (see Coles & Brown, 2001). These practices were different each year, dependent on the group, but it was still noticeably Alf’s classroom because the students led what they did. Alf, however, came to use his growing awareness of, say, “getting organised” as a way of working, for instance inviting students to display one instance each of a tessellation of a pentomino on the board and the class discussing what is the same and what is different. Our experience was that observing and taking multiple views lead to awarenesses linked to actions, supporting the process of learning. This process of patterning relates to the biological basis of being and so learning seemed to feel natural for the children and for the teacher.

When Alf was appointed at the University in 2010, we became colleagues working with prospective teachers and continued co-teaching, planning together before teaching and reflecting afterwards. Similarly, although the group of prospective teachers at the University of Bristol was made up of different individuals and practices, the strong meta-practices (Going meta) gave a similar feel to the group each year. The teachers were supported in identifying labels that described issues emerging from their early experiences of teaching (Staying with the detail) such as, “How do I know what they already know?” (Making it real). Mathematical behaviours noticed in the classroom because teacherly mathematical behaviours, recognised in an initial teacher education setting. Our conversations became ritualised in what we came to call reciprocal narrative interviewing supporting each other to get to new labels that could support our future learning (Staying with the detail; Making it real; Going meta).

We took our conception of reciprocal narrative interviewing, in part, from the work of Petitmengin (2006). We recognise four protocols that inform our roles as interviewers, the first three described in Petitmengin’s article as follows:

1. Stabilising attention. A regular reformulation by the interviewer of what the subject has said, asking for a recheck of accuracy (often in response to a digression or judgement). Asking a question that brings the attention back to the experience, e.g., How did you do that?
2. Turning the attention from “what” to “how” (never “why”).



3. Moving from a general representation to a singular experience. This is what we would term "story" with our prospective teachers, a re-enactment, reliving the past as if it were present. Talking out of experience, not from their beliefs or judgements of what happened, often involves the student teachers in a move to the present tense. Staying with the detail is important, a maximal exhaustivity of description that allows access to the implicit.

To these actions we added a fourth protocol developed from our experiences of working with prospective teachers:

4. Getting to new category labels. After dwelling in the detail, telling stories and exploring without judgement or digressions, the invitation, when narratively interviewing or working with a group of prospective teachers, is to invite statements of what is being worked on (relevance). These statements might be different for each member of the group, since the attention of individuals might have been on different aspects of the stories. New category labels might be identified to work on, such as "responding to individuals" or "how do I know what the students know?" that will link to learning new actions. (Brown & Coles, 2019, pp. 177–178)

We now draw out some links between the reciprocal narrative interviewing protocol above and Tracy's five methodological dimensions for becoming a mathematics teacher educator (Table 1). The first and third protocols, stabilising attention and moving from a general representation to a singular experience, are both mechanisms for "staying with the detail." Stabilising attention is about supporting a focus on experience, avoiding too quick an interpretation of that experience. General representations (often linked to immediate interpretations or judgments) tend to arise from existing ways of seeing/doing/being and so offer little opportunity for learning. Turning attention from what to how and getting to new category labels relate to "making it real" and "going meta." When, in a classroom, Alf offered to the class the meta-comment, "This group are getting organised, and being organised is an important part of becoming a mathematician," he was likely offering the students a new category label for their experience. He was commenting about their actions (hence a meta-comment) and in a manner that is intended to influence future actions (of that group and the rest of the class). Similarly, in a teacher education setting, an MTE hearing three related stories from the experiences of prospective teachers and offering the group the meta-comment, "What seems to link these stories is the challenge, how do I know what they [the students] already know?" This is again offering what is likely a new category label for the prospective teachers. Having identified such a label, the group work on a range of actions in the classroom which could support (in this case) "knowing what they know." And, again in a parallel with the classroom, such meta-comments and lists of associated strategies are intended to offer prospective teachers prompts for trying out new actions in their classrooms, expanding the space of possible things for them to try out and do.

Tracy's methodological dimensions (Helliwell, 2021) operate at the levels: learning to teach mathematics, learning to teach mathematics teachers, and researching the process of learning to teach mathematics teachers. Our history of collaboration also crosses all three levels, and we have tried to point to elements of self-similarity across these levels, which are present in the dimensions. The classroom stories (of Alf's meta-comments, for example) also suggest the relevance of Tracy's methodological dimensions (Helliwell, 2021) to the process of learning mathematics as we have tried to label (e.g., via ideas of what it is to be in the process of becoming a mathematician).

Illustration 3: Alf

In Brown et al. (2018), I described the following incident with a prospective teacher (the incident is also discussed in Coles, 2020, pp. 17–18).

A prospective teacher (in my tutor group) emailed me to ask if we could meet after school one day to discuss difficulties he had been experiencing at his placement school. By way of background, this teacher was in the middle of a 12-week placement at a high-achieving city school. He had previously spent a six-week placement at a rural school with much more mixed levels of attainment. At that first



school, he had been judged as being at a “Pass” level by the school mentors at one of the four Review Points of the PGCE course. We met soon after the email and I invited the prospective teacher to talk to me about the difficulties he was experiencing. This provoked a number of stories of incidents in school. I was aware of listening with a sense of what I might be able to offer. I suspect I made little comment in between the description of incidents. I have lost the details of these stories except for the one that provoked a response in me. This story concerned an incident with another mathematics teacher in the staff room and what the prospective teacher expressed as being on the receiving end of a social rudeness. With this, an awareness crystallised: in all the stories of incidents, one thing he was not talking about was the students he was teaching. I expressed this awareness with the suggestion; it was as if his concerns were all centred around his relationships with the other teachers. I advised that he place energy and attention in his relationships with the students he teaches and forget about how he thinks the other teachers are reacting to him. Soon after expressing this awareness, our meeting ended. I had no further contact from this prospective teacher, in relation to difficulties in school. His profile (as judged by the school) improved over the next (and final) two assessment points. Towards the end of the year I asked him what, if anything, he felt had changed in the second half of the course. He mentioned several factors as having made it easier for him to deal with issues and incidents in school and teach more effectively: one was meeting a friend who had some similar difficulties he had experienced; and one was the meeting described above. (Brown et al., 2018, p. 18)

I used this incident to reflect on Tracy’s methodology for becoming a mathematics teacher educator (Helliwell, 2021). At the time of the incident, I was an MTE with ten years’ experience in the role. In asking the prospective teacher to describe their difficulties there is an invitation for them to “stay with the detail” of their experience. As a MTE, I need to feel my way into what is occurring in the school, to be able to offer anything in response. I do not believe I can ever know what someone else is experiencing, but I do believe I can enter into a situation and understand, from my perspective, something of its possible tensions or dilemmas. As the prospective teacher is talking, as I report above, I felt an increasing sense of unease or “dissonance.” They were not talking in a manner that I am used to prospective teachers talking. A reason for this dissonance was not immediately apparent to me. I was aware of not having “conviction” about what was transpiring. There was a need, for myself to “stay with the detail” of what I was experiencing in response to the stories, as I supported the prospective teacher in staying with the detail of what was happening at school. With the story of the social rudeness, an awareness crystallised (“Finding conviction”) for me and, with that, the possibility of “Going meta.” I reflected back to the prospective teacher my awareness about the conversation we were having (that it was focused on adult relationships not relationships with students). I suspect that reflecting back of a “meta” awareness, was all that was necessary at this point for the prospective teacher to try something new in school, along with the suggested strategy of placing his attention and energy in his relationship with students (“Making it real”, in the sense of offering a mechanism to support enacting new practices).

There are several aspects to note, in relation to the way enactivism is part of initial teacher education at the University of Bristol. The first thing to note is that, after ten years of work as an MTE, it is still highly relevant to consider myself in a process of “becoming a mathematics teacher educator.” In other words, that level of Tracy’s methodology is never completed or finished. Secondly, the self-similarity across levels of communication is apparent and one of the key insights I take from enactivism as applied as a methodology and in communication. For instance, “Staying with the detail” is as important for me, as an MTE, as it is for the prospective teachers with whom I am working. “Going meta” is an almost constant internal prompt and aim, as an MTE, and it is also an idea we offer to prospective teachers in relation to their teaching (i.e., looking for opportunities for “going meta” in talking with their students). Indeed, the prompt I offer in the incident above is one that will potentially support the prospective teacher in a meta-awareness about their actions in school (e.g., “Am I thinking about adults, or children?”). So, “Going meta” as an MTE is a mechanism that may occasion prospective teachers in going meta in their practice in school.

The incident points to the aims of our initial teacher education course and the personalisation of the journey into becoming a mathematics teacher. The conversation with the prospective teacher was



not one I entered into with any sense of how it might go, or what might be needed. Working with Laurinda, some years ago, on psycho-analytic links to enactivism, teaching mathematics and teacher education, we were struck by Bion's (2013) admonition to analysts to approach conversations with analysands without "memory or desire" (first published in 1967). The point here is about a non-attachment to how the analysand (or, in our case, prospective teacher, or student) has been in the past (no memory) and an openness to who that person is becoming (no desire). I do have my own conviction about ways of being that will support prospective teachers to learn. It is out of this conviction that I notice the difference, or dissonance that I report back in the incident above. The teacher in the incident had clearly recognised that something was not working for him in school. His actions lacked effectiveness, in that context. In response to his expressed need, I offer my own awareness which is then up to him to do something with or not. Where the new actions will take him is unknowable at the time of the conversation.

The Process of Coming to Know Something New

In the illustrations above we have provided insights into how enactivism underpins our professional life as MTEs, via a methodology for becoming, which is as relevant to us now as it was when we first entered into the work of mathematics teacher education. In this section, we illustrate our ongoing use of this methodology to support our learning. In other words, we continue to explore our guiding question, "How is enactivist learning done?" by offering an illustration of how Tracy's methodological dimensions (Table 1) and the narrative interviewing protocol described above support our continuous adaptation and development as MTEs.

Thus, as part of the process of writing this article, we engaged in a four-way reciprocal narrative interview. We each initially shared a piece of writing, less than 500 words, focused on our current research. We then had an hour's meeting where we enacted this four-way process, and drawing on Tracy's methodological dimensions. The structure for this discussion was simple. Each person had at most ten minutes to be the interviewee and the other three participants could a) ask questions to work on what they did not understand (Staying with the detail), b) try out what they thought they had understood the interviewee to say (Using dissonance, Finding conviction), c) ask for an example, returning to detail all actions supporting the interviewee in developing new awarenesses (Making it real, Going meta). These new awarenesses are often recognised following a pause for the interviewee followed by more energetic speech, often a smile or a laugh. Word limit constraints mean that there is not sufficient space here to include all four conversation extracts, but Tracy's extract is included because we felt it best illustrated energetic speech and the arising of new awarenesses. Presented next is what Tracy wrote in preparation for the narrative interview, which is followed by the extract from the transcript, and then a reflective discussion.

Mathematics Teacher and Mathematics Teacher Educator Identity: Tracy

Recently, I have been thinking about the notion of identity, specifically, mathematics teacher and mathematics teacher educator identity. During my PhD, my research was framed around "becoming an MTE," and I avoided (although not deliberately) the notion of identity, at least explicitly. My focus was on developing a way of being, doing, and knowing as an MTE and so I felt no need to introduce another notion at that point. Attending a symposium on teachers' professional identities hosted by Jeppe Skott at Linnaeus University in 2023, I felt much of the content of the various presentations resonated with my own research, even though different words were being used to those I would use myself. During the symposium, debates emerged on the onto-epistemological status of identity, whether it is a purely narrative construct or a more participatory phenomenon grounded in experience. Aware of my own growing sense of identity as an MTE, but unsure how this experience might be explained from an enactivist perspective, I began to wonder how an enactivist might conceptualise identity. And so, together with Andreas Ebbelind, we set out to write an article drawing on enactivism



as a theoretical basis (see Helliwell & Ebbelind, 2024) for the special issue of the journal *Nordic Studies in Mathematics Education* associated with the symposium.

Drawing on Varela's work, identity is understood from an enactivist perspective as "an emergent property of a complex, distributed process mediated by social interactions" (Varela, 1999, p. 62). At the same time, identity is described as "a unitary quality, a coherence of some kind" (Varela, 1997, p. 73), brought forth by an observer. This coherence arises through an individual's coupling with the environment, "to give rise to what appears to an observer a purposeful and integrated whole" (p. 83), as if a coordinating agent were at the centre. Varela (1999) referred to this immaterial self as a virtual self. I understood this to mean that personal identity, at any point in time, manifests itself as a sense of self, it is the observer that imposes an identity, *as if* it existed.

From the field of mathematics teacher education, "identity work" has been defined as "a teacher's active and intentional work on their identities through various reflective activities" (Lutovac & Kaasila, 2018, p. 254). This idea became useful to me in reframing my doctoral work. Identity work, rather than identity, with its emphasis on process and deliberate reflecting, seemed to align well with the narrative-enactivist methodology I developed during my PhD, without committing to identity as something substantial or fixed. By revisiting the key narrative-enactivist principles formulated as part of my thesis and integrating David's use of multiple perspectives, Andreas Ebbelind and I conceptualised MTE identity work from an enactivist perspective (Helliwell & Ebbelind, 2024) as a set of guiding principles: focusing on process (as opposed to outcome); orienting towards relationships and attending to the other; working from and with multiple perspectives; privileging non-rational forms of knowing; engaging in a process of becoming self-aware; facilitating the letting go of ego-centred habits; and enabling new ways of making distinctions.

Extract from narrative interviewing

We pick up the narrative interview part-way into the time focusing on Tracy's writing. Tracy had been elaborating on her writing (above) and made a comment about the possibility of identity-work being non-rational, in the moment in which it occurs.

- Laurinda: You suddenly said in the moment it's non-rational. So when does identity work take place?
- Tracy: I guess it is deliberate but ... Oh, that's such a good question. ... Retrospectively, after something has been experienced, but I don't know if that's always the case. Maybe it's happening in the moment. ... So, yesterday I taught the new group of prospective teachers for the first time. It was their first day, and at one point I wanted the group to be quiet so I could talk to them. And they weren't quiet to begin with because they were, because they were really energised and my voice was quiet, and they were energetically talking to one another. My immediate response was just to wait, and eventually they noticed and went quiet. I didn't have any concern about getting them quiet, but I found myself launching into a story about how, on the previous day, my children were sharing experiences of different classes at their school. One of the things my son said was how a certain teacher gets a class to be quiet by using a clapping rhythm that the students clap back. Then I made some kind of meta-comment about them having to find their own ways to get children quiet when they're in school, but from then on, I kept using the clapping method mentioned by my son. That had not been a thing I'd ever done before. So, there was some sort of changing of behaviours that I recognised in myself, and I was aware of this difference in the moment. I suppose it was my awareness of this moment as one that they are likely to experience for themselves in school and seeing it as an opportunity to make it explicit. So, was identity work happening then? I think potentially there was some change that I was aware of that I was quite deliberate about which meant I then repeated that same thing.

Discussion

Laurinda asked Tracy a question, stabilising attention, which comes out of her listening and repeating what Tracy had said back to her. There was a pause followed by Tracy energetically stating, "That's such a good question", after which she accesses (or stays with) the detail of a story, reliving it, all said

with energy and laughter. We interpreted Tracy, here, as toying with the idea of identity work taking place in the moment of teaching, which according to Lutovac and Kaasila (2018) occurs through reflection. Through re-living the detail of her teaching from the day before, she was “critically reviewing practice” (Making it real) but with the awareness there was also a sense of critically reviewing practice occurring in the moment of teaching. Through Tracy’s story, we interpreted that there had been no pre-planning about meta-commenting (Going meta) to the prospective teachers about ways of getting a class quiet. Indeed, we conjectured that had the process of Tracy getting them quiet been smooth, that there might have been no provocation or need for a meta-comment about the process. No doubt there was awareness, for Tracy, that prospective teachers would soon be facing their own dilemmas of how to get classes quiet (a dissonance perhaps). What Tracy recalled (in the detail of her story) is being moved to then comment about what had just taken place, and then her using an unfamiliar strategy, to get the class quiet from that point on. There is, therefore, potentially evidence of identity-work taking place in the moment of teaching at two different levels. There is the level of teaching behaviours to get a class quiet, where Tracy exhibits new behaviours (and hence, some element of identity-work might be said to have taken place). But there is also the level of mathematics teacher educator awareness, where Tracy is perhaps also recognising something of an increased familiarity with meta-commenting on events with prospective teachers and of moving away from using her own (classroom) teaching behaviours, towards creating space for an exploration of a range of teaching behaviours as responses to issues that arise. We interpreted, in the story, Tracy’s growing conviction as a mathematics teacher educator and her capacity to be open to and share explicitly new behaviours arising in the moment of teaching. In sharing this awareness with the group of prospective teachers, Tracy is inviting them to engage themselves in identity work, about the teachers they want to become.

In this illustration of the process of coming to know something new, we offer a moment of methodological enactment in which the dimensions are used in a reflexive manner to occasion learning through conversation among MTEs. Laurinda’s question arises out of her own “Staying with the detail” of what Tracy had been saying, that is, attending closely to the words. There is also, perhaps, a sense of “Using dissonance” in making use of an awareness that what Tracy was saying about identity-work in the moment seemed to imply it was possible for identity-work *not* to take place in the moment. Presented here is, potentially, an example of listening to what is “not” said (see Coles, 2020 for a description of Laurinda’s practices of noticing nots).

Final Thoughts

In this article we have offered and then illustrated a methodology for becoming MTEs, by way of responding to the question, how is enactivist learning done? This methodology, developed by Tracy, has strong links to the ideas developed by David, in 1996, around enactivism as a research methodology (REF). In Tracy’s development of this methodological work, she distinguished: methodology for researching; methodology for learning as an MTE; and methodology for teaching (as an MTE). We view these three levels, collectively, as a methodology for becoming an MTE. We interpret the same dimensions (using dissonance; staying with the detail; finding conviction; making it real; and, going meta) as relevant across these levels. For example, we use the notion of “staying with the detail” as an important dimension in our teaching (supporting teachers to do this), in our learning (supporting each other to do this) and in our researching (exemplified by our use of that dimension in this article).

For the four of us authors, enactivism is a theoretical framework that also, given knowing is doing, supports developing methodological actions to prompt learning. We design research projects to have multiple views built in through collaborative groups of researchers with individual projects. Individual researchers might document new awarenesses, local theories, which are effective actions for them given their focus. Insights from the research of individuals can be used within the group, forming a consensual domain. Talking together, for example in reciprocal narrative interviewing, to identify common strands across personal experiences creates possibilities for what can be reported in formal



research, while supporting others in exploring new ideas, expanding the space of their possible. This is our response to the question of, “How do we adapt and learn?” What we learn through research, exemplified in this article by our four-way narrative interview, is part of a process, and does not have to be the same for each of us. What is important is that new awarenesses are linked to actions and will affect future behaviours. Tracy will be more attuned to opportunities in the moment to support her prospective teachers in what David calls teacherly behaviours. What we all shared through the discussion is methodological actions to support each of us learning, exploring the extension of two-way reciprocal narrative interviewing to a four-way virtual space.

Looking over the use of the methodological dimensions, we are struck by the energetic nature of the learning that seems to follow. This can perhaps be observed most clearly in the transcript of Tracy responding to Laurinda’s question, above, “I guess it is deliberate but ... Oh, that’s such a good question.” There is a start and a pause where the re-thinking of assumptions is palpable. So, one sense we have of enactivist learning is a sense of energy, relevance and an orientation towards the future (Making it real).

Conclusion

The methodological dimensions (Helliwell, 2021) can operate to occasion spaces for learning in which prospective teachers, or MTEs have opportunities to work with their felt experiences and arrive at new awarenesses that are meaningful within their own contexts and in relation to what seems important to them, rather than what is determined by someone else. From an enactivist perspective, learning cannot be understood independently of the histories, embodied experiences, and environments through which knowing emerges; consequently, the dimensions are not intended to produce particular forms of learning or to prescribe what should be noticed or understood, but to provide a language for noticing and discussing how professional learning emerges through teaching, researching, and dialogue (Brown & Coles, 2012; Helliwell & Embelin, 2025; Varela et al., 1991).

We recognise that we have not discussed context explicitly in this article. This is important because, although enactivism draws on a biological basis for being, it does not imply that knowing or learning occurs independently of context. Rather, what is brought forth emerges through the dynamic relation between the individual and their environment (Varela et al., 1991). We therefore do not offer the methodological dimensions as being relevant in every context. In the contexts in which we work, in the United Kingdom and Norway, they have enabled us, as MTEs, with prospective teachers to work across a range of contexts. At a meta-level, the dimensions are deliberately formulated to support conversations that can move across contexts, while leaving open how they might be taken up and enacted elsewhere. In this sense, they contribute to mathematics teacher education as both a resource for supporting the professional learning of MTEs and a methodological lens for studying it, extending the growing body of research on MTE expertise as both practice-based and developed through the dynamics of interaction and reflection (Helliwell et al, 2026b; Potari, 2026). The extent to which the dimensions might be relevant in other MTE contexts therefore remains an open empirical question.

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Ethical approval

Ethical approval for the research was granted for Helliwell's doctoral research by the Research Ethics Committee at the University of Bristol, School of Education.

Competing interests

The authors declare there are no competing interests.

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