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How Do They Measure Up? Primary Pre-service Teachers' Mathematical Knowledge of Area and Perimeter

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This paper reports on the results of three different investigations into pre-service teachers' understanding of the mathematical concepts of area and perimeter. Different test instruments were used with three cohorts from two universities in order to identify pre-service teachers' understandings and common misconceptions. The results indicated that many pre-service teachers across the cohorts had a procedural understanding of area and perimeter, displayed similar misconceptions to their student counterparts, and were limited in their ability to demonstrate examples of the mathematics knowledge required to teach these topics. The findings add to the limited field of research into primary pre-service teachers' understanding of area and perimeter, particularly within an Australian context and across institutions.

Introduction

There has been continuing interest in understanding and describing the mathematical content knowledge (MCK) and pedagogical content knowledge (PCK) of primary mathematics teachers (e.g. Chick, Baker, Pham, & Cheng, 2006; Hill, Ball, & Schilling, 2008; Ma, 1999; Rowland, Turner, Thwaites, & Huckstep, 2009; Shulman, 1987). Building on the work of Shulman (1987), researchers have attempted to construct frameworks as a means to understanding the complex relationship between types of knowledge required for teaching (e.g., Chick, Baker, et al., 2006; Hill et al., 2008; Rowland et al., 2009). Such frameworks have been useful in interpreting both in-service and pre-service teachers' MCK and PCK, with concerns being raised consistently about the limited content knowledge of teachers, across a range of mathematical domains. What is less clear, however, is the impact (if any) this has on teachers' PCK and what instruments would be suitable for investigating such a relationship.

Adler, Ball, Krainer, Lin, and Novotna's (2005) survey of international research found that teacher educators frequently report on their own pre-service teachers; however, as Menon (1998) pointed out, there are few studies of pre-service teachers' knowledge of perimeter and area. Our own review of literature found that studies reported involved small sample sizes (e.g., Baturo & Nason, 1997; Menon, 1998; Reinke, 1997), with few Australian studies (e.g., Baturo & Nason, 1996; Ryan & McCrae, 2005/2006) and limited studies across different universities or countries (e.g., Berensen et al., 1997). Although the recent Teacher Education and Development Study in Mathematics (TEDS-M) (Tatto et al., 2012) assessed primary and secondary pre-service teachers' MCK and PCK across 17 countries, Australian pre-service teachers were not included. Goos, Smith and Thornton's (2008) review of pre-service teachers' mathematics education suggested that future research could include qualitative case studies across universities as a means to create evidence across cases. This paper addresses this suggestion and reports on a comparison of three different cohorts of pre-service teachers' MCK and PCK in relation to area and perimeter from two different universities. Although the cohorts were compared using different test instruments, the findings revealed that similar misunderstandings were demonstrated, consistent with school students' difficulties as identified in the literature (e.g., Ryan & Williams, 2007).

Review of Literature

Knowledge for Teaching

Teachers use and need different types of knowledge for teaching (Chick et al., 2006; Hill et al., 2008; Ma, 1999; Rowland et al., 2009; Shulman, 1987). Knowledge for teaching mathematics is important as it underpins teachers' decisions about which examples or representations to use, what connections to make during a lesson, and how to respond to student thinking (Rowland et al., 2009).

Shulman's (1987) theoretical framework listed seven categories that have become the foundation for describing the knowledge base for teaching. He described content knowledge as a central feature and the "amount and organisation of knowledge in the mind of the teacher" (p. 9), and PCK as an amalgamation of content and pedagogy. A teacher's PCK is needed to teach different mathematical topics, making it comprehensible to learners; it is also necessary for understanding student misconceptions, knowing how topics are organised and taught, as well as influencing the ability to adjust lessons catering for all learners (Shulman, 1987). In particular, these two classifications of Shulman's teacher knowledge have been developed through other frameworks of teachers' knowledge (e.g., Ball, Thames, & Phelps, 2008; Chick, Baker, et al., 2006; Ma, 1999; Rowland et al., 2009; Tatto et al., 2012) extending our understanding of teaching mathematics.

Building on the work of Shulman (1987; 1998), as well as their own research, Ball and colleagues proposed a framework for distinguishing the different types of knowledge required for teaching mathematics: *Domains of Mathematical Knowledge for Teaching* (Ball et al., 2008). Their framework consisted of two broad categories: subject matter knowledge and pedagogical knowledge (see Figure 1). Within subject matter knowledge a teacher can demonstrate Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), and Horizon Content Knowledge (HCK). CCK is not exclusive to teachers; any adult may have well developed CCK but most likely will lack the knowledge required to teach it (Hill, Ball, & Schilling, 2004). SCK is unique to teaching (Ball, Bass, & Hill, 2004; Chick et al., 2006; Schoenfeld & Kilpatrick, 2008) and refers to the range of mathematical knowledge such as procedural knowledge, procedural fluency, conceptual knowledge, and mathematical connections (Ball & Bass, 2003). HCK includes a peripheral vision of mathematics; a teacher with this knowledge demonstrates understanding of the complexities of mathematical topics, has advanced knowledge, possesses a broad understanding of mathematical ideas and connections, and links their content knowledge with curriculum that their students know and will know in future years (Ball et al., 2004; Ball et al., 2009; Ball et al., 2008). The PCK section of the Domains of Mathematical Knowledge for Teaching framework is consistent with Shulman's definition of PCK as a blend of content and pedagogical knowledge, but it has been extended using three sub-domains: Knowledge of content and students (KCS), Knowledge of content and teaching (KCT), and Knowledge of content and curriculum.

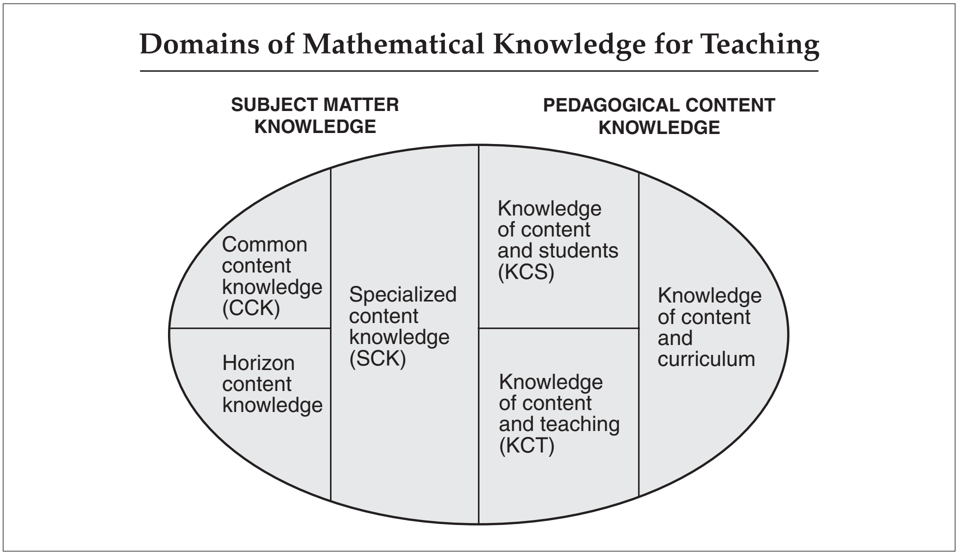


Figure 1. Domains of mathematical knowledge for teaching framework. (Ball et al., 2008, p. 403)

Chick, Baker, et al. (2006) also designed a framework for investigating mathematical PCK, and used it to describe the different PCK held by teachers when comparing their responses to different mathematical topics. Of particular relevance to this paper is the section of this framework described as Content Knowledge in a Pedagogical Context (see Table 1). The first of the five categories, *Profound Understanding of Fundamental Mathematics* (PUFM) relates to the breadth, depth, and thoroughness of understanding that many Chinese teachers demonstrated in Ma's (1999) study. Other elements of the framework share similarities to the knowledge identified by Ball et al. (2008). Mathematical Structure and Connections, for example, has a similar focus on the connection between mathematical topics, as does Ball et al.'s (2008) HCK, while "Procedural Knowledge" and "Methods of Solution" could be seen as being included in Ball et al.'s) SCK.

Table 1
Content Knowledge in a Pedagogical Context (Chick et al., 2006 p. 299)

PCK Category	Evident when the teacher…
Content Knowledge in a Pedagogical Context	
Profound Understanding of Fundamental Mathematics	Exhibits deep and thorough conceptual understanding of identified aspects of mathematics
Deconstructing Content to Key Components	Identifies critical mathematical components within a concept that are fundamental for understanding and applying that concept
Mathematical Structure and Connections	Makes connections between concepts and topics, including interdependence of concepts
Procedural Knowledge	Displays skills for solving mathematical problems (conceptual understanding need not be evident)
Methods of Solution	Demonstrates a method for solving a mathematical problem

Researchers have made use of these frameworks, or adaptations thereof, to report on aspects of teachers' and pre-service teachers' PCK (e.g., Watson, Callingham, & Nathan, 2009). Chick, Pham, and Baker (2006), for example, found the framework useful for interpreting teachers' understanding of the subtraction algorithm, while Watson et al. refined PCK to include recognition of key mathematical ideas, anticipation of student answers, and the employment of content specific strategies. As the Chick, Baker, et al. (2006) framework has been identified as being particularly useful in research with teachers (Bobis, Higgins, Cavanagh, & Roche, 2012), we have drawn largely on this framework to describe the subtle differences within our study of pre-service teachers' MCK and PCK for the concepts of area and perimeter.

Developing an Understanding of Area and Perimeter

Area refers to the measure of the two-dimensional space inside a region (Van de Walle, Karp, & Bay-Williams, 2012), while perimeter is a measure of length involving the distance around a region (Reys, Lindquist, Lambdin, Smith, & Suydam, 2012). Area and perimeter are often a source of confusion for students, due perhaps to both involving regions to be measured, or because students are taught formulae for both concepts at the same time, and therefore tend to confuse the formulae (Van de Walle et al., 2012). Ryan and Williams (2007) found that almost one-third of 13 year-olds used the perimeter formula rather than the area formula when finding a missing dimension. Similarly, 2007 NAEP (National

Assessment of Education Progress) results showed that only 39% of fourth-grade students could accurately calculate the area of a carpet, 15 feet long and 12 feet wide (Van de Walle et al., 2012). Other difficulties associated with area and perimeter include accuracy with measuring shapes with diagonal sides, conversion between square units (Ryan & Williams, 2007), conservation of area and perimeter (Ma, 1999; Murphy, 2012), and use of inappropriate units when calculating area and perimeter (Yeo, 2008). The source of these errors may be attributable to students' tendencies to think about these measures in terms of the measure rather than the concept. A greater focus on developing a meaningful understanding of measurement concepts, through using a sequence such as identifying the attribute, comparing and ordering, using informal units, using formal units, and then finally looking at formulae and application (Van de Walle et al., 2012) may address the over-emphasis on formula (Murphy, 2012). Other key conceptualisations include the notion that length and area are continuous quantities (Yeo, 2008); many classroom examples provide static representations which can lead to misconceptions about the conservation of area and perimeter such as the notion that as the perimeter increases, so too will the area (Murphy, 2012). Although this can be true (when the increase of the perimeter is caused only by the increase of only one pair of opposite sides of a rectangle, the area of the figure will increase as well), it does not hold true when the lengths of both sides of a rectangle are increased.

Pre-service Teachers' Understanding of Area and Perimeter

Difficulties with understanding area and perimeter are not restricted to school students. Ma (1999) for example, found that 8% of Chinese teachers and 9% of American teachers accepted without doubt, the claim that "as the perimeter of a closed figure increases, the area also increases" (p. 84). Yeo (2008) also reported that teachers confuse area and perimeter, and assume a constant relationship between the two measures. Concerns within pre-service teacher education are even more prevalent, with studies indicating that many pre-service teachers have poor conceptual understanding of area, relying on rules and formula, and have difficulties in explaining why these formulae work (Baturo & Nason, 1996; Berenson et al., 1997; Menon, 1998; Reinke, 1997).

Baturo and Nason (1996), found that some pre-service teachers had poor knowledge of area, including knowing that the area of a two dimensional shape when cut and rearranged will remain the same. During interviews, many provided responses which were incorrect and rule-dominated. Berenson and colleagues' (1997) international study of pre-service teachers' understanding of area required pre-service teachers to design a lesson plan introducing area to middle year students. The findings showed that many of the pre-service teachers had a primarily procedural knowledge of area, which was reflected in procedural and formula-dominated lesson plans.

Like Berenson et al. (1997), Murphy (2011) also asked pre-service teachers to design lesson plans, and participate in a follow-up interview; she also asked them to respond to four tasks, designed to ascertain their subject knowledge of area. The four pre-service teachers in this study had different strengths and limitations in their understanding of the topic. Of particular interest was Charlotte, who demonstrated limited subject knowledge, in that she made errors in using the formulae for calculating areas and confessed that she never knew when to use cm^2 or cm^3. Of the four participants, she was seen as having the most limited understanding of the topic. Her lesson plan, however, aimed to help children realise the concept of area as the amount of space, using investigative approaches as a way of measuring area using different units, rather than explicitly focusing on counting squares.

The recent TEDS-M (Tatto, et al., 2012) report of pre-service teachers' MCK and PCK included a reference to their understandings about area and perimeter. The report indicated a probability of greater than 0.70 that pre-service teachers would be able to solve "routine problems about perimeter", but would have "difficulty reasoning about multiple statements and relationships among several mathematical concepts … and [difficulty] finding the area of a triangle drawn on a grid" (p. 136). They also determined that although the pre-service teachers "were generally able to determine areas and perimeters of simple figures" (p. 136), they "were likely to have more difficulty answering problems requiring more complex reasoning in applied or non-routine situations" (p. 137).

Methodology

This study combines the results of three different projects that were conducted with three different cohorts of pre-service teachers from two different universities. Following independent collection and interpretation of data, the authors recognised similarities in results and conceptualised this paper as a comparative study that adds to the limited literature in this field. As a combined study, quantitative and qualitative methods were used to analyse a selection of pre-service teachers' responses to similar measurement items, focusing on their knowledge of perimeter and area. Essentially the three projects investigated the nature of a selection of pre-service teachers' MCK in relation to area and perimeter. In addition to this, an investigation was also undertaken with two smaller cohorts of pre-service teachers into what ways (if any) this knowledge impacted on pre-service teachers' PCK.

Participants

The 17 pre-service teachers from University A were in their final year of a four-year Bachelor of Education (BEd) course (Foundation to Year 12). Within this cohort there were three mathematics majors who would qualify to teach Foundation to Year 12 mathematics. All pre-service teachers from University A had previously completed three primary mathematics education units during the first two years of their course. During the course, the pre-service teachers had undertaken 104 days of Professional Experience including 62 days in a primary setting and 42 days in their discipline specialisation in a secondary setting. They had agreed and volunteered for a larger longitudinal study, and the sample of 17 pre-service teachers was a manageable size for the larger study, hence limiting the sample size for the current study. Ethics permission had been granted as part of a doctoral thesis, providing the first author with more detailed demographic data than that of University B.

There were two different cohorts from University B. The first cohort of 222 pre-service teachers was in the second year of a four-year BEd course (Foundation to Year 8). At the time of the study they had completed one education unit focusing on early childhood and primary pedagogy for teaching mathematics. They had undertaken two Professional Experience placements with a total of 25 days. Having had previous anecdotal evidence that pre-service teachers' understanding of area was limited, the researcher determined that a large sample size would provide evidence of the breadth of the issue. In addition, as the data involved short answer written responses, rather than interviews, it was manageable to read and interpret the 222 responses. The second cohort of seven pre-service teachers, were in the final year of a four-year BEd course (Foundation to Year 8). They were enrolled in their third primary mathematics, education unit of study and had undertaken 45 days of Professional Experience over three years. The sample size was deemed appropriate in that the project was originally conceptualised as part of a BEd Honours study and involved conducting, transcribing, and analysing seven 30-40 minute interviews. Table 2 summarises the total number of pre-service teachers who were selected and volunteered for the study from each institution.

Table 2
Total of pre-service teachers, institution and test instruments for each cohort

University	Cohort	Number of participants	Instruments used
University A	Fourth-year BEd (Foundation-Year 12)	17	One-on-one interview (see Figure 1)
University B Cohort 1	Second year BEd (Foundation-Year 8)	222	Test question
University B Cohort 2	Fourth-year BEd (Foundation-Year 8)	7	One-on-one interview (see Figure 2)

Instruments, Procedure and Data Analysis

University A. Seventeen pre-service teachers participated in a one-on-one interview with the first author during the second semester of the final year of their course. The interview was based around two questions (see Figure 2), and was about 30 minutes in duration. The first question assessed pre-service teachers' MCK of perimeter and area, identifying if they could correctly explain the difference between these two measurements, while the second question focused on the relationship between area and perimeter, and was adapted from a similar item in Ma's (1999) study.

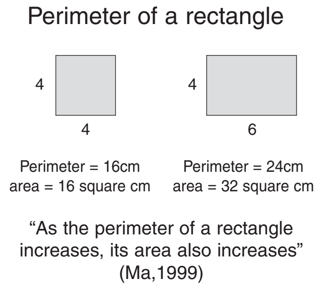
Question 1	Question 2	
Imagine you are teaching area and perimeter. Can you tell me the difference between the two?	Imagine that a student in your class says, "I think if the perimeter of a rectangle increases, its area also increases." What would be your response?	

Figure 2. Question 1 and Question 2 used in one-on-one interviews for University A

The 17 pre-service teachers' interviews were audio taped and transcribed afterwards. Transcriptions for each response were sorted and coded; identifying four different categories of responses (see Table 3). These categories were used to score and order all responses for both questions ranging from zero to three. Table 3 lists a description of the scoring codes and examples of responses received for both question types. In relation to Chick, Baker, et al.'s (2006) framework, a score of one relates to methods of solution, a score two relates to procedural knowledge and a score of three involves elements of mathematical structure and connections.

University B Cohort 1. The first cohort of 222 second-year pre-service teachers had all undertaken an exam as part of a primary mathematics education unit. These pre-service teachers had two hours in which to complete the exam, of mostly short answer responses. Pre-service teachers were allowed to refer to class notes and the text book, *Elementary and middle school mathematics: Teaching developmentally* (Van de Walle et al., 2012). Calculators were not permitted. Ethics approval was given to use the data from the exam results. Written responses, which were illustrative of the range of answers received, were selected, and these participants gave informed consent for the data to be included.

Pre-service teachers were asked to respond to the following: "John said that whenever you increase the perimeter of a rectangle, the area also increases. Susan says this is not true. Who is correct? Support your argument with a diagram". This question was similar to Ma's (1999) area and perimeter item, and to the second question asked in the one-on-one interview for University A. All participants attempted to answer the question. All responses were marked by the second author and coded using the coding rubric in Table 4. This rubric ranged from zero to four, with zero being incorrect and four demonstrating a correct response, which included examples and justification. The rubric also provides a measure of participants' MCK, particularly in terms of Chick, Baker, et al.'s (2006) elements of Mathematical Structure and Connections, Procedural Knowledge and Methods of Solution.

Table 3
Coding for one-on-one interview questions from University A

	Score = 0	Score = 1	Score = 2	Score = 3
Description of response	Unable to provide response correct	Some correct mathematical understanding but incomplete response	Correct response using procedural knowledge or lacking mathematical connections	Correct explanation justifies and/or understands the concept or process
Example response Question 1	Area is length plus width. Perimeter is length times width	Perimeter is outside of an object. Area is the inside of an object	Perimeter is adding length and width. Area is length times height	Perimeter is the distance around the outside of the shape. Area is the amount of space contained within the shape.
Example response Question 2	Accepted student's hypothesis but did not explain why	Accepted student's hypothesis, but used diagram/s to justify	Identified student was incorrect and explored area and perimeter of different rectangles to identify one example to show student was incorrect	Knew assumption was incorrect and could justify their response drawing on more than one example

Table 4
Coding rubric for exam answers to Q. 11 from University B Cohort 1 (N=222)

Score = 0	Score = 1	Score = 2	Score = 3	Score = 4
Incorrect response (John was correct)	Correctly identified Susan as correct	Correctly identified Susan with limited explanation and/or diagram	Correctly identified Susan as correct; some explanation and at least one example provided	Correctly identified Susan as correct; provided sound explanation and examples/counter examples to justify answer

University B Cohort 2. The second cohort of seven fourth-year pre-service teachers was in their final year of a BEd primary course. As part of a larger study, 20 pre-service teachers had undertaken a 15-item mathematical skills test that was designed to assess the mathematical attainment of beginning teachers and to identify errors due to misconceptions. From this original sample, seven pre-service teachers volunteered to take part in the second part of the study, involving a one-on-one interview. This interview was structured around six primary students' work samples, constructed by the researchers. For the purpose of this paper, the pre-service teachers' responses to the work sample shown in Figure 3 have been analysed and discussed. As in Ryan and Williams (2007), this item was designed to determine whether or not the students tended to use the formula for perimeter, rather than area, when finding a missing dimension. The interviews took about 50 minutes, and required the pre-service teachers to interpret all six work samples. They responded to the following interview questions:

- State whether or not the student's response was correct or incorrect
- What does this tell you about this student's thinking?
- Explain what you might do as a teacher to address this.

The final question was only asked if the pre-service teacher was aware of the misconception the student had written.

Finally, participants were asked to give reasons why some students confused area and perimeter. All interviews were transcribed, and commonly occurring themes identified and highlighted. For example, a common approach was to focus on the numbers in the problem, rather than explicitly identifying the underlying cause of the misconception. Illustrative examples of some of the approaches used are discussed later in the next section.

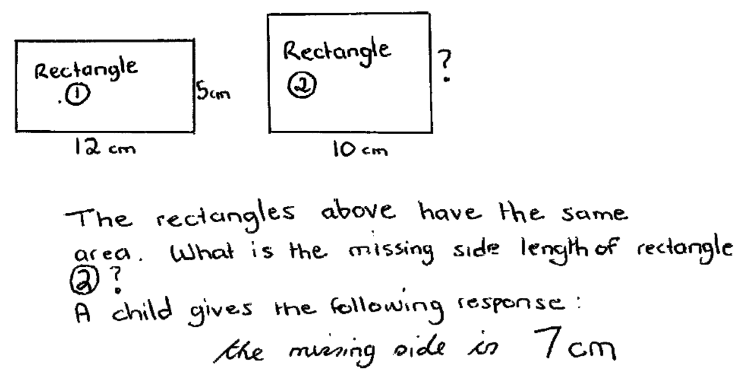


Figure 3. Area and perimeter work sample

Results and Discussion

University A: The Difference between Area and Perimeter

Question one (see Figure 2) required the 17 fourth-year pre-service teachers to explain the difference between area and perimeter. Table 5 provides the number of preservice teachers awarded each of the four codes (see Table 2) used to categorise the range of responses.

Table 5
University A fourth-year pre-service teachers' responses to Question 1 (N=17)

Explanation	Score =0	Score=1	Score=2	Score=3
Perimeter	2	5	0	10
Area	1	11	4	1

Of the 17 pre-service teachers, ten were able to provide a correct explanation of perimeter. Responses included mathematical language that classified perimeter as a measurement of length, with an example of an accurate response being "Perimeter is the edge—the outside of something [and] you measure all the sides and plus them."

A further five provided partial explanations of perimeter, but did not state that it was a measure of the total length. For example, "Perimeter is the outside of an object" or "Perimeter is length and width." There were only two pre-service teachers who could not explain the definition of perimeter correctly, with one confusing it with area and the other one stating, "I can't remember."

In Table 5 the responses for explaining area demonstrated that most (11/17) of the pre-service teachers had some knowledge but did not provide a complete definition. They referred to the "space inside the shape" and failed to include that area is the measure of this space. Later, they demonstrated a correct method for calculating the area of a rectangle.

There were four who received a rating of 2 for their answer, indicating a procedural explanation to explain area. There was a tendency to describe area as multiplying two sides with no clarification that this was a method used to calculate the measurement of the space within a rectangular shape. There were no attempts to explain the concept of area, with answers indicating knowledge of a rule for finding the area of a rectangle.

Mathew received a rating of 3 for providing an accurate explanation of both area and perimeter.

> Perimeter is the edge the outside of something. If you are thinking of a pool it is the path around the outside of the pool. Area is the amount of space within a 2D shape or the surface. Perimeter is just the outside of the 2D shape. We measure perimeter, there are different ways of measuring perimeter, you can just measure all four sides and plus them together. If it is a rectangle you can

> measure two sides and times them by two and then plus them. The area of a rectangle you can do length times width, [or] one times one side would equal the area of the inside to the rectangle.

This explanation supports evidence of "Content Knowledge in a Pedagogical Context" (Chick, Baker, et al., 2006) as the pre-service teacher can "deconstruct content to key components: Identifies critical mathematical components within a concept that are fundamental for understanding and applying the concept" (p. 299). Although wordy, Mathew's explanation shows evidence of MCK and includes knowledge of correct mathematical terms and understanding of the concepts, along with the processes described for calculating both measures. For example, he defined perimeter and explained how to calculate the perimeter for a rectangle. The terminology is simplistic but two methods for calculating the perimeter of a rectangle were accurately provided. During the interview he also explained that he had completed an activity with his students during his school placement. This explanation provides further evidence of his MCK and PCK and resulted in a rating of 3.

We did this lesson earlier in the year. We gave them grid paper and asked them to keep the area the same. How does that change the perimeter … ones that perimeter were the same and how does that change area. They obviously worked out the longer the skinner the shape a lot more perimeter you can get.

University A: Relationship between Area and Perimeter

Question 2 required pre-service teachers to discuss their response to a statement about a perceived relationship between the perimeter and area of different rectangles. All 17 fourth-year pre-service teachers from University A attempted this question. Table 6 provides a summary of their responses and coding from zero to three (see Table 3).

Table 6
University A fourth-year pre-service teachers' responses to Question 2 (N=17)

Question 2	Code 0	Code 1	Code 2	Code 3
Now imagine that a student in your class says, "I think if the perimeter of a rectangle increases, its area also increases." What would be your response?	2	6	5	4

Table 6 shows that two pre-service teachers were unable to identify the student's misconception, that if the perimeter of a rectangle increases, its area also increases. A further six were able to draw some examples exploring the perimeter and area of various rectangles, but incorrectly concluded that the student was correct. Their responses indicated a lack of MCK for understanding and making connections with a range of rectangles to solve the problem correctly. They tended to draw regular rectangles which did not assist them, and showed a lack of confidence in providing a convincing argument.

Five pre-service teachers explored a range of examples and identified that the student's statement was incorrect. They drew on their MCK to make connections by representing a range of different sizes of rectangles to solve this problem correctly. This suggested evidence of SMK as they started to think about more than one solution and made connections between the area and perimeter of different sized rectangles. They used their MCK to reason through examples by sketching rectangles to test and check theories.

Observations showed that at least one of the pre-service teachers was able to elaborate by explaining their understanding of the relationship between area and perimeter for different quadrilaterals. Other pre-service teachers, however, reached the answer after drawing a range of different rectangles and were only just convinced.

Julie was one of the five pre-service teachers who explored the example and discovered a correct solution by drawing diagrams. She was surprised by the question and stated during her interview, "A grade four says that, [Question 2] that is mind blowing… OK this is a tricky one isn't it?" Initially Julie was considering the student was correct, "I am going to say yes it does." She looked at the examples of rectangles recorded in the question. Next she drew some examples of rectangles discovering the perimeter can increase but the area can stay the same. "Look here this has increased [perimeter]… OK so the answer is no." After comparing some rectangles she was able to draw on her MCK to justify that the student was incorrect.

A further four preservice teachers convinced the interviewer that they clearly could draw on their MCK to interpret that the student was incorrect. They understood the student's misconception like a known fact, showed no hesitations during the interview, and could elaborate as to why they knew this. Some had completed a similar problem during their course work and others remembered completing a similar problem when assisting a student during their field experience teaching in a primary school.

Shelly, for example, was able to identify the misconception and also provided some appropriate suggestions for assisting the student:

> Tell them to go and test it… What happens if you change the shape of your rectangle? Maybe give them something to make different shaped rectangles. I think maybe keep the area the same and then change the rectangle around.

Her response indicates developing SMK through identification of appropriate teaching approaches and evidence of PCK as she "Deconstructs Content to Key components: Identifying critical mathematical components within a concept that are fundamental for understanding and applying the concept" (Chick, Baker, et al., 2006 p. 299) as applied to the area and perimeter of rectangles.

University B cohort 1: The Relationship between Area and Perimeter

Question 11 was similar to that used in University A and required 222 pre-service teachers to explain the relationship between the perimeter and area of rectangles. Table 7 shows a total of responses received from the pre-service teachers using the coding rubric shown in Table 4.

Table 7
Summary of scores for exam item (N=222)

Rating	Score=0	Score=1	Score=2	Score=3	Score=4
Number and percentage (N=222)	159 (72%)	8 (4%)	27 (12%)	17 (7%)	11 (5%)

A total of 72% of pre-service teachers incorrectly identified that John was correct, indicating that whenever you increase the perimeter of a rectangle, the area also increases. Like the teachers in Yeo's (2008) study, their responses clearly indicate a strong misguided belief that there is a constant relationship between the two measures. This belief prevailed despite the pre-service teachers having been given a similar problem in tutorials, a similar practice exam question, and a link to an interactive website which also explored the relationship. They also had the opportunity to refer to class notes and the textbook to assist with answering the question. The following are typical examples of the types of responses received that were coded as 0:

> John is correct as whenever the perimeter increases, the area has to increase as well.

> John is correct because you cannot increase the area of a shape without the perimeter increasing.

Although many responses included diagrams, the majority depicted two rectangles with sides, for example, of 4 cm and 2 cm, and 5 cm and 2 cm. Such examples modelled an increase in both the perimeter and area of the second rectangle. Although their answers did demonstrate that a relationship exists, the answers showed a strong tendency to present one example to "prove" a case, rather than exploring different examples or drawing on MCK to seek a counterexample.

Table 7 shows that the remaining 28% identified that Susan was correct but their responses varied in quality and depth of their justification. Better responses included a number of examples and often a counter example, or mentioned that while John may be correct sometimes, it does not hold true for all situations. Figure 4 shows an example of a response that received a rating of 3 as it includes more than one example.

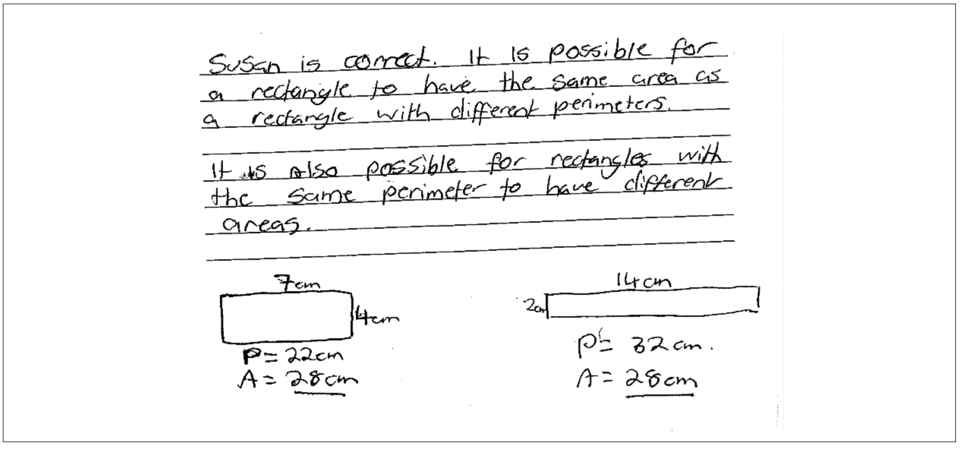


Figure 4. Pre-service teacher example of response to Question 11 University B Cohort 1

The answer is essentially correct (although the areas have been measured in centimetres, rather than square centimetres), and appropriate diagrams have been provided to justify that the area of two different rectangles can be the same but their perimeter different. A second set of rectangles could be illustrated demonstrating the inverse: same perimeter, different area. Only 5% of all responses received a score of four, meaning that they correctly identified Susan as correct and showed examples to justify their answer. For example:

> Susan is correct. The perimeter of a rectangle with sides of 24 and 2 centimetres (see a) [drawing of rectangle] is 52 centimetres and the area is 48 cm^2. However, the perimeter of a rectangle with sides of 6 cm and 8 cm (see b) is 28 cm, but has the same area, 48 cm^2. Therefore, increasing the perimeter does not always increase the area [The answer included two diagrams which correctly illustrated the dimensions of the two different rectangles].

> Susan is correct. Though, frequently when the perimeter increases in length, the area is also larger, this is not always the case. For instance, both these rectangles [drawings of rectangles, with dimensions of 8 and 5, and 4 and 10] have the same area of 40 cm^2 yet the perimeters are different. Although the perimeter has increased from 26 [cm] for the first rectangle to 28 [cm] for the second rectangle, there has been no corresponding change to the area as John indicates. Therefore John is right in that an increase in perimeter can result in a greater area; however Susan is more correct in qualifying that this is not always the case.

Unfortunately, however, most of the pre-service teachers in this cohort provided responses that were either incorrect, or limited in terms of justification and explanation. The results from both universities' cohorts show that the majority of pre-service teachers in the study could not provide a convincing argument that the area of a rectangle does not necessarily increase when the perimeter is increased. The findings are similar to Ma's (1999) in that only one of the US teachers in her study successfully examined the student's proposition and attained a correct solution.

University B Cohort 2: Responses to Area Work Sample

Seven pre-service teachers completed the final area and perimeter problem. This problem required pre-service teachers to respond to a constructed student work sample, interpret the student's response, and explain why some students demonstrated confusion with area and perimeter. All seven pre-service teachers recognised that the student's response in the work sample (Figure 3) was incorrect and that the missing height of the second rectangle was 6 cm, not 7 cm. Many responses made no mention of the terminology, "area and perimeter" to justify their responses. Instead responses focused on descriptions related to the formula, or to the operations of addition and/or multiplication. For example, five of the seven responses received made no mention of area or perimeter, and instead focused on the use of the operation of addition instead of multiplication, as the following shows:

> Oh I get it now, ok so 12 plus five equals 17 so I guess if you are trying to add them they say 10 and 7 makes 17 so they are not multiplying them they are just adding them.

> I think that all they've done is gone the difference between 12 and five is seven and therefore this one over here will have to be 7.… Ahh I get it now ok so 12 plus five equals 17 so they are not multiplying they are just adding.

Two pre-service teachers did refer to area, and identified that the student's answer showed a lack of fluency with calculating the area of a rectangle. For example, Ann stated that "Um, they probably don't realise that the area is length times the breadth". Perhaps not surprisingly, then, the teaching strategies suggested by a number of participants focused on "showing" the student the procedure for calculating area and perimeter, as the following response from Janet shows:

> Ok I'd need to work back through measurement so she knows that what we are measuring is the mm… area I'm getting all flustered now with the area and perimeter. So she needs to know it's the area inside the rectangle and to do that we are not going to add one side and another side but we need to multiply these two sides [points to the dimensions of the first rectangle shown in Figure 3] in order to tell us the area inside.

Although two participants attempted to deal with the attribute of area and length, the description and explanation of their strategies lacked clarity and did not relate specifically to the task in question, as Jackie's response illustrates:

> I'd probably do it hands on with like a desk and things, getting them to just focus on figuring out area, so measure the length and width of their desk and find the area from that and then we might do that with other things in the room.

One participant, Sarah, suggested the use of informal units perhaps suggesting a conceptual, rather than a procedural approach, to address the student's error:

> Even if they need to use informal units and say like I have a book or a manila folder [picks up a manila folder that is sitting on the desk beside her] and I measure the perimeter of it and say how many blocks does it take to cover this compared to how many blocks it takes to go around the edge so they can see the difference between what the perimeter is and what is actually taken up by the area.

While Sarah's answer shows a recognition of the use of concrete materials, her identification of blocks as a measuring unit for perimeter is problematic, as care needs to be taken when counting the blocks around the corners; also the use of the same unit for both area and perimeter potentially could also contribute to confusion between the two concepts.

Mia mentioned the use of grid paper, with the justification being that:

> They haven't got the understanding that area is length times width, because otherwise they would have gone 12 times 5 is 60, so I know that the area of rectangle one is 60 centimetres, so for the area of rectangle 2 to be 60 centimetres, what do I have to times 10 by and then get that number of the side measurement, which is then 6… but to help them and aid their understanding, you could get grid paper and get them to colour and know that is covering 60 squares…

Mia's explanation shows that she has provided an appropriate reason for the student's error and that grid paper would be an appropriate material to utilise. Her explanation, however, lacks clarity and tends to focus on a procedural approach based around the formula for calculating area, rather than emphasising the concept of area as the space inside a region.

The seven participants were also asked to speculate on why students may confuse perimeter with area. Many responses showed that they had an intuitive understanding of why this occurred, but had difficulty articulating this into an explanation, as the following examples show:

> Um I don't know because I think like… kids think it's hard like how do we know just what that whole space [points to the area inside the rectangle] equals just from knowing what the outside edges are. Yeah, because I still think that too. (Jackie)

> I'm not sure if it could be possibly be how its labelled in that we've just got umm the one measurement there [points to the length of one of the rectangles in Figure 3] and one measurement there [points to the width of the rectangle] as well you can see that that might lead to it being just about 2 sides so we've got to consider this side and this side as well [points to the other two unmarked sides] rather than just these two if they are in that additive frame of mind. Yeah, um, is that making sense? (Janet)

Interestingly, the above responses focus on calculating area and perimeter, rather than the concepts of area and perimeter, and make no mention of teaching approaches contributing to errors. As mentioned earlier, one of the sources for the confusion may be traced to the concurrent teaching of the two concepts (Van de Walle et al., 2012) or an over-emphasis on teaching the formula for both, rather than the concepts.

One pre-service teacher actually referred to this, but her reasoning behind the explanation was rather interesting:

> I mean you learn them [perimeter and area] at the same time; as you progress, you are only retaining ten per cent of what you learn in the first place; you do your perimeter then area then volume together; I guess the thought that if I add those together I can get the right response.

Like the other two cohorts, this third cohort of pre-service teachers indicated a strong tendency to think of area and perimeter in terms of using a formula to calculate answers, rather than two measurement concepts that involve covering and length respectively. The findings suggest that the participants possessed a procedural knowledge of area and perimeter, but demonstrated a limited understanding of the attributes of the two concepts and the relationship between them. Their procedural understanding then limited the potential of their suggested teaching strategies to assist the student with developing a conceptual understanding of area and perimeter. This was illustrated particularly through Mia's response, who suggested using a 6 by 10, or a 12 by 5 grid "to show that it is 60 squares", but did not attempt to link the dimensions of the array to the total number of squares.

Conclusions and Implications

This study reported on three cases examining pre-service teachers' knowledge of perimeter and area. The authors used three similar instruments to assess pre-service teachers' MCK of perimeter and area, and found that they revealed limitations in this knowledge, consistent with the findings from the literature.

Most of the pre-service teachers within this study were able to calculate the perimeter and area of rectangles, as they used this knowledge to provide answers for the exam or interview questions. This knowledge drew on their MCK. However, when the fourth-year pre-service teachers from University A were asked to explain the difference between area and perimeter, just over one half could correctly explain the term perimeter, only one third provided a correct definition for area, and most of these gave a procedural explanation. This would be inadequate for the knowledge expected to teach the topic effectively.

Arguably it would be expected that pre-service teachers bring to the course a sound MCK, in order to fulfil entry requirements for a tertiary course. The results of this study, however, identified that, like many primary school students, some pre-service teachers have misconceptions related to knowing the difference between perimeter and area. The results show, disturbingly, that these misconceptions are still prevalent in the final year of their study. As teacher educators, therefore, we need to be cognisant of this, and provide opportunities for pre-service teachers to address the gaps in their MCK with these topics. This is not necessarily easy to accomplish. The findings for Cohort 1 from University B demonstrated that 72% of the pre-service teachers could not provide a correct answer, despite taking part in investigations that focused on this, and having access to tutorial notes and the unit's textbook. Perhaps what is required is greater provision for pre-service teachers to examine their own misconceptions, diagnose student misconceptions, and be provided with opportunities to engage in professional conversations about these issues.

Although involving a smaller sample size and a different university, half of the fourth-year students at University A had the same misconception that there was a constant relationship between the perimeter and area of rectangles. The findings are consistent with those noted by Ma (1999), and indicate that perhaps pre-service teachers intuitively believe that such a relationship exists. This seems to be a particularly strong conviction, indicating that explicit teaching and extended investigations may be necessary to counteract such beliefs.

The instruments and the scoring rubrics used in both universities were suitable for classifying pre-service teachers' responses in terms of identifying their incorrect knowledge, showing some understanding, identifying correct procedural understanding, or exhibiting connected knowledge that justified responses and drew on conceptual understanding. Such instruments may prove useful to other teacher educators.

Pre-service teachers' explanations across the cohorts also showed a tendency to use limited mathematical terminology as well as using descriptions relating to procedural knowledge. Participants from the second cohort from University B, for example, tended to describe area as "length times width", rather than as the space inside a region, indicating a reliance on formula.

Only a small number of second-year pre-service teachers from University B and similarly 25% of fourth-year pre-service teachers from University A, could justify their responses and provide a convincing explanation of the relationship between area and perimeter. This finding is of concern as graduate standards require pre-service teachers to communicate clearly and accurately when designing a lesson and teaching these concepts (Australian Institute for Teaching & School Leadership (AITSL), 2011). Such planning would require graduate teachers to draw on their PCK to design an activity for their students, similar to that provided by Sarah from University B. She suggested a conceptual method of teaching rather than a procedural approach (albeit with limitations) to assist a student having difficulties with the comparison of area and perimeter, which has been identified as an effective teaching approach (e.g., Clarke & Clarke, 2002).

Discussion of student work samples and identification of errors has been shown to be an effective method for eliciting pre-service teachers' understandings (e.g., Ryan & Williams, 2007). Examples of pre-service teachers' responses and the tools used to code responses could be shared with future cohorts of teachers when teaching this topic. The pre-service teachers could then discuss and code the answers as a means of developing their understanding of how a teacher begins to develop their MCK for teaching.

The data drawn from each study provided results that could be compared across cases to compare the three cohorts of pre-service teachers. Through our comparisons, we have recognized that pre-service teachers have similar strengths and weaknesses in terms of their MCK and PCK in relation to area and perimeter. Along with adding to the limited field of research in this area, it is hoped that this study could form the foundation for future studies and comparisons of test instruments of other topics identified as difficult for pre-service teachers.

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