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Prior Study of Mathematics as a Predictor of Pre-service Teachers' Success on Tests of Mathematics and Pedagogical Content Knowledge

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There remains a lack of empirical evidence about the relationship between the level of mathematics studied at high school and within tertiary degrees and primary school pre-service teachers' success in curriculum subjects. Further, there is little evidence to inform the structure and delivery of mathematics teacher preparation. In this study, the content and pedagogical knowledge of pre-service primary teachers were examined, as was their view of the effectiveness of a unit of study based on mathematics content and pedagogy. The cohort comprised 122 graduate diploma primary teacher preparation students; the unit's assessment required them to know the mathematics they were expected to teach as well as describe how to teach it. It was found that the level of high school mathematics undertaken was highly correlated with success in the teacher education unit designed to prepare prospective teachers to teach primary (elementary) mathematics. The findings have implications for enrolment in pre-service primary teacher preparation courses as well as for the structure of mathematics curriculum units.

Introduction

The study reported here examined the level of mathematics content knowledge that pre-service teachers brought to primary (elementary) teacher preparation. The importance of pre-service teachers' knowledge of subject matter has been recognised as central to their teaching (e.g., Ball, Hill, & Bass, 2005; Goulding, Rowland, & Barber, 2002; Silverman & Thompson, 2008). Internationally, a number of authors have expressed concern that many pre-service teachers have learnt limited mathematics at school (e.g., Adler, Ball, Krainer, Lin, & Novotna, 2005; Henderson & Rodrigues, 2008). Yet it has also been reported that teacher preparation sometimes does not focus on remediating deficiencies in teacher knowledge of mathematics because there are so many competing agendas (Kane, 2005). A number of authorities have identified as a research priority an investigation of what pre-service teachers know and how best to equip them to teach primary mathematics (Ball, 1988; Goulding et al., 2002; United States [U.S.] Department of Education, 2008). With this background in mind, this study examines one mathematics curriculum unit of study in an Australian university to examine what knowledge the pre-service teachers arrived with, how it was related to their previous study, and how they improved through completion of the unit.

Importance of Subject Area Content Knowledge in Primary Teacher Preparation

In 2002, Goulding et al. made the following comment about the mathematical subject knowledge that pre-service teachers bring to teacher preparation: "For pre-service teachers … what they bring to training courses would seem to be critical" (p. 690). The authors believed that, in the main, tertiary teacher education courses did little to modify pre-service teachers' content or pedagogical knowledge in relation to mathematics teaching. The authors held that mostly, pre-service teachers would teach as they were taught. The reason for this was that pre-service teacher education units were a relatively weak intervention, in part because of the time demands in university education due to competing priorities (Kane, 2005). The ineffectiveness of initial training upon subsequent pedagogy was also reported by Askew, Rhodes, Brown, Wiliam, and Johnson (1997). Goulding et al. (2002) believed that effective teacher preparation ought to be based upon empirical evidence, including knowledge of the mathematical understandings with which pre-service teachers entered teacher preparation programs and how various programs impacted on their competency and confidence.

There is considerable debate about what constitutes critical knowledge for the preparation of pre-service teachers. For example, the recently released *Professional Standards for Teachers* (National Standards Expert Working Group, 2010) in Australia lists seven key standards, only one of which relates to a knowledge of content and how to teach it. Within this one standard there are nine sub-standards that relate to knowledge of: skills and pedagogy; stages of development; current research related to remediation; different communication strategies; sequencing and links to broader curriculum; assessment; reporting; ICT usage; and knowledge of Australia's Indigenous peoples. Addressing the list of priorities above illustrates the diversity of competing demands that Kane (2005) reported as leaving little time for transforming students' understanding of mathematics and how to teach it. Among all the standards and sub-standards of skills and pedagogy, content seems to be de-emphasised. This might be because there is an assumption that prospective teachers entering teacher education programs understand primary mathematics concepts, an argument noted by Henderson and Rodrigues (2008).

Mathematics Curriculum Knowledge

In regard to "skills and pedagogy", the importance of content knowledge in the teaching of mathematics has long been recognised as central to successful teaching at all levels (e.g., Ball et al., 2005; Ma, 1999; Osana, Lacroix, Tucker, & Desrosiers, 2006; Shulman, 1987, 1999; Warren, 2009). This relationship was articulated by the U.S. Department of Education (2008, p. 37): "Teachers must know in detail the mathematical content they are responsible for teaching and its connections to other important mathematics, both prior and beyond the level they are assigned to teach."

How knowledge to teach mathematics is best developed in primary teacher preparation courses is a matter for debate. Ball et al. (2005) list some of the most common recommendations:

- that teachers study more mathematics, either by requiring additional mathematics course work or a subject matter major;
- that there be a focus on mathematics methods course work, particularly related to the mathematics expected of the classroom teacher and curriculum materials; and
- that prospective teachers be chosen from selected colleges, anticipating that they are more likely to succeed in mathematics teaching "betting that overall intelligence and mathematics competence will prove effective in producing student learning" (p. 16).

Ball et al. (2005) question whether teachers need knowledge of advanced calculus or linear algebra in order to teach secondary, middle, or elementary school students. The assumption is that the study of more advanced mathematics ought to become decreasingly less relevant to mathematics teaching towards the lower grade levels. Knowledge of calculus seems less relevant to the teaching of counting than to middle school algebra. There is some research to support this assumption. Ma (1999) noted that it was possible to pass advanced courses in mathematics without understanding how they might inform the teaching of primary mathematics but that, none the less, a deep conceptual knowledge of mathematics plays a vital role in mathematics teaching and learning. At a macro level, most researchers agree with the U.S. Department of Education (2008, p. xxi) statement: "It is self-evident that teachers can not teach what they do not know."

Knowledge of mathematics content and how to teach it are intertwined in complex ways (Shulman, 1999). Shulman (1987) used the term pedagogical content knowledge (PCK) and described it as an intersection of subject knowledge and pedagogical knowledge. Askew et al. (1997) reported that highly effective teachers had knowledge and awareness of inter-relations between the areas of the primary mathematics curriculum they taught. However, "being highly effective was not associated with having an A level or degree in mathematics" (p. 5). Ma (1999) also noted that high levels of teacher content knowledge do not necessarily imply that individuals understand the material in a way that enables them to impart or teach it to students. Ma describes what is needed to teach as profound understanding of fundamental mathematics (PUFM). That is, teachers need to understand the material and ways of representing it to students. This has recently been described as mathematical knowledge of teaching (MKT) (Silverman & Thompson, 2008). Essentially, PCK and MKT are dependent upon a fundamental understanding of underlying mathematical structures (Silverman & Thompson, 2008). Goulding et al. (2002) suggested that there is a direct correlation between subject matter knowledge (SMK) and teaching mathematics, with teachers with strong SMK being more likely to be assessed as strong numeracy teachers and teachers with low SMK being more likely to be assessed as weak numeracy teachers. Goulding et al. reported that higher levels of pedagogical subject knowledge were linked to the systematic presentation of new ideas and making explicit links between different representations (verbal, concrete, numerical, and pictorial). Ball and McDiarmid (1988) argued that teachers' subject knowledge influenced the nature of questions they asked their classes, the types of tasks they allocated students, and teachers' ability to respond to questions.

Hill, Rowan, and Ball (2005) found that teachers' mathematical knowledge was significantly related to student achievement gains in first and third grades. In particular, teachers with higher content knowledge produced the students who demonstrated the greatest improvement. Hill et al. (2005) also noted that the total number of mathematics methods and mathematics content courses taken as part of teachers' pre-service and post-service graduate higher education were highly correlated. They were surprised to find that teachers' mathematical content knowledge predicted student gains in mathematics even in first grade. Hill et al. recommended content-focused professional preparation and pre-service programs as valid ways to improve student achievement.

A number of authors have noted that the level of pre-service teachers' mathematics and PCK is very important since there is little development of this on school placement (e.g., Brown, McNamara, Hanley, & Jones, 1999). The explanation for this is that mathematics PCK becomes subsumed in the pragmatics of general pedagogic concerns and that supervising teacher mentoring focused on classroom management, especially when their mentees are in survival mode. Once a teacher commences classroom practice there is likely to be limited opportunity to develop deeper mathematical PCK, reportedly in part because collaboration between teachers is limited (e.g., Bakkenes, De Brabander, & Imants, 2011; Weissglass, 1994) and there is a tendency for teachers to be resistant to change (e.g., Cuban, 1984; Gregg, 1995).

Diverse Approaches to Mathematics in Primary Teaching Preparation

It is to be expected that different pre-service preparation programs have different emphases upon mathematics curriculum and different ways to meet the various certification standards. Even within the domain of mathematics curriculum education, the focus upon content and pedagogical content knowledge compared to other content domains differs between institutions and even within an institution. These differences include considerable differences in the contact time allocated to mathematics curriculum across institutions.

In some jurisdictions there are multiple pathways to primary teacher certification. For example, New York State has five pathways (Boyd, Grossman, Lankford, Loeb, & Wyckoff, 2009) with a range of mathematics prerequisite requirements prior to teacher preparation entry. Most primary school teachers in Australia complete an undergraduate degree, usually full time over 4 years. This pathway is common across many countries, including China (Li, Zhao, Huang, & Ma, 2008). The alternative pathway in Australia, the United Kingdom, USA and elsewhere is a graduate diploma, usually completed in 1 year subsequent to the completion of an undergraduate degree. Throughout Australia students have increasingly favoured the 1-year, 18-month, or 2-year graduate pathways, with enrolments increasing proportionally at the expense of 4-year undergraduate degrees in a number of universities including the author's institution. An analysis of the time allocated to learning to teach mathematics across five teacher preparation institutions (University of Melbourne; University of Sydney; Griffith University, Gold Coast and Mount Gravatt Campuses; Queensland University of Technology) ranged from 90 hours down to only 24 hours of contact. About 40 hours of contact time was found in three institutions. Another provider, the Wesley Institute, offers online courses with no face-to-face contact time. None of the above have a pre-requisite level of mathematics content knowledge for enrolment.

It is difficult to readily determine how the allocated hours of contact for mathematics-related teaching are used, in part because course outlines tend to be generic in nature and do not list what is actually taught. In some institutions there is greater emphasis on theories of learning and social issues; in others the focus is on specific pedagogical approaches to teaching the content for the primary years. Henderson and Rodrigues (2008) suggest that the relative lack of focus on content and specific pedagogy for mathematics is because there is "an assumption that skills possessed need simply to be added to pedagogical content knowledge and other curriculum knowledge to produce effective teachers" (p. 104). Thus, in some Australian states, for example Queensland, there may be no systematic accounting of what is taught about teaching mathematics or what standards content or PCK is attained upon graduation.

Further, Australian primary teachers are not at present required to undertake registration examinations. Instead, state-based accrediting bodies review university course structures, and students are accredited on the basis of assessments of their university. The added criterion is that the student demonstrates "reasonable classroom practice," a judgment made by the primary school in which the pre-service teachers gain classroom experience.

Testing prior to registration exists in New York State, where prospective teachers must pass specific tests (e.g., New York State Liberal Arts and Science Test – LAST, and Assessment of Teaching Skills-Written-ATS-W, and possibly an appropriate Content Specific Test – CST). However, these tests do not focus on PCK, not even the content essential for teaching primary mathematics. Henderson and Rodrigues (2008) report that in England and Wales, teachers must achieve a minimum standard in numeracy, literacy, and information handling before qualifying. Similar standards are required in some Australian states, for example New South Wales.

Rationale for this Study

Particular shortcomings in the research literature confirm the need for this study. First, there is limited empirical research to guide primary preparation providers as to what level of mathematics ought to be considered essential for entry into primary teacher preparation courses (e.g., Goulding et al., 2002). Second, there is limited research into how mathematical understanding is best developed in primary teacher preparation programs and what relationships exist between the pre-service teachers' content knowledge and PCK. These problems have been recognised for some time. For example, Ball (1988) reported:

> This lack of attention to what teachers bring with them to learning to teach mathematics may help to account for why teacher education is often such a weak intervention – why teachers, in spite of courses and workshops, are most likely to teach math just as they were taught. (p. 40)

More recently, Ball et al. (2005) reported the problem described above remains, and that part of the reason for the limited empirical data informing these questions is that "testing teachers, studying teaching or teacher learning, using standardised student achievement measures – each of these draws sharp criticism from some quarters" (p. 45). The U.S. Department of Education (2008) noted:

> Most studies have relied on proxies for teacher's mathematical knowledge (such as teacher certification or course taken) [and that] existing research does not reveal the specific mathematical knowledge and instructional skill needed for effective teaching … Direct assessments of teachers' actual mathematical knowledge provide the strongest indication of a relation between teachers' content knowledge and their students' achievement. (p. xxi)

In short, empirical data on the depth or extent of pre-service teachers' content knowledge are relatively scarce; this is also the case in Australia. However, it is generally accepted internationally that many primary school teachers have less than ideal mathematical knowledge upon which to base their pedagogy (e.g., Ball et al., 2005; Brown & Benken, 2009; Ma, 1999). Such deficiency has also been reported in Australia (e.g., Masters, 2009). Further, although a lack of confidence in mathematics and teaching mathematics has been documented (e.g., Bursal & Paznokas, 2006; Henderson & Rodrigues, 2008), ways to remediate this situation in teacher preparation units have received scant attention.

Aims of this Study

The study had two guiding questions:

1. What relationships exist between high school and prior tertiary subject selection of mathematics and pre-service teacher success on primary mathematics content and pedagogical content knowledge?
2. What relationships exist between demonstrated content knowledge and demonstrated pedagogical content knowledge upon completion of a particular pre-service teacher mathematics preparation unit of study?

Methodology

The method chosen for this study was mixed-mode. Data were used inferentially and qualitatively, that is, the raw data were examined to determine the relationships between the variables. The following data were collected from the pre-service teachers:

1. The level of mathematics studied at high school. (Survey)
2. The form of mathematics studied during their undergraduate degrees or prior tertiary study. (Survey)
3. The level of mathematics upon entry to the course as measured by a standard Year 9 test of numeracy (MCEETYA, 2009). (Pre-test)
4. The level of mathematics upon exit from the course as measured by a standard Year 9 test of numeracy (MCEETYA, 2009). (Post-test)
5. A measure of pre-service teachers' ability to describe how they would teach specific mathematics to primary students. This was in effect an estimate of students' PCK at exit. (Post-test)

Test Procedures and Analyses

The pre-tests were administered in the first week of the mathematics curriculum unit and the post-tests in the last week of tutorials. The pre- and post-test NAPLAN data and the students' PCK were mapped to the pre-service teachers' prior mathematics learning. The relationships between prior study and student content and PCK tested in the mathematics curriculum unit were analysed using an analysis of variance. PCK was assessed upon completion of the unit; there was no pre-test of PCK since the specific pedagogy for teaching the number and algebra components of primary mathematics had not been taught to students.

Subjects

Almost the entire cohort of 129 students from the Graduate Diploma in Primary Education 2010 participated in the study (n=129 for the pre-test and n=122 for the post-test). The percentage of females at the start of the study was 85%. The majority of students had completed high school since 2000 and with few exceptions had undertaken a degree before commencing teacher pre-service education. The cohort was chosen on the basis of convenience: the researcher had the opportunity to collect data from its members. The subjects of this cohort were similar in entry numeracy and exit results to cohorts in the past two years. These pre-service teachers may well be similar to student intakes for similar courses at other teacher preparation institutions at least in the state of Queensland, potentially across Australia, and internationally such as in the United Kingdom. In Australia and the United Kingdom at least, teacher preparation courses do not stipulate pre-requisite knowledge of mathematics.

Curriculum Unit Description

The curriculum course structure included the teaching of numeration, whole number computation, fraction computation, and introductory algebra, and there was an emphasis on teaching proportional reasoning across the strands of number, space, and measurement. Teaching sequences emphasised the use of specific language to make links between various models, material and diagrammatic and symbolic representations. This approach to teaching and learning mathematics is supported widely (e.g., Goulding et al., 2002; Reys, Lindquist, Lambdin, & Smith, 2009; Van de Walle, 2007). The explicit approach has the support of a number of education bodies (e.g., U.S. Department of Education, 2008) and mathematics education researchers (e.g., Kirschner, Sweller, & Clark, 2006).

The underlying goal of the curriculum unit was to teach the underpinning mathematical concepts to the pre-service teachers while teaching them how to teach the concepts. For example, by modelling how to teach division with the use of specific language, materials, and linking these representations to symbolic recording, it was anticipated that the pre-service teachers would understand division as well as know how to teach it. The curriculum unit in this study had been approved by the teacher registration body in the state (Queensland College of Teachers, 2006) as meeting the requirements for teacher preparation such that the graduating students are eligible to be registered as teachers in the state of Queensland.

Instruments

Categorising the level of mathematics studied at high school

Assessing and categorising the level of high school mathematics was relatively unambiguous since each level was described to the students. The categorisation mirrors the form of mathematics studied at high school. Students who cease study of mathematics at Year 10 or 11 generally have had limited exposure to abstract mathematics associated with algebra, proportional reasoning in number and geometry contexts, or logic associated with proof. These students who had not completed any senior mathematics were classified as Level 1.

Students who study senior Mathematics A similarly have limited exposure to abstract mathematics; rather, they study units that focus on the application of mathematics in financial contexts, applied geometry such as navigation or building construction and plans, and relatively simple presentation and analysis of data. Mathematics A does not assume knowledge of calculus and the applications of algebra and geometry are relatively simple. Students who had studied Mathematics A or its equivalent were classified as Level 2.

Pre-service teachers who completed Mathematics B or its equivalent were classified as Level 3. Mathematics B (or its equivalent) is generally the minimum level of school mathematics needed to enter science-based courses at tertiary institutions and is undertaken by about 20% of senior school students in

Australia (Barrington, 2006). Mathematics B or its equivalent typically has core units such as introduction to functions, rates of change, periodic functions and applications, exponential and logarithmic functions, optimisations, integration, and statistics. The subject matter is mostly calculus and there is some statistics including hypothesis testing.

At a higher level, students who study senior Mathematics C study core topics including groups, real and complex number systems, matrices and applications, vectors and applications, the application of calculus, and a range of optional topics including linear programming, conics, dynamics, and advanced periodic functions and exponential functions. Generally only students who intend to enter tertiary study associated with the hard sciences such as engineering, actuarial studies, or pure mathematics study Mathematics C. Barrington (2006) reported that across Australia about 10% of graduating high school students complete Mathematics C type courses. Students who had studied Mathematics C were classified as Level 4.

The completion of the various levels above provides a reasonable guide to the level of mathematics undertaken, and presumably understood, by the students. For example, a student can gain a pass result in Level 1 or 2 with very limited understanding of abstract mathematics, proof, algebraic processes, or even good number sense. This is not the case with Levels 3 and 4. It is for this reason that most tertiary institutions assume the equivalent of Levels 3 or 4 knowledge for entry to most science-based tertiary courses and frequently offer bridging courses for those lacking in this level of mathematical competence.

Categorising the form of mathematics studied at university

Assessing and categorising the level of tertiary mathematics studied was problematic. It was difficult to estimate accurately the level of tertiary mathematics embedded in courses that varied from “mathematics associated with nursing,” “mathematics associated with health sciences,” “health science statistics,” or “business mathematics.” The categories of tertiary mathematics levels that emerged from the tertiary data were “no mathematics,” “health science statistics,” “business mathematics,” mathematics associated with business, accounting, or economics, and “advanced mathematics” associated with the study of subjects including physics, engineering, and computer sciences. “No mathematics” indicates that the tertiary experience did not add to the mathematics the students learned in high school. “Health science mathematics” tends to be dominated by specific mathematics associated with measurement and is not very dissimilar from aspects of Mathematics A in terms of the level of abstraction required. It is to be expected that “business mathematics” might extend upon what students had studied in high school mathematics to Levels 2 and 3, since a typical business degree contains up to three 10-credit point subjects in research methods and statistics as well as two or three subjects in which mathematics plays an important role, for example accounting-based subjects or economic modelling.

Assessing entry and exit content knowledge (numeracy)

In order to gain a measure of students' content knowledge of mathematics at the beginning of the course, students completed the 2009 Year 9 NAPLAN non-calculator test (MCEETYA, 2009) under examination conditions. At the end of the course the students completed the second of the two Year 9 NAPLAN tests. In both instances the pre-service teachers were not allowed to use a calculating device. A test analysis of the NAPLAN items shows that, due to the structure of test items developed by MCEETYA (2009), students with a reasonable knowledge of primary computation ought not to have been disadvantaged by not having access to a calculating device (Norton, 2009). NAPLAN test papers are designed to assign students to particular band levels, and thus test a range of difficulty levels with questions that are of a standard lower than what is expected of a year level as well as some more challenging questions. Teachers of upper primary years would be expected to teach most of the concepts tested in these tests and few educators would argue that teachers do not need to know at least middle years mathematics.

Assessing Exit Pedagogical Content Knowledge

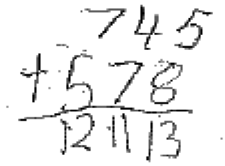
In order to assess students' grasp of PCK, students completed 10 questions under examination conditions. The structure of the written exam is presented in Table 1.

Table 1
Structure of the Post-Test Exam including Extended Answer Questions

Question	Concept	Marks
NAPLAN Year 9 Numeracy test		/31
	Short answer test of PCK	
1	Teaching naming numbers	/5
2	Teaching the addition concept	/5
3	Teaching subtraction with renaming	/7
4	Teaching the multiplication algorithm	/7
5	Teaching the area model of multiplication	/7
6	Teaching the division algorithm	/7
7	Teaching fraction and decimal representations	/7
8	Teaching mixed number subtraction	/7
9	Teaching problem solving in the context of fractions and decimals	/7
10	Teaching algebra problem solving	/10
Total		/100

Two sample items testing PCK for a lower primary and a middle primary concept are presented in Figure 1 and Figure 2.

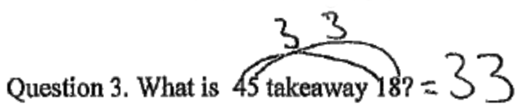
A Year 3 student carried out the following addition.



a) What was his conceptual error and what teaching might have led to that error?
b) Set out a teaching sequence clearly linking materials and formal symbols with clear connecting language.

Figure 1. Question 2

Examine the student working below showing the computation 45 - 18.



a) What teaching and strategies might have led to this method? What are the limitations of the method?
b) Set out a teaching sequence clearly linking materials and formal symbols with clear connecting language.

Figure 2. Question 3

Appendix A contains a sample of a good script where full marks were awarded for Question 3. The solution presented in Appendix A illustrates that the pre-service teacher is able to recognise error patterns in student scripts and design a teaching sequence to assist in remediation of this error. The pre-service teacher's solution in Appendix B illustrates that the student can link the equivalent representations of 75%, .75 and ¾. The marking criteria are documented in Appendix C.

SPSS 18 was used to undertake all analyses. Significance was assessed with type 1 error, α = 0.0.5 for 2-sided tests, and significance set at significant * < 0.05, highly significant ** <0.01, very highly significant ***<0.001.

Comments on the Instruments and Potential Limitations

There were additional hours of study related to mathematics curriculum after this unit, but they were limited and the focus was upon general pedagogical principles, planning, and designing assessment rather than providing specific strategies for the diagnosis and remediation of key aspects associated with the number strand, which was the focus of this course.

The use of the NAPLAN tests as a measure of numeracy has the support of the Department of Education and Training (2010a, 2010b). The authors cite consistent matching of scores in sample and test populations of up to a million students in any year. NAPLAN reports student achievement according to bands, that is, raw scores are scaled to 1 with a mean of 500 and standard deviation of 100. In this study raw scores are used, but this does not detract from the validity of the results or the comparisons made.

It could be considered that the test of PCK is problematic in that it essentially asked pre-service teachers to replicate the pedagogy for teaching numeration, algorithms, and problem-solving models that they had studied in lectures and workshops. However, the use of the instruments such as those described above is supported by Council of Australian Governments [COAG] (2008) who reported valid teacher assessment should not be remote from what teachers do in the classroom. In terms of the teaching and assessment approach, most teacher educators would concur that systematic linking of various representations of mathematical concepts is central to teacher planning (e.g., Goulding et al., 2002; Reys et al., 2009; U.S. Department of Education, 2008; Van de Walle, 2007). Thus, it is reasonable to expect teachers to be able to describe what they would get students to do, what language they would use, what materials they would use, and how they would assist students to connect various representations of mathematical concepts. From this point of view the test of PCK has content validity.

Results

Level of Mathematics Studied at High School and at University

The first level of data reporting and analysis seeks to answer the first research question:

> What relationships exist between high school and prior tertiary subject selection of mathematics and pre-service teacher success on primary mathematics content and pedagogical content knowledge?

Initially the data on high school and tertiary mathematics are presented, then pre-service teachers' results in tests of upper primary content and mathematics PCK are documented. The levels of senior high school mathematics completed by commencement of pre-service teaching, and the level of tertiary mathematics undertaken, are documented in Table 2.

Table 2
High School and Tertiary Mathematics Completed (N=119)

University Mathematics	Categories of high school mathematics completed by pre-service teachers			
	Level 1	Level 2	Level 3	Level 4
None (54.4%)	7.5%	34.4%	9.2%	3.3%
Health statistics (5.9%)	3%	1.5%	0.8%	0%
Business mathematics (31%)	1.5%	9.2%	17.6%	2.5%
Advanced mathematics (8.4%)	0%	0.8%	2.5%	5%
Total	12.6% (*N*=15)	45.9% (*N*=55)	30.1% (*N*=36)	10.8% (*N*=13)

The survey data indicate that most students had studied relatively low levels of high school mathematics (about 59% Level 1 or 2), about a third had studied intermediate mathematics (Level 3), and about 11% had studied advanced mathematics, that is, both Mathematics B and C (Level 4). Most students had not studied any mathematics as part of their tertiary courses, about 37% had completed mathematics as part of health sciences or basic business statistics, and few (8.4%) had studied advanced mathematics at a tertiary institution.

The following results are reported in terms of previous high school mathematics, without taking into account any tertiary mathematics studied by students. The possible effects on tertiary mathematics results are discussed at the end of this section.

Results on Tests of Content and Pedagogical Content Knowledge

In the sections below students' results on the test items are reported and major findings described. The pre-service teachers in this study were found to have a level of mathematical understanding not significantly different from the average Year 9 student in the state of Queensland.

In regard to the first research question, the relationship between the level of high school mathematics studied and success on a test of primary mathematics content, the data indicate that higher levels of high school mathematics are associated with higher scores on both the pre- and post-test NAPLAN tests and the written test of PCK (see Table 3). In terms of the pre-test of content knowledge, the mean differences between Level 1 and 2 students were 1.91 marks (p=0.940); between Level 2 and Level 3 students it was 3.15 (p=0.008); and between Levels 3 and 4 the mean difference was 2.85 (p=0.401). There was an

Table 3
Outcomes on Content Knowledge and Pedagogical Content Knowledge Tests

High school mathematics	Pre CK/31		Post CK/31		Post PCK/69	
	mean	(sd)	mean	(sd)	Mean	(sd)
Level 1 No senior	15.63	(5.42)	17.90	(6.82)	43.53	(14.13)
Level 2 (Maths A)	17.54	(4.11)	19.45	(4.59)	47.63	(11.25)
Level 3 (Maths B)	20.69	(4.56)	22.94	(3.61)	52.69	(9.53)
Level 4 (Maths C)	23.54	(4.70)	25.27	(3.66)	60.42	(4.75)

increase in NAPLAN scores for each high school mathematics category, which was statistically significant (df, 21, 117; F= 4.734; p= 0.000). Analysis of scripts indicated that upper primary concepts such as division of two-digit numbers, operations with fractions, and questions related to proportional reasoning were the most challenging to the pre-service teachers, especially for pre-service teachers with high school mathematics at Levels 1 and 2.

It is worth noting that the variation of scores was much more extensive among students who completed lower levels of high school mathematics. This was the case for each assessment instrument. The data indicate that more mathematics studied in high school was not only associated with higher marks on these tests, but that this was consistently the case.

The data in Table 4 sum up relationships between the pre and post-tests of content knowledge and the post-test of PCK and levels of high school mathematics completed.

Table 4
Summary of ANOVAs on the Pre-Test for Content Knowledge (CK), Post-Test for Content Knowledge and Post-Test for Pedagogical Content Knowledge According to High School Mathematics Studied

Test	Df	*F*	Sig	Comment
Pre-CK	2, 114	12.497	<.000	There was no significant difference between scores of Level 1 and Level 2 groups. Students who studied more advanced mathematics (Level 3 and Level 4) achieved much higher scores.
Post-CK	2, 117	17.474	<.000	There was little to distinguish between Levels 1 and 2 and between Levels 3 and 4, but the latter groups had much higher scores than the students who studied lower levels of high school mathematics.

Test	Df	*F*	Sig	Comment
Post-PCK	2, 117	11.032	<.000	There was no difference between those who studied Level 1 to Level 2. Level 3 students' scores were significantly better than Level 1 but not Level 2 and while Level 4 were significantly better than Levels 1 and 2 students' scores, they were not significantly better than Level 3 students.

Those pre-service teachers who had studied low levels of high school mathematics equivalent to Mathematics A (Level 2) were similar to those studying no mathematics in senior years of high school (Level 1). Further, the effect of studying high school mathematics at Level 3 – which contains a strong emphasis on calculus – was indistinguishable from that of studying at Level 4. That is, doing the extra abstract mathematics at high school did not seem to confer any advantage; passing the equivalent of Mathematics B was sufficient.

The study of the equivalent of Mathematics B at high school seems to be a defining feature of success on tests of primary content and PCK, which is explaining how to teach it. This finding is supported by data contained in Appendices D and E showing the ranking of the top and bottom quartiles. Almost without exception, students who had studied to Level 3 (Mathematics B) at high school occupied the top quartile of results. The data in Appendix D illustrate that, when final content and ability to explain how primary mathematics is taught is tallied and students ranked according to this total, almost universally the top 30 students had studied calculus and most of the top 20% of students had also studied advanced or business mathematics at university. The top ranked pre-service teacher who had studied high school at Level 1 (Year 10) was ranked 14th overall. However, it should be noted that this student had studied computing mathematics at university and was graded with a distinction. The highest ranking achieved by a pre-service teacher who had studied Level 2 mathematics was 12th overall and s/he had achieved a credit in university statistics. Of the top ranked pre-service teachers, a colleague and experienced mathematics educator who moderated the course results commented, "Wow, I agree this student really knows a lot about how to teach the various concepts." The data in Appendix E illustrate that students who had studied Level 1 and Level 2 high school mathematics dominate the bottom quartile.

The second aspect of the first research question focuses on students' selection and completion of tertiary mathematics courses and probes any relationship that might exist between this and their subsequent success on the pre-service tests of content and PCK. It is very difficult to make much of this because so many variables are unknown. It is not known exactly what content was taught in the tertiary courses or how well it was learnt. Importantly, it is not known how tertiary study might be associated with an increased primary mathematics content of PCK mark or what interaction might exist between the tertiary mathematics studied and the level of mathematics studied at school. Most of the top 20 students had studied statistics of some form and a few had studied engineering or computing. Still, the data in Appendix E indicate that most of the bottom quartile had not studied mathematics at university. However 13 out of 30 had studied some form of statistics, sometimes associated with finance or health science. It was clear that this study of basic statistics did not compensate for not having done at least Level 3 high school mathematics.

The second research question sought to examine the relationship between demonstrated content knowledge and PCK upon completion of the course. Most students who studied the higher levels of mathematics at high school achieved relatively well on all tests. High scores on mathematics content were associated with high scores on pedagogy. It could be said that these pre-service teachers were sufficiently literate to explain what they understood. Similarly, pre-service teachers who did not know the mathematics could not explain it no matter how many non-mathematics-based subjects they had undertaken at a tertiary level.

The 10 students who achieved less than 50% on the examination were granted a supplementary examination after several weeks of further study. All students who attempted the supplementary examination attained at least 50%. The student who scored 23% on her first attempt at describing pedagogy subsequently attained 88% on similar tasks.

Discussion and Conclusions

The review of pre-service teacher program requirements and outlines indicates that within Australia, and internationally, there is considerable diversity in terms of what is taught and what time is taken to teach it. Face to face learning time varies form zero for study options offered online to close to 100 hours. Without access to their examination scripts it is difficult to determine what is taught in the various courses and what emphasis there is upon content and PCK. Readers are asked to decide for themselves if the findings have any relevance to their own situation.

In regard to the first research question, this study begins to document what content and PCK pre-service teachers from one postgraduate unit on one campus at one institution have demonstrated. As a cohort, the students entered the unit with content knowledge similar to the average Year 9 student (age 13 to 14 years). Relatively low levels of mathematics prior to entry to primary teaching preparation are not unique to this sample: Adler et al. (2005, p. 361), for example, reported that in many countries "prospective elementary teachers have learned limited mathematics in school." This finding supports the concerns expressed by Henderson and Rodrigues (2008) who reported teachers' understanding of mathematics was shaped by school and informal experiences and that teacher education programs tend to assume that prospective teachers bring with them sufficient mathematical understanding to enable them to promote effective classroom practice.

The data presented here show that most pre-service teachers who have completed limited mathematics study in high school, know less when they commence tertiary teacher preparation study and exit with lower levels of content and PCK than other pre-service teachers. That is, having completed no senior high school mathematics, or having studied mathematics without calculus, is strongly associated with lower marks on tests for primary mathematics content and PCK.

A few pre-service teachers who undertook Level 1 and Level 2 high school mathematics did achieve high scores on the pre-service teacher preparation tests. This may be because many Queensland students who are quite good at mathematics have received advice not to take the equivalent of Mathematics B and C unless they intend to enter tertiary courses that specifically require these, such as engineering and the hard sciences. This is especially the case with Mathematics C. A further factor that discourages mathematically capable students from undertaking the more exacting mathematics subjects is that low-level mathematics subjects (Mathematics A) have the same weighting reward as high-level mathematics subjects (Mathematics B and C) for tertiary entrance. In terms of final tertiary entrance ranking scores, a student with high achievement in Mathematics A might well gain similar credit to a person with a high achievement on the much more demanding Mathematics B or C subjects. These channelling factors may help to explain the wide range of mathematical achievement among Level 2 pre-service teachers.

It is difficult to determine what effect undergraduate tertiary study of mathematics has upon the level of relevant mathematics a pre-service teacher brings to teacher preparation. This is in part due to the observation that most pre-service teachers who studied relatively rigorous tertiary mathematics associated with science, finance, or computing had previously studied high school mathematics at least to Level 3. However, the data indicate that the study of tertiary mathematics associated with health sciences or statistics did not seem to compensate for the lack of study of Level 3 mathematics in high school. In short, if a pre-service teacher did not study mathematics to Level 3 and did not study advanced mathematics at university, but rather did no tertiary mathematics or only mathematics units associated with health sciences such as nursing or basic statistics, it was highly likely they would fail or nearly fail tests of primary content and PCK, even after 40 hours of focused tertiary learning. There is a substantial body of research indicating that teachers' confidence in teaching is strongly correlated to their confidence with the subject matter of mathematics (Ball, 1988; Bursal & Paznokas, 2006) and teacher confidence affects their practice (e.g., Stipek, Givven, Salmon, & MacGyvers, 2001).

Almost half the pre-service teachers exited this unit with relatively strong knowledge of content and how to teach it (refer to Table 4). Some possible contributing factors include the structure of the unit, its content, how it was taught, the time it was implemented, and the nature of the intake.

In regard to the second research question, the results indicate that pre-service teachers who were proficient at mathematics were effective at explaining how to teach it. This finding provides empirical support for the arguments of those who consider there is a strong link between content knowledge and teaching knowledge (e.g., Ball et al., 2005; Goulding et al., 2002; Ma, 1999; Silverman & Thompson, 2008; U.S. Department of Education, 2008). It is interesting that a test designed to assess primary and lower middle school students' knowledge of mathematics (NAPLAN) should be such a strong predictor of success on a pre-service test of PCK and overall success on the curriculum unit. This finding supports the claims from the Department of Education and Training (2010a, 2010b) that the NAPLAN tests are a reliable assessment of primary and middle school mathematics across a range of student ability.

The study raises another interesting question. Why would the study of calculus, particularly advanced calculus, be such a robust predictor of high marks on both content and pedagogical knowledge tests designed for primary students? There is no evidence in the data to answer this question and the finding contradicts earlier research (e.g., Askew et al., 1997). It may be that those pre-service teachers who had selected to study high levels of high school mathematics were in the main generally more competent or more intelligent. Alternatively, the study of advanced mathematics may have assisted these pre-service teachers in becoming analytical beyond the domains of calculus or more advanced statistics, such as in concise writing of explanations about how to teach mathematics. A third and related possibility is that knowing calculus helped these pre-service teachers to quickly develop a profound understanding of primary and early middle year mathematics, particularly in regard to the content of the NAPLAN tests.

The data indicate there is merit in exploring the use of the level of high school mathematics completed as a partial filter for teacher preparation programs. At least knowing the level of high school mathematics completed by the applicant would alert the tertiary preparation provider to the need for additional testing in order to signal the need for early intervention. Widely available tests such as NAPLAN could be used to provide additional data.

The major finding of this study suggests the following recommendations for further study. First, a more in-depth study of the relationship between content and pedagogical knowledge is needed. Second, ongoing research into the effectiveness of various mathematics pre-service teacher programs is warranted, as are instruments to study progress. The data indicate that further research is needed on the content, duration, and delivery methods of units preparing pre-service teachers to teach mathematics. It is clear that in this and potentially many other instances, too little is done too quickly for the many students who enter teacher preparation with limited mathematical background.

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Appendix A
Sample of a good response to a PCK question.

18
27
45

Question 3
Examine the student working below showing the computation of 45- 18.

3 3

Question 3. What is 45 takeaway 18? = 33

(a) What teaching and strategies might have lead to this method? What are the limitations of the method? (1 mark)

This student may have been taught using rainbow facts and also taking the smallest from the biggest. This method does not teach the concept of renaming and students do not understand the algorithm.

(b) Set out a teaching sequence clearly linking materials and formal symbols with clear connecting language. (4 marks)

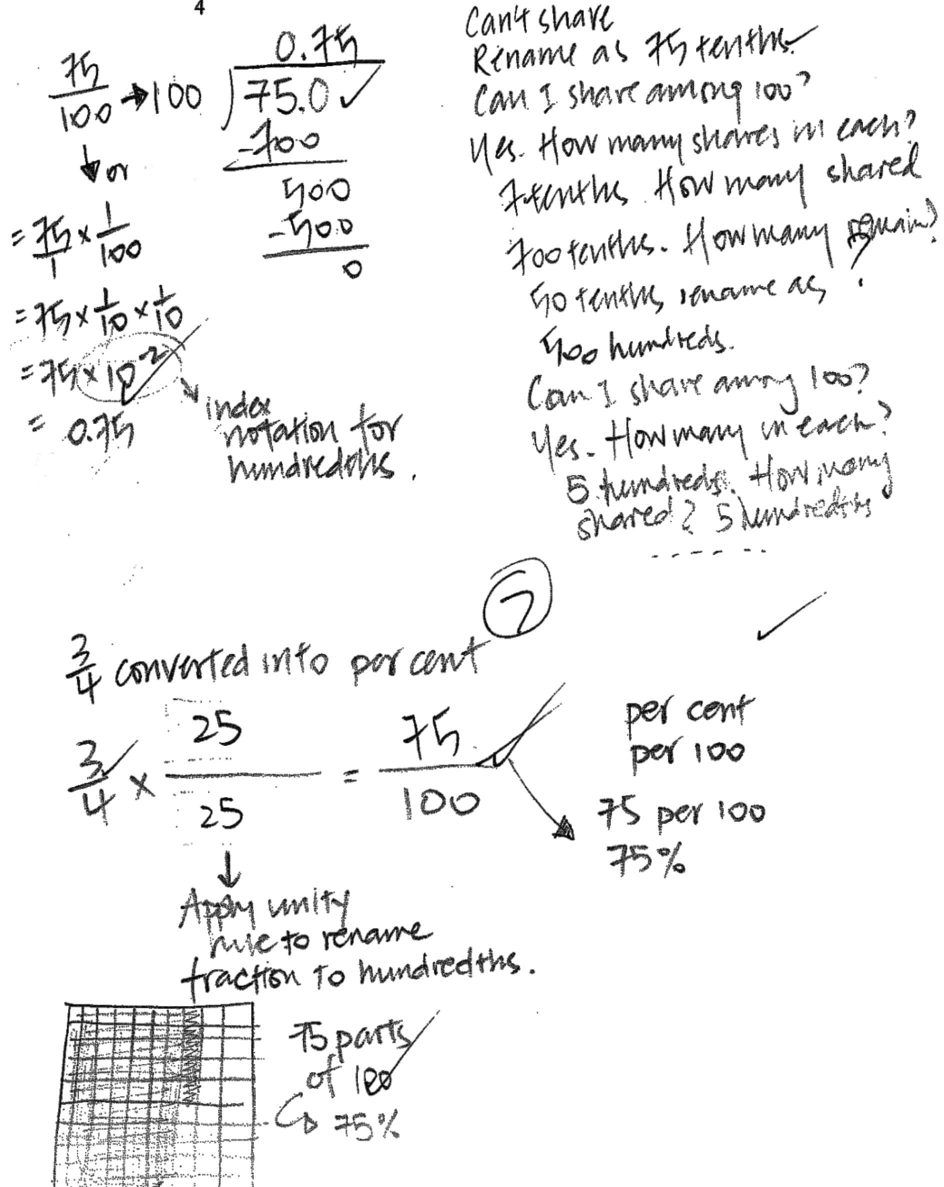
Language	Materials	Symbols
Takeaway the ones. I have 5 ones can I takeaway 8 ones? NO! Rename one of the tens as 10 ones. Record the 3 tens and 15 ones.	Tens / Ones	3 15 45 − 18
I have 15 ones, can I takeaway 8 ones? YES! Do it. 15 takeaway 8 is 7. Record.	Tens / Ones	3 15 45 − 18 7
Takeaway the tens. I have 3 tens, can I takeaway 1 ten? YES! Do it 3 tens takeaway 1 ten is 2 tens. Record	Tens / Ones Takeaway	3 15 45 − 18 27
	Tens / Ones Takeaway	

45 takeaway 18 is 27

Appendix B

Question 7 (7 Marks)

Explain how you would develop and understanding that $\frac{75}{100}$ was equal to 0.75 and $\frac{3}{4}$ and 75%.



Appendix C

Marking Criteria for Written Examination Tasks assessing PCK.

Grade	Criteria description
A	The problem is fully solved. The response shows evidence of interpretation, analysis, identification of assumptions, use of appropriate strategies and procedures for teaching while showing initiative. All choices and explanations are justified and all steps well explained. Teaching has been explicit with appropriate use of various representations. Full marks.
B	The problem is fully solved. The response shows evidence of interpretation, analysis, identification of assumptions, use of appropriate strategies and procedures for teaching while showing initiative. There may be minor errors in choices and explanations or justification of steps contains minor omissions. Teaching has been explicit with only minor omissions in the use of appropriate use of various representations. High marks e.g., 8/10 or 6/7 etc.
C	The problem has been solved. However, while there is evidence of use of appropriate strategies for teaching; justification, explanations or use of representations, they have not been appropriate in significant ways or choices and explanations have not been well explained. A peer would likely have difficulty following the teaching steps. Approximately half marks.
D	The problem has not been solved. There are significant flaws in methodology for working out the solution and explaining its teaching with poor communication or lack of use of appropriate use of representations. Few or zero marks.

Appendix D

Raw Scores on Tests, High School Mathematics and Tertiary Mathematics Studied for <u>*Top Quartile*</u>*, Ranked According to Final Test Score.(Ranked according to final test score).*

N1/31	N2/31	Written /69	Final %	High School Mathematics	Math Level	Tertiary Mathematics
22	28	66.5	94.5	Maths B (HA)	3	None
25	30.5	63.5	94	Maths B, C	4	Commerce statistics
24	26	67.5	93.5	Maths B	3	Statistics
27	30	63.5	93.5	Maths B, C	4	BSC Hons physics
25	26.5	66.5	93	Maths A (VHA)	2	Business finance (D)
28	27	65.5	92.5	Maths B, C HA	4	none
26	28.5	63.5	92	Maths B (Vic)	3	none
23	26	65.5	91.5	Maths B	3	Statistics
14	25	66	91	Maths B, C	4	Finance statistics
29	29.5	61	90.5	Maths B, C Dist	4	Engineering
28	27	63	90	Maths B C VHA	4	Economics
17	26.5	62.5	89	Maths A (HA)	2	Statistics Credit
26	24.5	62.5	87	Maths B	3	none
21	23	63.5	86.5	Year 10	1	Computer maths (D)
18	25	61.5	86.5	Maths B	3	none
25	26	60.5	86.5	Maths B	3	Probability credit
24	29	57.5	86.5	Maths B, C (D)	4	MSc.
27	25	61	86	Maths B	3	Statistics (D)
20	23.5	62.5	86	Maths B	3	Physics (D)
22	23	62.5	85.5	Maths B (HA)	3	none
22	26	59	85	Yr 12 Canada adv	2	none
23	23	61.5	84.5	Maths B (HA)	3	Business risk assess
19	22	62	84	Maths A	2	None
26	26	57.5	83.5	Finite math Canada	3	Statistics maths
16	20	63	83	Year 11	2	none
25	27.5	55.5	83	Maths A (HA)	2	none
15	19.5	63	82.5	Maths A	2	none
18	20	60.5	80.5	Maths B	4	none
28	26	53	79	Maths B	3	Statistics (Pass)
16	23	54.5	77.5	Maths B	3	Accounting

Key:
N1 – Score on pre-test for -content knowledge with Year 9 NAPLAN test.
N2 – Score on post-test for content knowledge with Year 9 NAPLAN test.
Written – Score on post-test for content pedagogical knowledge
VHA – very high achievement
HA – high achievement
D – distinction

Appendix E

Raw Scores on Tests, High School Mathematics and Tertiary Mathematics Studied for Bottom Quartile (ranked according to final test score)

N1	N2	Wri	%	High School Math	Lev	Tertiary Mathematics
21	24	44	68	Maths A (VHA)	2	none
9	19	46.5	65.5	Year 10	1	none
20	22	42.5	64.5	Maths B Canada	3	Statistics
24	27.5	36.5	64	Maths B 1968	3	None
24	20	43	63	Maths B (LA)	3	Statistics
15	23	38.5	61.5	none	1	none
15	19	42	61	Maths B	3	Statistics
20	17	44	61	Maths A HA	2	none
20	20	39.5	59.5	Maths A	2	Statistics
18	14	45	59	Maths A	2	none
23	17	42	59	Maths B	3	Statistics for fin
11	13	44	57	Maths A	2	Intro to account
15	18	37.5	55.5	Math B Canada	3	Statistics
8	19.5	33	52.5	Maths A fail	2	none
14	15	37	52	Maths A HA	2	Statistics
12	13	39	52	Maths A credit	2	none
15	15	35.5	50.5	Maths A	2	none
	17	33.5	50.5	Maths A	2	none
15	13	37.5	50.5	Maths A	2	none
20	16	34.5	50.5	Year 10	1	none
16	19	31	50	Maths A	2	none
12	24	24	48	Maths A	2	none
14	12	34.5	46.5	Maths A (SA)	2	Statistics
12	8	38	46	Year 10	1	none
25	19.5	23	42.5	Maths A fail	2	Statistics
13	13	29	42	Maths A	2	none
17	17	24.5	41.5	Year 10	1	Statistics
	9	31	40	Maths A (SA)	2	none
5	8	22	30	Year 10	1	Statistics for psy
14	7	23	30	Year 10	1	Biostatistics health
13	7	15	22	Year 10 (SA)	1	none

Key:
N1-Score on pre-test for content knowledge with Year 9 NAPLAN test.
N2- Score on post-test for content knowledge with Year 9 NAPLAN test.
Written-Score on post-test for content pedagogical knowledge
VHA- very high achievement
HA- High achievement
D- Distinction
SA- Sound achievement
LA- Low achievement