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"I Know You Have to Put Down a Zero, But I'm Not Sure Why": Exploring the Link Between Pre-Service Teachers' Content and Pedagogical Content Knowledge

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This paper reports on an investigation into pre-service teachers' mathematical content knowledge and their ability to interpret students' responses to a multi-digit multiplication task and make subsequent appropriate teaching decisions. Using a combination of quantitative and qualitative methods, the researchers tested aspects of the mathematical knowledge held by a volunteer group of twenty final year pre-service primary teachers. A volunteer sample of seven pre-service teachers were involved in a follow-up interview, where they were provided with hypothetical student work samples, including one using the long multiplication algorithm, and asked to analyse the student's mathematical thinking and make suggestions as to appropriate teaching approaches. The results indicated that the pre-service teachers in the study had an instrumental understanding of the long multiplication process that impacted on their ability to both recognise and address students' mathematical errors. This study provides an insight into the lack of content knowledge of a small sample of pre-service teachers with respect to multiplication of two and three digit numbers and subsequent lack of pedagogical content knowledge for teaching this topic.

Keywords: pre-service teachers • content knowledge • pedagogical content knowledge • multiplication • errors

Concerns about pre-service teachers' limited mathematical knowledge are widespread in the research literature (e.g., Ball, 1990; Lange & Meaney 2011; Ryan & Williams, 2007), along with the acknowledgement that effective teachers require both knowledge of students' mathematical ideas and thinking as well as knowledge of mathematical content (e.g., Ball & Bass, 2000; Ball, Lubienski & Mewborn, 2001; Hill, Ball & Schilling, 2008). The seminal work of Shulman (1986) identified several categories of knowledge important for teaching, and since then researchers have continued to build on and re-define these categories (e.g., Hill et al., 2008; Ma, 1999), including the development of frameworks designed to conceptualise the various aspects of teacher knowledge. Of particular relevance to this paper is Shulman's definition of pedagogical content knowledge (PCK) which refers to "an understanding of how particular topics, problems, or issues are organised, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction" (Shulman, 1987, p. 8). In addition to PCK, reference is also made to mathematical content knowledge (MCK), and teachers' knowledge of content and students (KCS) which Hill et al. (2008, p. 375) define as "content knowledge intertwined with knowledge of how students think about, know, or learn this particular content". Ball (2000) and Ma (1999) both advocate that teachers need to understand the mathematics they are required to teach, along with an ability to provide sound explanations of mathematical ideas. Research, however, suggests that many primary pre-service teachers experience similar difficulties with foundational mathematical skills and concepts as the students they are required to teach (e.g., Ball, 2000; Ryan & Williams, 2007).

This paper reports on some of the findings from a larger study that aimed to investigate the nature of the relationship between a selected number of pre-service teachers' MCK and aspects of their PCK, across a range of mathematical domains. Within the mathematical domain of rational number, the pre-service teachers' responses to a two-digit multiplication task are discussed in detail in this paper. The multiplication task was selected because there are few examples in the research literature of pre-service teachers' understanding of this, it was similar in nature to the item given to practising teachers in Ma's study, and the researchers had discussed the pre-service teachers' responses to other items elsewhere (e.g., Maher & Muir, 2011; Maher & Muir, under review). The study differs from others in the literature in that the majority of the data are gathered through interviewing pre-service teachers as they interpret students' approaches to mathematical tasks and are then asked to identify appropriate teaching strategies to address students' errors and misconceptions.

Review of the Literature

Shulman (1987) provides useful classifications of teachers' knowledge. According to Shulman, the teacher has a special responsibility in relation to content knowledge and should possess depth of understanding in order to communicate what is essential about a subject and be able to provide alternative explanations of the same concepts or principles. "We expect teachers to understand what they teach, and when possible, to understand it in several ways" (Shulman, 1987, p. 14). Ma (1999) used the term Profound Understanding of Fundamental Mathematics (PUFM) to describe what some of the teachers in her study had, compared with other teachers who did not have this understanding. While these teachers possessed solid mathematical content knowledge, it went beyond the ability to compute correctly, to being aware of the conceptual structure of mathematics and being able to teach it to students. Ma's study and others (e.g., Mewborn, 2001) have found that although primary teachers generally have a command of the facts and algorithms that comprise school mathematics, many lack conceptual understanding of this mathematics. The literature reveals that this lack of understanding encompasses different mathematical topics, including quotitive division, fractions, ratios, area, perimeter, units of measurement, proof, place value, and decimals (Mewborn, 2001). There is little evidence in the literature to identify how much or what type of content knowledge a primary school teacher requires (Hill, et al., 2008) but many primary teachers express considerable lack of confidence in their own knowledge and understanding of mathematics (Stephens, 2000). The American Council on Education (ACE) stated that "a thorough grounding in college-level subject matter and professional competence in professional practice are necessary for good teaching … students learn more mathematics when their teachers report having taken more mathematics" (as cited in Mewborn, 2001, p. 28).

It appears, however, that simply having studied mathematics at a higher or even advanced level before undertaking teacher training does not necessarily equate with having strong content knowledge. In their wide-scale study into effective teaching of numeracy, Askew, Brown, Rhodes, Johnson, and Wiliam (1997) found there is a lack of evidence to support a positive association between formal mathematical qualifications and pupil gains and that even teachers with high level mathematics qualifications, displayed knowledge that was compartmentalised and framed in terms of standard procedures, without underpinning conceptual links. Content knowledge is important, as "you cannot teach what you do not know" (Rowland, Turner, Thwaites & Huckstep, 2010, p. 22), but studies which look beyond formal qualifications and consider different types of knowledge are arguably more useful. Ma's work and others (e.g., Ball, 2000; Ball et al., 2001) have also highlighted that many pre-service teachers have weak understandings of many of the mathematical skills and concepts that they are required to teach, and tend to adopt primarily procedural approaches when identifying teaching strategies (e.g., Chick, Pham & Baker, 2006; Maher & Muir, 2011).

Teacher Knowledge Frameworks

A number of researchers (e.g., Hill et al., 2008; Baker & Chick, 2006; Chick et al., 2006; Rowland et al., 2010) have created conceptual frameworks describing aspects of teacher knowledge. Figure 1 shows Hill et al.'s domain map for mathematical knowledge for teaching. Each of the six portions of the diagram is a proposed strand of mathematical knowledge for teaching (MKT). Subject Matter Knowledge refers to mathematical knowledge, with no knowledge of students or teaching being entailed. This is distinguishable from Specialised Content Knowledge (SCK), which is described as the mathematical knowledge that allows teachers to engage in particular teaching tasks, including how to represent mathematical ideas and provide mathematical explanations for common rules and procedures. The right side of the oval is closely aligned with Shulman's definition of PCK and contains knowledge of content and students (KCS), content and teaching (KCT), and knowledge of curriculum. KCS is focused on teachers' understanding of how students learn particular content, but is separable from knowledge of teacher moves, such as how to remedy students' errors. The authors contend that KCS is different from teachers' subject matter knowledge and that a teacher may have strong knowledge of the content itself, but weak knowledge of how students learn the content or vice versa.

The framework devised by Rowland et al. (2010) builds upon Shulman's different knowledge types and was developed through extensive observation of mathematics lessons. The four dimensions of the framework are named as foundation, transformation, connection, and contingency. The foundation dimension refers to the knowledge and beliefs the teacher possesses, while the other three dimensions refer to the way in which knowledge is brought to bear on the preparation and conduct of teaching. The purpose of the framework was

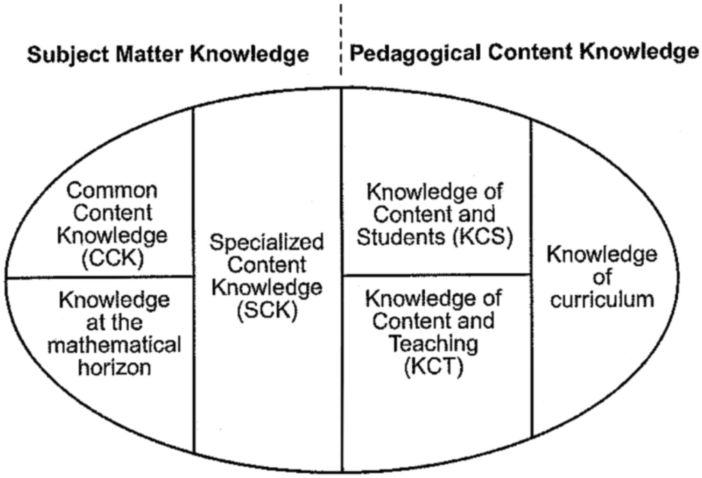

Figure 1. Domain map for mathematical knowledge of teaching (Hill et al., 2008, p. 377).

to provide a mechanism for interpreting and reflecting on mathematics lessons and while the study discussed in this paper did not involve direct classroom observation, aspects of the framework, such as the pre-service teachers' abilities to *transform and make connections* influenced the construction of the theoretical framework used to interpret the pre-service teachers' responses to the tasks.

Chick et al.'s (2006) framework was also helpful in the design of our own theoretical framework. Their framework is divided into three parts: (a) *Clearly PCK* includes those aspects which are most clearly a blend of content and pedagogy; (b) *Content Knowledge in a Pedagogical Context* includes those aspects drawn most directly from content; and (c) *Pedagogical Knowledge in a Content Context* includes knowledge which has been drawn most directly from pedagogy.

Within the context of this study, the researchers were particularly looking for evidence from the Clearly PCK category, including the pre-service teachers' abilities to discuss or use teaching strategies for teaching a mathematical concept, and their ability to discuss or address student misconceptions.

Pre-service Teachers' Mathematical Knowledge

As previously mentioned, concerns about pre-service teachers' content knowledge are expressed widely in the literature (e.g., Baker & Chick, 2006; Ball, 1990; Chick et al., 2006; Lange & Meaney, 2011), with weaknesses noted across a range of mathematical topics. Ball (1990) identified that pre-service teachers have difficulties with the concept of fractions and the meaning of division of fractions, while Stacey and her colleagues (Stacey et al., 2001) found that decimal numeration was a source of difficulty for pre-service teachers, with one in five pre-service primary teachers demonstrating inadequate knowledge of decimal numeration with the related risk of transferring flawed understandings to students. Chick et al. (2006) used their PCK framework to study primary teachers' responses to a question about the subtraction algorithm. They found that although the teachers seemed to have a solid knowledge of the subtraction algorithm, few had strategies for helping students recognise the existence of a problem.

Research has also shown that pre-service teachers' mathematical knowledge is characterized by a tendency to adopt primarily procedural approaches (Chick, et al., 2006; Maher & Muir, 2011), and demonstration of instrumental, rather than relational understanding (Skemp, 1978). Skemp used the term *relational understanding* to refer to the understanding associated with the underlying principles of a particular mathematical idea, whereas instrumental understanding involves following rote learnt rules and procedures, that is "rules without reason" (Skemp, 1978, p. 9). He argued that while *instrumental understanding* may allow students to obtain the correct answers to certain mathematical tasks, it is likely to prove limiting in terms of providing a sound foundation for teaching others.

The Multiplication Algorithm

Research into teachers' knowledge of multiplication has focused primarily on the algorithm, with the findings indicating that while the teachers could often accurately perform the algorithm to achieve the correct answers, they often could not explain the place value concepts behind the algorithm (Ball et al., 2001). The algorithm for multiplying large numbers is derived from the process of decomposing numbers into "expanded form" and multiplying them in parts (Ball, et al., 2001). According to Booker (2011), students experience more difficulties with the multiplication algorithm than any of the other operations. These errors typically relate to little understanding of one example of the multiplication algorithm in which two lines of working out are recorded followed by an addition of these partial products. Errors also result when place value is not considered, with renaming and when zeroes occur. Students will typically "forget" to place the zero in the second line of the working out, as demonstrated in the example below, indicating a lack of understanding of place value or a reliance on rules.

$$\begin{array}{r} 36 \\ \times\ 13 \\ \hline 108 \\ 36 \\ \hline 144 \end{array}$$

Ball et al. (2001) found that in their studies with pre-service teachers that many of them explained the algorithm in procedural terms, using language such as "lining it up correctly", "moving the numbers over", and "adding a zero" (p. 444). They also found similar results when using the same multiplication task with practising teachers (Ball, et al., 2001). When Ma (1999) provided her teachers with a work sample showing a student forgetting to "move the numbers" (p. 28), she found that while the USA teachers focused on the location of digits, the Chinese teachers were able to provide explanations of the multiplication algorithm that used the distributive property to explain the role of place value. The teachers in Ma's study who provided procedural explanations for the error tended to focus on columns and digits and how to move the numbers, rather than why. Many admitted that they could not actually identify 'why', stating, "I can't remember why you do that. It's just like when I was taught, you just do it" (p. 31). These teachers also tended to provide procedural teaching strategies, such as reminding students of "the rule" or providing them with lined paper to make the lining up easier. In contrast, many of the Chinese teachers could provide conceptually based explanations for the error, and discussed the students' errors with relation to place value and the distributive law. Ma (1999, p. 48) cites the following of a typical example of an explanation by one of the Chinese teachers:

> I will tell the students that since the 4 in 645 represents 4 tens, therefore, 123 multiplied by the 4 equals 492 tens. 492 tens, where should the 2 be lined up? Of course with the tens place. ... The digits at the ones place of these three numbers (615, 492, and 738) actually represent three different values. One represents ones, one represents tens, and the other one represents hundreds. Your problem is that you didn't notice the difference and saw them all as representing ones. (Ms. G.)

Furthermore, many identified conceptually-based teaching strategies such as explaining the rationale behind the algorithm, separating the problem into three sub-problems (123 x 645 constitutes 123 x 600, 123 x 40, and 123 x 5) and providing them with some simpler diagnostic problems. Ma (1999) found that with relation to the multiplication algorithm, 61% of US teachers and 8% of Chinese teachers were not able to provide authentic conceptual explanations for the procedure, with even conceptual explanations often varying in their depth and rigour. Of particular relevance to this paper, Ma also found that none of the teachers whose knowledge was procedural described a conceptually directed teaching strategy, and not a single teacher was observed that would promote learning beyond his or her own mathematical knowledge.

Methodology

This study used primarily qualitative data to determine whether or not a selection of pre-service teachers' MCK impacted upon their ability to identify student errors across a range of mathematical domains and then implement appropriate teaching strategies to address these errors. For the purpose of this paper, the execution of the long multiplication algorithm, was the focus of the investigation, with the following research questions being identified:

1. What is the nature of pre-service primary school teachers' MCK relevant to primary school mathematics, specifically in the domain of multi-digit multiplication?

2. To what extent can pre-service primary teachers identify students' errors in the execution of the long multiplication algorithm?
3. To what extent can pre-service primary teachers identify appropriate teaching strategies to address students' mathematical misconceptions in this area?

The Study

This study had two phases. The first phase involved the completion of a fifteen-item test instrument consisting of nine multiple choice and six short answer questions relating to mathematical content relevant to the Australian primary school curriculum. The multiple choice items were selected from a sample ACER Teacher Educational Mathematics Test (TEMT) designed to test the mathematical attainment of beginning pre-service teachers and to uncover errors due to misconceptions. The short answer test items were adapted from other studies involving research into the MCK held by pre-service and in-service primary school teachers and was largely informed by the work of Ma (1999), Ball et al. (2001), Ryan and Williams (2007) and Stacey, et al. (2001). Twenty final-year Bachelor of Education pre-service teachers volunteered to take part in this first phase, with seven of these indicating an interest in participating in the second phase of the study.

The second phase consisted of individual interviews that were structured around four key questions or instructions relating to six student work samples, with one of them being an example of a student's error made in performing the long multiplication algorithm. The work samples were constructed by the researchers, and adapted from previous studies on students' mathematical thinking in various topic areas (Ma, 1999; Ryan & Williams, 2007; Stephens, 2006). The mathematical concepts that underpinned the tasks reflected those on the test instrument, and in the larger study included items on interpreting the results of coin tosses, ordering decimal numbers and finding the area and perimeter of a rectangle. Table 1 shows the item discussed in this paper from the test instrument, the corresponding work sample description, and the primary questions asked. The interviews took approximately one hour and were audiotaped.

Each interview began with the researcher asking the participant to identify whether or not the student's response was correct. If an error was identified, then the participant was asked to provide the correct response and to identify possible reasons for the error. Depending upon the participant's response, further clarifying questions were asked, such as 'What does this work sample tell you about the student's understanding of this multiplication algorithm?' Participants were asked how they would assist the student, with further clarifying questions asked if participants mentioned general teaching approaches such as using manipulatives or explaining the algorithm.

The interviews were transcribed and analysed using an adaptation of the framework for analysing PCK. This was based largely on the work of Chick et al. (2006) and Ball et al. (2001) (see Table 2). Table 2 shows that the three categories were (a) knowledge of content; (b) knowledge of learning; and (c) knowledge of teaching. The first category, *Knowledge of Content*, is essentially MCK, and encompasses what Rowland et al. (2010) term "foundational knowledge" as well as Ma's (1999) "Profound Understanding of Fundamental Mathematics" (PUFM). *Knowledge of Learning* relates to Ball et al.'s (2001) Specialised Content Knowledge in that it required participants to examine and understand student solution methods to problems. *Knowledge of teaching* refers to PCK, and particularly aligns to Chick et al.'s (2006) descriptor of Student Thinking and Ball et al.'s (2001) terms of KCS and KCT.

Table 1
Test Item, Work Sample and Questions Related to Long Multiplication

Test item	Corresponding work sample	Questions asked
Q. 11 123 x 645 equals	Some students have completed the following items (a, b, d and e not discussed here) c) 47 x23 141 94 235	Please examine the items and decide whether or not they are correct or incorrect. If incorrect, please provide correct response. What does this tell you about the child's thinking? How would you assist this student?

Participants' responses were then assigned a rating between 0-2, based on the rating descriptors for the three domains of knowledge. Table 2 describes the ratings from 0 to 2 for each of the three knowledge domains.

To assess the appropriateness and reliability of the rating scheme, a second coder read and rated the interview transcript for each participant. Any discrepancies were discussed and a mutual decision was made as to the rating assigned.

The coded responses for each of the seven participants were compared with their individual responses to the test items administered in the first phase of the study. Of particular interest was any relationship between the participants' MCK as demonstrated in the test and the interview, and the nature of their responses in relation to their interpretation of the student work samples and strategies for addressing student mathematical misconceptions.

Table 2
Rating Descriptors for each Knowledge Domain

	Rating descriptors		
Knowledge domain	0	1	2
Knowledge of content (MCK)	Could not solve problem correctly	Solved problem but with limited fluency and/or confidence	Solved problem, demonstrating procedural proficiency and/or relational understanding
Knowledge of learning (SCK)	Did not identify error in student's thinking	Identifies error but no or limited explanation given	Identifies error in thinking and provides appropriate explanation
Knowledge of teaching (PCK/KCT)	No teaching strategy identified	Identifies teaching strategy with no or limited reference to instruction and/ or focus on instrumental understanding only	Identifies appropriate strategy or approach for developing students' relational understanding of mathematical concept.

The results obtained from the interviews in relation to the multiplication work sample are discussed in the following section. While it is acknowledged that this is a small sample size, the interview situation did provide for the researcher to probe further into the pre-service teachers' thinking and a comparison of their responses indicated that there were similarities in their thinking, indicating that some generalisability would be reasonable.

Results and Discussion

Of the twenty participants who attempted the multiplication item on the test instrument, only six provided a correct response. Incorrect responses ranged from answers of 1095 to 80 335, with all incorrect responses being different.

Figure 2 shows Jacky's incorrect response to the item, and represents the common difficulties participants had with remembering the steps of the algorithm.

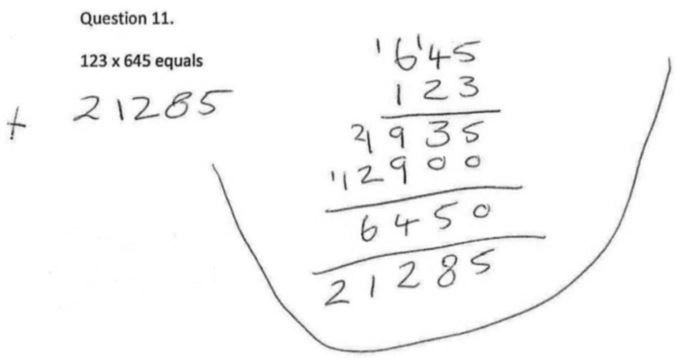

Figure 2. Jacky's response to test item.

Of the fourteen participants who recorded an incorrect response, four of them participated in the second phase of the study. Table 3 summarises the seven participants' responses to the multiplication item on the test instrument and the student work sample. The participants' ratings of 0, 1, or 2 are included for each of the three categories as determined by their answers in the interview. The incorrect responses received on the test instrument are highlighted in bold font. N/A has been used to denote that the participant did not identify the student's answer as incorrect and therefore did not progress any further in the interview. The data in Table 3 relate specifically to test item 11, in which the problem was 123 x 645 equals … ? The correct response for this item would have been 79 335.

Table 3
Participants' Responses to the Multiplication Items

	Knowledge of Content (MCK)		Knowledge of Learning (SCK)	Knowledge of Teaching (PCK/KCT)
Participant	Test Question 11	Interview: Solve problem	Interview: Identification of student error	Interview: Identification of teaching strategy
Ann	correct	1	1	1
Mia	correct	1	1	1
Sarah	correct	2	1	1
Janet	**60 815**	2	1	1
Larissa	**79 235**	0	0	N/A
Courtney	**61 035**	0	1	0
Jacky	**21 285**	0	0	N/A

Knowledge of Content

Question 11 on the test instrument and the participants' ability to solve the corresponding problem in the interview provided an indication of their knowledge of content. Courtney, for example, attempted to perform a combination of the standard algorithm and an informal method to answer the test item (see Figure 3).

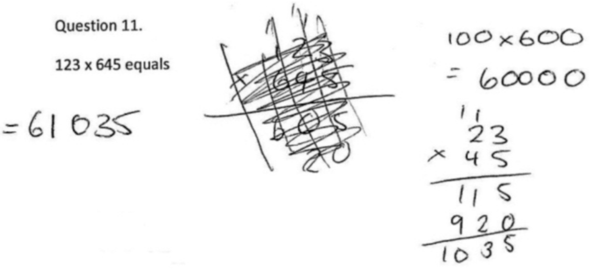

Figure 3. Courtney's response to test item Q11.

When attempting to solve the related item (47 x 23) in the interview situation, Courtney again failed to carry out the algorithm successfully, although this time she seemed to make a computational error, rather than a procedural one (see Figure 4).

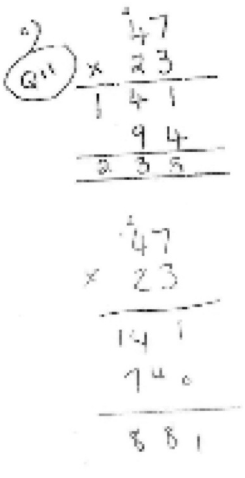

Figure 4. Courtney's response to interview item Q11.

Like Courtney, Larissa made a computational error in her execution of the multiplication algorithm in the test, but then indicated in the interview that the student had carried out the algorithm successfully and provided the correct answer to the multiplication. As previously indicated, Jacky did not correctly solve the item in the test or the interview, and like Larissa, thought that the student had provided a correct response.

Interestingly, Janet did not provide the correct answer to the multiplication item in the test, but could correctly execute the procedure to achieve a successful result in the interview. For both items she attempted an informal algorithm, and used partitioning to break the numbers up. As Figure 5 shows, however, she incorrectly applied the distributive property, which resulted in the answer of 60 815. Perhaps the inclusion of 3-digit numbers confused her, as she was able to accurately use the distributive property to show that 47 x 23 could be solved by multiplying 47 x 3 and 47 x 20 in the interview. She later explained that:

> I would have seen that you multiply the 47 by 3 and the next step I multiply it by 2, rather than thinking I am multiplying the 47 by the units and then I will multiply the 47 by the tens ... so my thinking has evolved a little bit [from the test item] so it's that pulling apart and deconstructing that I've started using in my normal life now. (Janet)

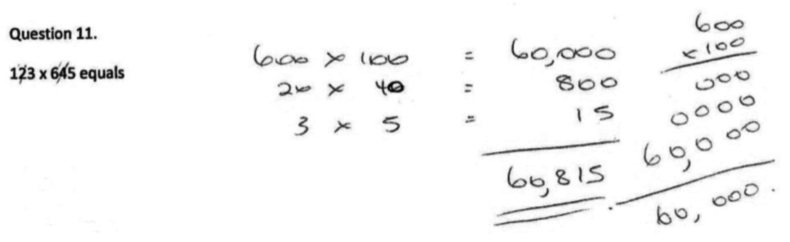

Figure 5. Janet's response to test item.

One of the other participants, Mia, achieved the correct answer in the test and the interview, but her explanation in the interview showed a reliance on procedure and limited confidence:

Mia:	Um, when I was doing this myself and I looked at it and I wrote cross out zero which means I don't even understand what that rule means
Researcher:	When you say cross out zero, what do you mean?
Mia:	Well with 47 times 23, I work um, from the 3 in 23 with 3 times 7 is 21 so you put down the one then 3 times 4 is 12 [pauses while she considers her own calculation]. Oh, sorry, I've carried the two from the 21, so there is 2 over there, so 3 times 4 is 12 plus 2 is 14, and it's just so automatic because it's just what I learnt when I was at school, cross the 3 out and put down a zero.

Researcher:	Why do you cross out the 3?
Mia:	Has it got something to do with the different values? So 3 is a different value to 2 and we need to move across to that column, or something maybe?
Researcher:	So the crossing out of the 3 is a signal to put down the zero?
Mia:	Yes, it's just automatic. Probably not the best way to teach it when I can't explain it myself!

In summary, the participants in the study showed limited ability to explain how and why the multiplication algorithm worked and two participants were unable to identify when it had been carried out incorrectly in a student's work sample. As the above exchange shows, Mia used procedural terms, such as "cross out zero" and "carry the two" to explain her working out, which is similar to the approaches used by both the pre-service and practicing teachers in Ball et al's (2001) studies.

Knowledge of Learning

Knowledge of learning related to the pre-service teachers' abilities to first recognise that an error had occurred in the work sample, and then identify reasons for the error. As Table 3 shows, while most participants could identify the student's error, they provided no—or limited—explanation for this. The following exchange conducted with Courtney is illustrative of the limited explanations provided:

Courtney:	Um, they've put that up there—the 21—then they've added these two together; then they've got three and four is fourteen, so they've plused the two from there; then they've moved down here and that's when they have come into trouble there.
Researcher:	Why?
Courtney:	Um, from what I can remember at school, you put the zero in um; and then 'cause you've got over here, you have used this. You have to start using this one here—the three—and you've multiplied it and then you start from here. So two sevens are fourteen. Um, and that's where they've lost me, um, [with] their thinking.

Similarly, Ann also found it difficult to identify the error beyond a procedural response, stating that, "All they have done is forgotten to put the zero down ... ". None of the participants received a rating of '2' for their answers, with their procedural responses again reflecting the findings from Ball's and Ma's studies, with a focus on what to do, rather than why.

Knowledge of Teaching

With regard to knowledge of teaching, participants' responses also reflected a tendency to focus on instrumental understanding, with none of the responses indicating an appropriate teaching strategy that focused on developing students' relational understanding of the multiplication algorithm. Many provided vague or general teaching suggestions as the following exchange with Courtney illustrates:

> Researcher: What could you do?
>
> Courtney: Um, they could use estimation. So they could take it to the closest, um, even numbers, um, and so make it ones that are maybe 50 by 20, maybe take those zeroes off and have a rough estimation of what it will be ...

Mia also talked about estimation and using a teaching strategy of rounding 47 to 50 but then concluded, "It's obvious that they understand the process that you do with double digit multiplication in that algorithm, but they have forgotten to put down the zero". It is interesting how she describes the students as *understanding the process* and "forgetting" to put down the zero, rather than focusing on the student's obvious lack of conceptual understanding of how the process works.

Ann mentioned that "You could do it as a class depending on how many students are actually doing that one wrong", but in contrast with many of Ma's Chinese teachers, there were no specific or explicit teaching strategies identified.

A similar item in the interview schedule which involved algorithms using addition and subtraction also produced procedural responses, and although MAB blocks were mentioned in relation to a teaching strategy, only two participants explained how the manipulatives would be used to represent the mathematical concepts underlying the procedures.

Conclusion

The participants' responses to the test item and the related interview task showed that some of them exhibited difficulties with carrying out the multiplication algorithm correctly, and with formulating sufficient explanations as to how the algorithm worked. It seemed that many of the participants held only an instrumental understanding of the process, and this in turn impacted upon their ability to identify student errors and therefore attempt to address this with appropriate teaching strategies. Some provided incorrect answers for the item in the test instrument and did not identify the student's work sample as being incorrect. As a result of this they would not have been able to help the student with developing an understanding of the process involved in using the algorithm to multiply two two-digit numbers.

Even participants who could execute the long multiplication process correctly tended to provide a limited explanation as to the student's thinking; and perhaps not surprisingly, this then restricted their ability to provide an appropriate approach or teaching strategy to develop conceptual understanding in students. The teaching strategies they suggested tended to focus on either procedural approaches, such as reminding them to put down the zero, or on general suggestions such as using estimation or rounding up of numbers.

The implications of this study point to the need for pre-service teachers to have stronger MCK, along with the influence that this would have on their PCK. While it seems obvious that having a strong MCK is necessary for effective teaching, it seems that for these pre-service teachers at least, there are areas such as long multiplication that pose particular problems and challenges. In the study, this manifested itself through participants' provision of procedural responses, indicating a lack of relational understanding of the long multiplication process or a PUFM in relation to place value in general. Furthermore, it is of particular concern that these pre-service teachers were in their final year of study and therefore unlikely to address their own limitations in this area before teaching others.

It is hoped that this study contributes further to investigating the link between MCK and PCK and identifying the aspects of pre-service teachers' knowledge that needs to be focused on in the delivery of education courses.

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