Mathematics Teacher Education and Development 2013, Vol. 15.1, 5–28

An Enactivist Perspective on Teaching Mathematics: Reconceptualising and Expanding Teaching Actions

Jo Towers
University of Calgary

Jérôme Proulx
Université du Québec à Montréal

We reject a trajectory approach to teaching that classifies "good" and "bad" teaching actions and seeks to move teachers' practices from one of these poles to the other. In this article we offer instead a conceptualisation of mathematics teaching actions as a "landscape of possibilities". We draw together terms commonly used in the literature to describe teaching strategies, and add our own, to offer an expanded view of teaching actions. We illustrate each with data extracts drawn from our various studies of mathematics teachers and classrooms, and explain how a range of teaching actions can be woven into a coherent teaching practice. Note that we are not talking about growth in teaching in this paper, nor about change in teachers' practice over time. We aim to simply talk about and conceptualise teaching in ways that can broaden our understanding of it.

Keywords: mathematics teaching • teacher actions • enactivism

(Models of) Teaching in the Literature

Teaching is referenced in many studies of mathematics education specifically, as well as in the broader field of education. Most of these studies examine teaching in order to see the repercussions of it for students, to isolate and capitalise on its effectiveness, and to grasp its potential for promoting learning (e.g., Bruce, Esmonde, Ross, Dookie, & Beatty, 2010; Cai & Wang, 2010; Desimone & Long, 2010; Friesen, 2009; Pesek & Kirshner, 2000; Puntambekar, Stylianou, & Goldstein, 2007). Many such analyses of teaching fall into what might be termed the "effective teaching" paradigm, although there are voices of caution that call for care in the way in which teaching effectiveness is linked to student progress (e.g., Bednarz, Fiorentini, & Huang, 2011; Davis, 2008; Davis & Sumara, 2010). Broadly speaking, though, in mathematics education researchers do not typically look at teaching for its own sake, to understand it better, to theorise it, and/or to know more about teaching as a phenomenon as well as about its possibilities. Jaworski (2006), for example, notes that, "… we have many theoretical frameworks or constructs related to mathematics teaching and its development, but nothing to compare with the big theories of learning" (p. 189). This results in a situation where researchers lack a nuanced language to describe teaching actions and a means to conceptualise particular teaching approaches, methods, strategies, and styles (indeed, even a means to distinguish between all of these constructs). This is what we aim to do here—to theorise teaching, based on a grounding theoretical framework and collected data from our studies. We do not aim to look immediately at its outcomes with respect to students' learning, but rather to broaden our capacity to conceptualise it in richly theorised ways, for the sake of teaching itself and in order to offer distinctions in its forms and new ideas about it.

Work has already begun in conceptualising teaching as a discipline (Loughran & Russell, 2007) and in responding to the perceived lack of a coherent theory for teaching in mathematics education (Jaworski, 2006). Many have used metaphors to define the work of mathematics teachers or mathematics teaching, in positive or negative ways. For instance, King (2001) uses the metaphor of "jazz improvisation" to describe conceptually-oriented teachers; Brousseau (1988) uses the one of the "theatre actor" to describe how teachers are continually confronted with paradoxes; Cooney (1988) discusses that of "broadcaster" of information; and Hoong and Chick (2007/2008) liken the "balancing act" of managing the various goals of teaching to tightrope walking.

One major group of theories has adapted popular theorising about *learning* styles to the phenomenon of teaching (e.g., Davis, 2004; Felder & Silverman, 1988; Hsieh, Jang, Hwang, & Chen, 2011; Nicoll-Senft & Seider, 2010). Another major grouping of theories establishes a difference between teaching that is 'student-centred' and that which is 'teacher-centred' (e.g., Schaefer & Zygmont, 2003; Schumacher & Kennedy, 2008) or between 'traditional' and 'innovative' pedagogies, though these opposing teaching styles are given many different names in the literature (e.g., Brodie, 2011; Ertesvåg, 2011; Hufferd-Ackles, Fuson, & Sherin, 2004; Pesek & Kirshner, 2000; Ramsey & Ransley, 1986; Smith, 1996). Another important grouping contrasts 'traditional' teaching approaches in mathematics with more contemporary forms of teaching. For example, Cobb, Perlwitz, and Underwood-Gregg (1998) combine radical constructivism and socio-cultural theorisations to describe what they call an "inquiry" approach to mathematics teaching. Further, Thompson, Philipp, Thompson, and Boyd (1994) compare "calculational" and "conceptual" approaches to mathematics teaching, discussing the richness but also the demanding role of the latter.

What most such framings have in common is their emphasis on the distinctions between extreme (polarised) conceptions of teaching. The usual implication drawn by proponents of such models is that teachers ought to move from one pole towards the other—that is, that "good" teaching can be adopted through a shift in teachers' pedagogical practices from what might broadly be framed "telling" to activity that is often referred to as "facilitation" (Davis, 1994, 1997). We term such models "trajectory" models of teaching and we acknowledge that they have some value as helpful markers, particularly for practitioners thinking about expanding their classroom practices. However, we also note that such trajectory models can be seen to have a tendency to trap teachers into believing that they should abandon all or most of their established or familiar practices in order to move forward towards more desired practices. Such a sentiment—one that many of us have heard from practising and preservice teachers—is expressed by a teacher with whom Davis and Sumara (2003) worked: "I know *I'm supposed to be facilitating* the learning, but sometimes it's just easier to tell [students] how to do things" (p. 131, emphasis added).

In this article, we aim to expand this polarised trajectory mindset and instead propose a conceptualisation of teaching as recursive and non-linear. We seek to understand teaching better by reinterpreting the middle space of such trajectories. In doing so, we also reject the notion that the middle ground is occupied by a simple averaging of two extremes into a hybrid practice incorporating parts of each teaching strategy related to the "traditional" or "reform" poles.

One difficulty we face in this endeavour is that some of the models of teaching currently presented in the literature tend to assume that discourses on learning (e.g., constructivism, behaviorism) can also be interpreted as discourses on teaching: behaviorist teaching, constructivist learning, etc. Davis and Sumara (2003), noting that not enough has been done to highlight the inherent difficulties of linking discourses on learning to prescriptions for teaching, suggest that rather than asking "How can we improve on what we're already doing [as teachers]?" researchers should be moving more towards asking "What are we doing when we claim to be teaching?" (p. 132). Our work takes up this challenge and offers a nuanced vocabulary for describing, first, what teachers *do*, rather than jumping immediately to considering what implications such actions have for learning, or how these teaching actions are legitimised by or associated with a particular learning theory. Our view is that learning theories, while not linked to a specific prescribed teaching approach, can however help us make sense of a teacher's actions. Thus, we offer in what follows the grounding theory that frames our view of learning and that will provide us with keys for interpreting, and talking about, mathematics teaching.

Enactivism as a Rationalising Basis

While we question the linkage of learning theories and teaching theories/ prescriptions, personally we have come to this interest in understanding the phenomenon of teaching from a shared history in studying classroom events through the lens of enactivist theory (e.g., Proulx, 2007, 2010; Proulx, Simmt, & Towers, 2009). Enactivism is an encompassing term given to a theory of cognition that views human knowledge and meaning-making as processes that are understood and theorised from a biological and evolutionary standpoint (from the work, e.g., of Maturana, 1988; Maturana & Varela, 1992; Varela, 1999; Varela, Thompson, & Rosch, 1991). From this perspective, our biology matters in the process of coming to know.

A key understanding in evolutionary thinking is that species 'fit' within their environment. The concept of fitting is not a static one in which the environment is constant and only the species evolves and continues to adapt. Maturana and Varela (1992) note that species and environment co-adapt to each other, meaning that each influences the other in the course of evolution. This co-evolution is called *structural coupling* by Maturana and Varela (1992) because both environment and organism interact with each other and experience a mutual history of evolutionary changes and transformations. It follows that the environment does not act as a selector, nor does it predetermine or cause evolution; rather, it is a trigger for the species to evolve, much as the species acts as trigger for the environment to evolve in return. Maturana and Varela explain that events and changes are occasioned by the environment, but they are determined by the species' structure.

> Therefore, we use the expression "to trigger" an effect. In this way we refer to the fact that the changes that result from the interaction between the living being and its environment are brought about by the disturbing agent but *determined by the structure of the disturbed system*. The same holds true for the environment: the living being is a source of perturbations and not of instructions (1992, p. 96, emphasis in the original).

Maturana and Varela call this phenomenon structural determinism, meaning that it is the structure of the organism that determines the changes that will occur in the organism, although such changes are triggered by the interaction of the organism with its environment. Hence, we understand that changes do not reside inside either the organism or the trigger; they come from (and are dependent upon) the organism's interaction with the trigger. Thus, the triggers from the environment are essential, but they cannot determine the changes. Hence, in teaching, we cannot make the assumption that instructional properties are present in and of themselves in the tasks offered or the teaching actions made, and that these will determine learners' reactions. Nor, though, can it be assumed that the responsibility to learn rests solely within the learner and simply needs to be facilitated.

This understanding renders problematic two often-heard solecisms in education—the ideas that a good student will learn no matter what kind of teaching is experienced and, on the opposite pole, that a weak student is unlikely to learn much no matter how directive the teaching. Enactivist thought helps to emphasise the critical role of the teacher as a trigger and also as one who co-adapts and learns him/herself and therefore alters the learning environment of the learner. Enactivist thought also helps to illuminate the reason why teaching strategies that have been shown to be 'effective' in particular circumstances do not always 'work' in new circumstances. The teacher and learners co-adapt and it is in the interaction between the learner and environment that learning happens, not because of the learning environment (or the learner). Simplistic interpretations of cause and effect in teaching and learning are therefore problematised in an enactivist interpretation. What will be learned cannot be pre-decided. Learning, then, cannot be determined by teaching but it is, nevertheless, dependent on teaching. The learner's structure allows the environment to be problematic—to occasion learning. The effects of the environment are not *in* the environment, but are made possible by the organism's structure in interaction with its environment. The teacher is therefore more than a "guide" or "facilitator" of learning; he/she is a fundamental part of, even an accomplice in, the learner's processes (Proulx, 2010). A teacher is a "full participant in the emerging cognitive structure of the learning unit" (Towers & Martin, 2009, p. 47) that includes all actors in the environment. In being structurally coupled with learners, the teacher becomes complicit (Sumara & Davis, 1997) in the learners' knowledge and is "within" the learners' cognitive acts such that teaching is a complex act of participation in unfolding understandings (Martin & Towers, 2011). Thus, we do not use enactivism to create a teaching theory, but mainly as a rationalising source to legitimise various sets of teaching practices.

Reconceptualising Mathematics Teaching

Over the years, many authors have described teachers' practices in mathematics classrooms, and some of the language used in these accounts still has currency today. A widely-used term to describe a broad set of practices that still seem prevalent in 21st century mathematics classrooms is telling (see, e.g., Chazan & Ball, 1999; Lobato, Clarke, & Ellis, 2005; Smith, 1996) or what Cooney (1988, 1994) calls broadcasting of information. This mode of teaching, also referred to in the literature by such terms as "traditional teaching" (Hufferd-Ackles, Fuson, & Sherin, 2004) or "instrumental teaching" (Pesek & Kirshner, 2000), contrasts with more open-ended, less teacher-directed modes all of which seek to encourage teachers to move away from a "telling" stance (and also given various names in the literature such as reciprocal teaching, constructivist teaching, reform teaching, inquiry teaching, and—shifting the locus of attention from teacher to learner—learner-centred teaching, problem-based learning, and facilitation of learning). Again, however, many of these ways of describing teaching appear as oppositional (such as "one can't be a reform-minded teacher if one tells") and the frameworks sometimes neglect to notice that there are many nuanced ways to "tell," just as there are many nuanced ways to "facilitate."

However, some recent work (e.g., Brodie, 2011; Chazan & Ball, 1999; Davis, 2004; Lobato, Clarke, & Ellis, 2005) does acknowledge a more nuanced terrain of mathematics teaching. Brodie (2011), for example, describes hybrid practices enacted by the teachers she studied as those teachers shifted towards reform practices. Similarly, Chazan and Ball (1999) call for a rethinking of what it means to tell, incorporating into the reform-minded teacher's arsenal a wider range of legitimate telling actions. Lobato, Clarke, and Ellis (2005) challenge the research and teaching community to "move away from a false choice between constructivist teaching approaches and telling methods" (p. 131) by re-embracing direct telling and reformulating it as "initiating". Such a reformulation, they claim, requires a reconsideration of telling in terms of (a) its function (rather than the form of the teacher move); (b) the conceptual rather than procedural content of the new information presented; and (c) its relationship to other actions in the classroom. In fact, Lobato, Clarke, and Ellis (2005) explicitly reject attempts to include traditional telling (even in judicious amounts) in an expanded set of teaching actions, arguing instead for a rethinking of its conceptual roots. For example, they suggest that it is possible for teachers to use a series of questions in such a way that the questions actually tell (what Bauersfeld, 1980, referred to as funneling), while a declarative statement (that might be disdained as traditional telling) might provoke new ways of thinking for students or encourage student explanations. Therefore these are not undesirable teacher actions when considered in the context of their effects rather than simply their form.

Each of these more nuanced readings of teacher actions enriches the ways and means that we researchers and teachers have to talk about mathematics teaching. Thus, in this article, we aim to enhance this understanding of mathematics teaching through offering a set of descriptors developed from our own studies of teaching and learning mathematics that we have found helpful as a nuanced way to think and talk about teaching. However, while useful to us (and therefore potentially to others) the contribution we offer here is not limited to this expanded vocabulary for teaching actions. We discuss in the following section some teaching actions that we have noticed teachers enacting and that we have questioned and attempted to rationalise in order to understand them *in the context in which they were produced*.

Hoong and Chick (2007/2008) have also noted the importance of not judging a teacher's moment-to-moment actions based solely on an observer's chosen framework of "good" and "bad" strategies, but instead taking into account the many conflicting goals of today's teachers (e.g., such goals as meeting curriculum objectives, complying with administrative directives, "sheltering" (Fritzen, 2011) vulnerable students, and so on). Therefore, in addition to a vocabulary of teaching actions, we offer a way to conceptualise a broad variety of teachers' actions as potentially legitimate, even in the most innovative and current conceptions of teaching. This positions our work as a means to understand, theoretically, why a movement back and forth through a variety of teaching actions is appropriate in contemporary classrooms without reducing interpretations of such recursivity either to hybridity of practice between two poles of desirable and undesirable practice or to stepping stones on the way to more desirable practice. In other words, in our framing, teachers do not sometimes act in ways that can be interpreted as directive (telling, funneling, etc.) because they are, even briefly, failing at being competent and innovative teachers. They sometimes act in these ways to do the right thing in context—for enriching students' learning in an environment in which they are structurally coupled to other learners over time scales that typically extend well beyond that of a research project and in a landscape of complex goals and commitments.

Categorisations of Teaching Actions

In this section, our examples are taken from multiple studies of learning and teaching in which we have engaged over the last fifteen years. The examples are intended to be illustrative of the teaching actions that we have identified and studied over this period rather than to represent the reporting of findings of a single study. Therefore, it is not appropriate (or even possible within space constraints) to describe the methodologies and methods of each of these studies. To offer some brief contextual information, though, each study was designed as an interpretive inquiry into teaching and learning in Kindergarten to Grade 12 settings spanning contexts in two countries (Canada and the United Kingdom) and in settings taught in either English or French.

We cluster teaching actions into three main categories/distinctions: (a) *Informing*; (b) *Orienting*; and (c) *Shepherding* (see Figure 1).

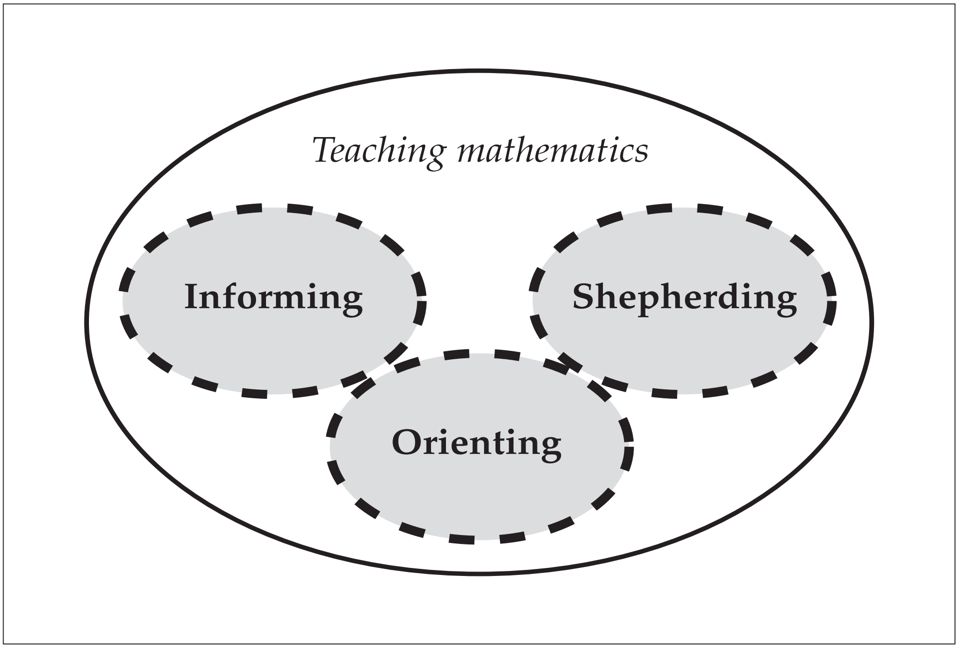

Figure 1. Categories/distinctions of mathematics teaching actions.

These three categories are offered as a way to highlight distinctions in the nature of the actions the teachers were enacting. They are not offered here in an intention, as we have mentioned above, to see them as a trajectory where Shepherding, for example, would be seen as an optimal category for which teachers should aim. Our research work in classrooms and professional development settings (e.g., Proulx, 2003, 2007; Towers, 1998) has led us to see all these categories of teaching as legitimate in the context in which they were produced, and this is what we aim to discuss in this article. We argue that each of the categories can be relevant as teaching actions and do not represent a level of teachers' expertise or capacities, but mainly a frame of mind or an orientation for intending to have students learn. Further, the dotted lines that separate each category emphasise the fact that these categorisations are not isolated from one another and can be enacted at different times during teaching and even in intertwined ways.

In the following section, we detail what each category means through offering illustrations taken from our studies but also through describing various nuanced teaching actions that would fit under each category. These sets of clustered teaching actions are not offered here as the exhaustive outline of all actions that could exist under each category, but mainly as a way of understanding how each category can be enacted. For ease of reference, at the end of this section we provide a table summarising the following analysis (see Table 1).

Informing Practices

We begin with a set of practices that we call teachers' *Informing* practices. Such practices tend to have at their core the giving of information. We include three examples of Informing practices: *enculturating*, *reinforcing*, and *telling*.

Enculturating. The first type of Informing practice is one we term *enculturating*. These actions induct students into the language, symbolism, and practices of the wider mathematics community. When engaging in enculturating, teachers typically refer to themselves and the students as belonging to a community of practice. They often use "we" or "our" in their speech about what ought to be done (mathematically) or they may refer to mathematics itself as an external body of knowledge against which the language or practice should be evaluated. Here are illustrations, all spoken by one teacher to a class of Grade 10 students in a Canadian high school:

Teacher: In mathematics, three words we assume everyone knows the meaning of are point, line, and plane.

Teacher: I want to be sure we're all talking the same language here. That's what we're doing.

Teacher: Notice our little symbol for parallel? The parallel lines with the little *l*, sort of like an exponent *l* up there.

Teacher: In mathematics there is no such thing as a z-angle.

Teacher: Why is that bad mathematical language I'm using?

Reinforcing. Another common Informing teaching action is what we term *reinforcing*, where the teacher places an emphasis on particular ideas to which students should pay attention within the classroom activity. This reinforcing can take the shape of nuancing or clarifying a point made in a discussion, or even, as Forman and Ansell (2001) call it, of re-voicing a student's contribution. In that context, the teacher may

> repeat, expand, recast, or translate students' explanations for the speaker and the rest of the class. The teacher revoices students' contributions to the conversation so as to articulate presupposed information, emphasise particular aspects of the explanation, or disambiguate terminology. (p. 119)

Here is an example (translated from French, the language in which the exchange took place) of a Grade 7 teacher who asks a student to explain how she arrived at "Fifty" for the following problem: "Luc has read 20 pages in 48 minutes. At this rate, how many pages will he read in 2 hours?"

Student: I did 48 times 2. And this 96, I still need 24 minutes to make 2 hours.

Teacher: So, 96, it lacks 24 minutes to get to 2 hours.

Student: Yes. And it says 2 hours. So, because it is like 20 pages for each 48 minutes...

Teacher: Oh, that is interesting; he reads 20 pages for each block of 48 minutes. Well done, well explained.

Student: So I figured he must read 10 pages in 24 minutes.

Teacher: Ok, this is very important, what she just said. If he reads 20 pages in 48 minutes, she tells us that we now have 24 minutes. And 24 is half of 48 minutes, so we have twice as less time, so he will read twice as less pages. So, if he reads 20 pages in 48 minutes, then he will read half. So you are saying it is 10.

Student: So, I did 20 pages times 2. Which is 40. And added 10 pages because 24 minutes were lacking. It gave me 50.

Teacher: Well done. [...] So, what she did, in fact, is to work with ratios. She says to herself that each time she has 20 pages, she has 48 minutes. So she looks at how many 48 minutes she has.

In this extract, the teacher re-explains the student's explanations by reformatting them: he clarifies them, corrects them, repeats them, reworks them, adds elements to them, etc. Through reinforcing, teachers can re-voice (Forman & Ansell, 2002; Forman, Larreamendy-Joerns, Stein, & Brown, 1998; O'Connor & Michaels, 1993) student contributions that are distributed throughout a discussion, thereby legitimising them but also, crucially, bringing coherence to these fragmented contributions as a single idea.

Telling. Another type of teaching actions related to Informing is what we term *telling*, similar to what Cooney (1988) has referred to as 'broadcasting' and actions that Brodie (2011) would call 'inform'. As such practices are well documented in the literature (e.g., Chazan & Ball, 1999; Davis, 1996; Smith, 1996), we have not included a further example here, but we note that telling includes explaining a concept, giving information, offering the correct solution to a mathematical problem or exercise, modelling one's own ways of solving problems, adding a new solution that students have not yet thought about, and so on.

Orienting Practices

A second clustering of teacher actions are those that we call *Orienting* interventions. Here, the teacher directs students' attention towards particular answers, solutions, or positions. It is a mode that differs from Informing by its attempts to involve students in the solution process by not directly giving the information but instead orienting students toward it and encouraging them to make sense of it. Some of these actions give the impression that the teacher is concerned with drawing information from students rather than (as with Informing) giving information to them. Brodie's (2011) "initiate" and "elicit" both reflect this same idea of trying to draw out information from students. Other Orienting actions have less of a flavour of drawing out and more of a flavour of shuttering off alternative paths, but usually without directly informing students of the preferred solution strategy or correct answer. To illustrate the breadth of this category, we present four kinds of teachers' orienting interventions: *clue-giving*, *blocking*, *pretending*, and *anticipating*.

Clue-giving. Teachers sometimes deliberately use clues to orient students toward specific pathways to the solution. We call this mode of intervention *clue-giving*. This kind of strategy has also been called "cued elicitations" by Edwards and Mercer (1987), where the teacher begins with a more open-ended effort to elicit insights and gradually (or sometimes not so gradually) bounds the responses to achieve the intended outcome. This activity has been discussed in Bauersfeld (1980), but in negative terms, as "funnelling" practices, and is reminiscent of the Socratic method one finds in Plato's Meno (see Fauvel & Gray, 1987, pp. 61–67).

Clue-giving is illustrated in the following episode, where two students have been trying to find the area of the figure shown in Figure 2 by attempting to find the area of the four inner triangles. They have asked the teacher for help and she has worked with them on their solution (finding the area of each of the four inner triangles) for a few moments. One of the students seems not to be following the conversation between the teacher and the other student and he asks for clarification:

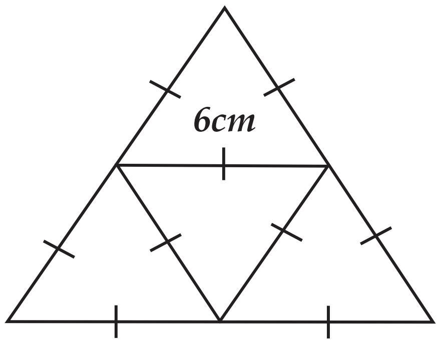

Figure 2. Triangle.

Student 1:	Are we looking at the whole triangle, or just this? [*Pointing to the upper small triangle*]
Teacher:	Depends which way you want to look at it. [*Pause*.] Could we look at the whole thing at once?
Student 2:	Yeah.
Teacher:	And if you did that, do you know how long this base is? [*Pointing to the base of the large triangle*.]#
Student 2:	Yeah.
Teacher:	How big would that be?
Student 2:	Twelve.

Teacher: Twelve. Could you drop a perpendicular from this vertex? [*Pointing to the top vertex.*]#

Student 2: Ohhhh!

Teacher: Down to that base?#

Student 2: Yeah.

Teacher: Yeah. And do you know this side? [*Pointing to the right-hand edge of the figure.*]#

Student 2: Yeah. OK. I get it.

Teacher: And you'd know this would be six. [*Pointing to the right-hand half of the base of the large triangle.*]#

Student 1: And that would be twelve. [*Pointing to the right-hand edge of the figure.*]

Teacher: And that would be twelve. So you could do it in one instead of four parts.

In this transcript, the lines marked # are examples of teacher clue-giving, heavily pointing to elements of the desired solution path.

Blocking. The second Orienting practice we call *blocking*. When blocking, a teacher is typically making efforts to prevent a student from following a certain (erroneous) path in the solution process. For example, during a lesson on formal two-column proofs in geometry, the teacher was challenged by a number of students who said that their friends in other classes had been shown mnemonics for remembering equivalent angles. These are probably familiar to the reader as Z-angles for alternate interior angles on opposite sides of a transversal, X-angles for vertically opposite angles, etc. The students wanted the teacher to show them these mnemonics in order to help them recognise the equal angles. The teacher objected, saying that such descriptors are not recognised by the markers of the provincial exam and so students may as well learn the correct terminology from the start. The students continued their appeal, promising that they would not use the mnemonics on the exam, just as an aid to their learning. The teacher continued to block this approach, closing the discussion with:

Teacher: Do not learn incorrect terminology and use it as a means of trying to learn the mathematics.

Pretending. A third form of Orienting intervention we have identified is one we call *pretending*. Here, the teacher takes a position or points to a solution that he or she does not necessarily agree with or knows is mathematically incorrect in order to provoke response or to engage students in argumentative discussion (in some cases, it could even be playing the devil's advocate). Drawing from data collected in a Grade 10 classroom, the following episode shows the teacher using this strategy to press the students for information about what constitutes the base of a figure (Figure 3).

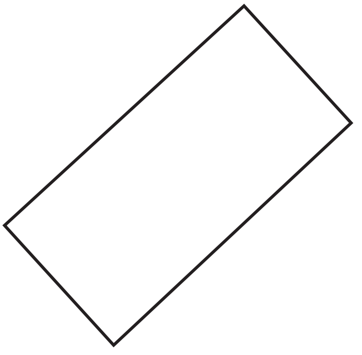

Figure 3. Rectangle.

Teacher:	[*Referring to calculating the area of the rectangle*]. Base times height. What do we mean by base?
Students:	The bottom.
Teacher:	The bottom. So, I could not find the area of something like that [*Figure 3*] because it doesn't have a bottom?
Student:	You have to cut it in half.
Teacher:	[*Sounding surprised*] You have to cut it in half? Why?
Student:	Because it doesn't have a base. Or no, you can just do the side. Do it on the side. Then …
Teacher:	[*Interrupting*] So the base doesn't have to be the bottom?
Student 1 :	No.
Student 2:	Something horizontal.
Teacher:	So we couldn't find the base of this [*Figure 3*] or the area of this because it doesn't have a bottom?
Student 1:	You can.
Teacher:	[*Inaudible*] this point. Is that what you're telling me?
Student 1:	It doesn't have to be the bottom.
Students:	It doesn't have to be the bottom.
Student 1:	A flat line thing.
Teacher:	Flat?
Student:	As long as you know both sides.
Teacher:	As long as you know both sides. As long as I know this side and this side [*pointing to two opposite sides on Figure 3*]?

Student 3: No, that side and that side [*pointing to two adjacent sides*].

Teacher: Oh, that side and that side! The base is basically one of the sides, right? It doesn't have to be the bottom, technically. We usually use the base because we set something down, but if you're talking about a geometric figure you can choose any side you want to be the base.

We also see the teacher's closing comment in this exchange as a reinforcing move, linked with the previous Informing category. This demonstrates how different teaching categories can be mobilised within a specific exchange and explains why we used dotted lines to circumscribe the categories in Figure 1.

Anticipating. The final kind of Orienting move to which we will draw attention we call *anticipating*. This move is directed at protecting students from error or removing the challenging aspects of a task to render it more accessible. This impulse has been described elsewhere (Andersson, 2011) as "sweeping the way" for students, likened to the way in which, in Curling, a sweeper sweeps the ice just ahead of the rock to ensure a smooth path.

The following example of anticipating is taken from a study of a class of ten and eleven-year-olds studying mathematics in a British secondary school. Two students had worked together to find (correctly) the area of the figure shown in Figure 4.

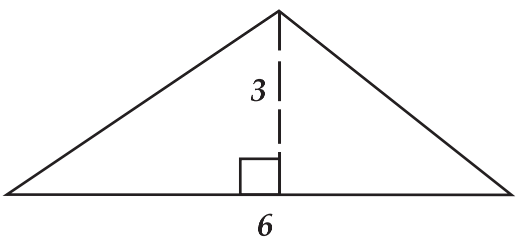

Figure 4. Triangle.

At this moment, the teacher came to their desk and asked them to describe their solution method, which they did. Next, instead of leaving the students to try the next problem (Figure 5) themselves, the teacher suggested that they turn the page upside down to view the next triangle. There was no evidence in the data that the students had struggled to solve the previous problem and no evidence that they could not solve the problem given in Figure 5, as they had had no opportunity yet to try it; so we read the teacher's intervention here as anticipating difficulty and acting to remove the challenging aspects of the task in order to protect the students from error. (Indeed, this was the rationale offered by the teacher when later analysing this teaching episode).

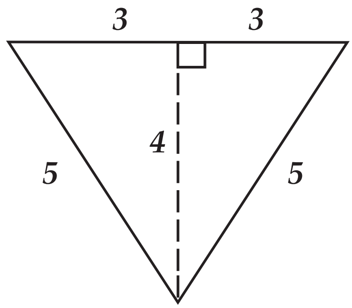

Figure 5. Inverted triangle.

Shepherding Practices

The third major clustering of teacher actions are those we call *Shepherding* interventions. These actions tend to be subtle—a form of coaxing and prompting, but towards the "possible" in the mathematical territory rather than, as with Orienting practices, towards a given or preferred solution route. In these episodes, students' voices may be more prominent in the discourse, and the familiar discourse pattern where the teacher takes a turn after each student speaker may be absent.

Inviting. The first of these practices we illustrate is *inviting*. Here, a teacher sees a mathematical possibility (which may or may not be an efficient route to a solution but nevertheless has mathematical integrity) and offers it as an avenue for exploration. For example, at the beginning of the example already offered of clue-giving (lines 14–29), we would categorise the teacher's intervention as inviting:

Student 1: Are we looking at the whole triangle, or just this? [*Pointing to the upper small triangle*]

Teacher: Depends which way you want to look at it. [*Pause*.] Could we look at the whole thing at once?

The teacher was aware that the solution path the two students had been following up to that moment had been to find the area of each of the four inner triangles. Student 1's clarification was the prompt for the teacher to suggest a new possibility —considering the figure as a whole. We propose that this was an inviting intervention as posed, but the students were unable to take it up as an inviting possibility because in this instance the teacher followed up by clue-giving.

Rug-pulling. The second Shepherding intervention we present is *rug-pulling*. This action has the effect of destabilising students' thinking in order to present new possibilities or extend the problem space. Rug-pulling actions shift the focus of students' attention to something that initially confuses them and requires them to reassess their mathematical process.

In the following example, two students have been set the challenge of finding an algebraic relationship between the size of an inner square "pond" and the number of square paving stones needed to surround it (see Figure 6 for a family of such ponds).

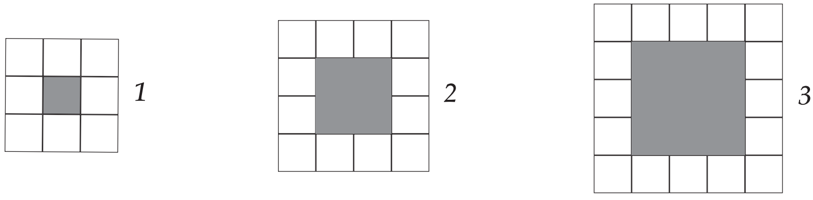

Figure 6. Family of ponds.

The two students rapidly solved this problem, moving quickly beyond drawing and counting to establishing a relationship between the length of the side of a pond and the number of paving stones required to surround it (4n + 4). The students seemed confident of their solution and satisfied that they were finished with the problem, so the teacher asked them to now consider rectangular ponds, and drew for them an example (Figure 7).

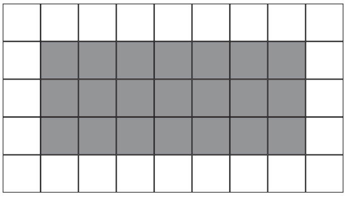

Figure 7. Teacher's drawing.

Unable to simply project from the context of the square ponds to the broader context of rectangular ponds, the students needed to *fold back* (Martin, 2008; Pirie & Kieren, 1994) to form a new image by drawing and counting ponds and paving stones of varying length (and eventually, but not immediately, varying width), tabulating their results, and generating a new generalisation involving two variables (length and width). Such destabilising of a solution process and the subsequent rebuilding of a more complex solution is frequently observed to follow teachers' rug-pulling interventions and serves an important purpose in deepening student understanding (Towers, 1998).

Retreating. The final Shepherding intervention to which we will draw attention is *retreating*. As its name suggests, this is a deliberate action on the part of a teacher to give students the space to ponder a problem or question. It is important to note that this action is a considered, pedagogical one, and not just an act of absence by the teacher. A teacher who is abruptly called away from a conversation with one student to manage a classroom disruption on the other side of the room, for example, would not be considered to be retreating.

Retreating can appear to be a non-action, or just the learned response of a busy teacher in a high-activity environment like a classroom, but we suggest it is an important component of teaching and requires the same sophistication of judgment as each of the other actions we have identified.

In the following episode of retreating, a ten-year old student was trying to find the perimeter of a given figure (Figure 8).

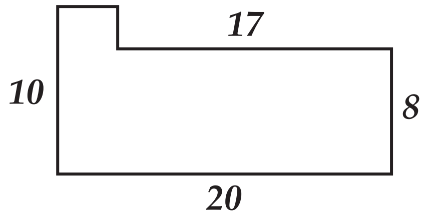

Figure 8. Polygon.

The student struggled, and asked for help, so the teacher worked through a method of finding the missing vertical length with the student. Some minutes later the student asked for help again—this time to find the missing horizontal length:

Teacher: Now in this case, we've got the whole length is twenty and that part is seventeen. So what's the missing bit?

Student: Oh no! Can I add them together?

Teacher: You don't want to be adding them together.

Student: Errrr [*pause*].

Teacher: Shall I leave you to think about it for a moment?

Student: Yeah.

Teacher: And I'll come back.

The data gathered in this case allowed insight into the teacher's reflections on her actions through a journal. She described her purpose in retreating as allowing the student to ponder the problem in her own time. The student was struggling with understanding the geometry and relationships of the offered figure and the teacher thought that this student probably did not understand that the broken lengths and their counterparts were equivalent (i.e., that in the vertical dimension, for instance, the length marked 8 and the unmarked vertical length are equivalent in length to the side marked 10). The teacher had worked through this relationship with the student, but was not confident that the student understood. Rather than abandon the problem, or work through it again, the teacher judged that this student may yet be able to see the relationship for herself, so retreated in order to give the student some time to think.

Summary

As a summary of the above explanations, Table 1 shows examples of various teacher actions that we have identified. The list we offer here is not exhaustive: it is meant to be illustrative of the kind of teacher interventions characteristic of each of the three clusters—*Informing*, *Orienting*, and *Shepherding* actions.

Table 1
Mathematics teaching actions

Categories	Sub-Categories	Explanations
Informing	Enculturating Reinforcing Telling	Giving information
Orienting	Clue-giving Blocking Pretending Anticipating	Directing students' attention
Shepherding	Inviting Rug-pulling Retreating	Supporting and coordinating the possible

We believe there is further work to be done in the mathematics education field to identify and document additional teacher actions, and we acknowledge the other advances being made in this domain (see, for example, Brodie, 2011; Siemon, Virgona, Lasso, Parsons, & Cathcart, 2004).

Discussion

What we have argued—implicitly, or even at times explicitly—through outlining the categories above is that each of these teaching actions can be seen to be aiming at student learning as the underpinning rationale that orients their enactment. To paraphrase Potvin (2004), a teacher needs to act with one fundamental and unique intention in mind: to foster students' learning. This intention has the potential for legitimising teaching actions that are not good or bad in themselves but are linked to a frame of mind that gains its relevance from its intention to have students learn. When the purpose of a teaching action is to make the world of students bigger or, as Sumara and Davis (1997) phrase it, to enlarge the space of the possible for students, we see that teaching action as legitimate. It is in this sense that we avoid placing these categories within a trajectory, one that would organise them from good to bad, novice to expert, poor to rich, etc. Enactivism helps us to see all these teaching actions as legitimate, because the point of our attention is the coupling of interactions in the teaching/learning space, and not the categorising or offering of a judgment on the action in and of itself.

Through the structural coupling of teacher, learner, and environment in the mathematics classroom, teaching actions "fit" their context—they are not to be seen as ultimate, nor perfect, as an external assessment could propose a different action for different reasons. Our point is that teachers' and students' actions co-adapt and it is through this exercise of practical judgment (Dunne, 1993) that a teacher learns how to act. The teacher's role, then, is one of full participant. He or she is a fundamental part of the learning organisation of the classroom and brings forth a world of significance with the learners (Kieren & Simmt, 2009) in which each of the above-described actions might be legitimate. Through their co-adaptation, they both become complicit in the events unfurling in the classroom.

Take, for example, the case of telling—one of the Informing actions. Such actions have often been rejected by mathematics education researchers as too directive (see, e.g., Davis, 1994; Smith, 1996). Yet telling may be done for many reasons: to offer a first example of how particular exercises can be solved, to answer a student question, to offer an idea that students have not thought about and that could enrich their understanding, etc. It is true that not every act of telling is a sound choice, and in the absence of any other kind of action by the teacher habitual telling might, as many researchers have documented, be restrictive and smothering for learners. By telling, though, teachers can offer their perception and interpretation of the mathematical issue at hand in order to have students look at the issue from that angle, an angle considered important by the teacher and that he or she may feel would help students make more sense of the mathematical concepts at play. It is in that sense that we claim that telling can be legitimate and even fruitful for the enrichment of the mathematics worked on in the classroom. Often, these issues—ones to which the teacher wants to ensure students pay attention—are the very reasons behind having chosen a specific task to solve in class, and the teacher has made the judgment that reining in and redirecting attention is appropriate at that moment. Within a rich, engaged mathematical space, instances of telling can be appropriate.

Another of the Informing actions—enculturating—might be similarly important, even in a classroom structured by mathematical inquiry. Students may inquire deeply into, say, the relationships between angles on a plane and may develop their own recording systems to document their inquiries, but nevertheless in order to communicate their ideas effectively with the wider mathematical world they will need to be introduced to, or informed of, the recognised grammar and notations of geometry. The artistry of teaching, of course, is not in choosing whether to share such information but in knowing when and how to intervene in the students' learning so that such actions fit this evolving landscape.

Similarly, the third example of an Informing action—reinforcing—has often been employed to ensure that important ideas are reinforced by the weight of the teacher's authority in classrooms where student voice has lesser value. Elsewhere, we have reframed reinforcing in terms of "placing an emphasis" (Proulx, 2007) on important ideas to which students might pay attention. In other words, reinforcing might help students to notice (Mason, 2002) mathematical structures and therefore has legitimacy even in contexts where distributed control is practised and valued (see, for example, those described by Davis and Simmt, 2003). In this sense, reinforcing is enacted in order to give more substance and richness to the issues explored, and also to point to particular aspects that the teacher senses might possibly be missed by students. We see similar impulses underlying the pretending teaching strategy. These actions are a way for the teacher to make sure that students have solid conceptual ground on which to base their answers, by intervening in the process through offering these playful solutions or questions.

Other actions, such as blocking, may also seem to stifle learning. As we have noted elsewhere (Towers, 2002), teachers' blocking interventions are often well-meaning and usually directed towards trying to move a student forward to a solution in an efficient manner. Sometimes, though, blocking may prevent a student from important sense-making activities such as folding back (Martin, 2008; Pirie & Kieren 1994) to build a richer, thicker image of the problem situation which may be necessary to structure and build the foundation for any conceptual move forward. In the case of blocking that we offered earlier, we see the students' appeals to be shown the angle mnemonics as their efforts to fold back to form images with which to think and the teacher's press forward towards correct terminology as blocking such folding back. We stress, though, that although the example we have given suggests that blocking may be a strategy to be avoided, this is not necessarily always the case. Abstraction, for instance, is an important mathematical practice and sometimes a teacher may legitimately block students' efforts to use concrete materials or specific images as a crutch for learning.

Enactivist thought reminds us that in teaching we cannot make the assumption that instructional properties are present in and of themselves in the tasks offered or the teaching moves made. "Knowledge is about situatedness" (Varela, 1999, p. 7) and in the classroom the context surrounding the decision to be made about what to do at a given moment "is not a 'noise' concealing an abstract configuration in its true essence" (p. 7), it is "both where we are and how we get to where we will be" (Varela, 1999, p. 7). Therefore, each of the kinds of teacher interventions we have described, we claim, are not "good" to do or "bad" to do, in general. Each may be legitimate in its appropriate context. Such thinking intensifies the difficulty of, say, guiding beginning teachers of mathematics. In this frame, it is not possible to provide a list of 'things to do' in a mathematics lesson—write the objective for the lesson on the board, give clear directions, distribute questions around the room, etc.—even though, at any one particular moment, any one of these might be a good thing to do. Nor is it possible to prescribe what ought *not* to be done (e.g., asking the same question repeatedly, not answering directly, allowing a student to struggle with a complicated solution when there is an easier way) as each of these actions might also be legitimate in a given situation. It is in this sense, we believe, that teaching is a complex practice (Bednarz, Fiorentini, & Huang, 2011); not because there are a lot of things happening all at once in a classroom and a lot of decisions to be made, but because the choices to be made are inherently ambiguous and conflicted and their outcomes cannot be reliably replicated from one context to the next.

Teaching, then, is not about the application of rules and principles; it is more about judgment in context, about adapting constantly to the context—a context where learners, teacher, and subject matter are structurally coupled.

> Thus truly ethical behaviour does not arise from mere habit or from obedience to patterns or rules … Intelligence should guide our actions, but in harmony with the texture of the situation at hand, not in accordance with a set of rules or procedures. (Varela, 1999, pp. 30–31)

Some interpretations of such teaching have formulated it as “facilitation” but we reject this perspective, as we've done in Proulx (2010), for it suggests that the work of learning is that of the learner, only, and that the teacher's role is a passive one—to simply set up the environment to 'allow for' learning. Certainly, the teachers role is to occasion learning (Kieren, 1995), but we see that role as more active than facilitation. The teacher must act, but must do so "intelligently" (in Varela's terms), and the landscape of actions we have proposed here point to the breadth of possibilities open to the teacher at any given moment.

Current efforts to improve schools by addressing the quality of teachers' teaching (see, for instance, Cobb and Jackson, 2011) rest upon assumptions, that may be articulated or implicit, about what constitutes "good" teaching. In attempting to shift teachers' practices, the full range of their current strategies must be explored and teachers must be assisted in finding ways to decide whether each of these actions is appropriate in the new context and if so when, why, and for whom. In this way, teachers may be better informed about the broad landscape of teaching actions available to them, be alerted to their current biases for particular practices within this landscape, and therefore be better positioned to make good judgments, in context, about developing their practices in concert with reform efforts.

Concluding Comments

In this paper we have presented a landscape of teaching actions, and clustered these within three categories—*Informing* actions, *Orienting* actions, and *Shepherding* actions. Based on our rationalising framework of enactivism, we have taken care not to organise these categories, and their associated teaching actions, in a trajectory from 'not recommended' to 'recommended.' This move contrasts with many of the current discourses on teaching, which promote particular teaching actions and eschew others.

We believe that our proposal offers mathematics teachers, teacher educators, and educational researchers an expanded vocabulary for describing and analysing teachers' classroom actions, but also a means to understand the whole landscape of teaching actions seen in classrooms. Our framework helps to show why teachers may move back and forth between actions that might be seen (in other frameworks) as both desirable and undesirable and hence explains the kind of practices that are seen as 'hybrids' between less sophisticated and more sophisticated poles of practice (see e.g., Brodie, 2011).

As we have offered only examples of the kinds of actions that exemplify each of our three categories, and do not claim to have presented a comprehensive account of all possible actions, we suggest that there is ample opportunity for further research that identifies additional teaching actions (and potentially additional categories of actions). In addition, further research would help illuminate the complex judgments that allow teachers to move flexibly within and between these categories. Of course, it will also be important in the future to study the ways in which the actions we have documented occasion student learning, though this was not our intent in this paper. Finally, we envision worthwhile research that might explore whether, and how, these categories and descriptions of teaching actions might be useful in learning to teach.

References

Andersson, A. (2011). A "Curling teacher" in mathematics education: Teacher identities and pedagogy development. *Mathematics Education Research Journal, 23*(4), 437–454.

Bauersfeld, H. (1980). Hidden dimensions in the so-called reality of a mathematics classroom. *Educational Studies in Mathematics, 11*, 23–41.

Bednarz, N., Fiorentini, D., & Huang, R. (2011). *International approaches to professional development for mathematics teachers*. Ottawa: University of Ottawa Press.

Brodie, K. (2011). Working with learners' mathematical thinking: Towards a language of description for changing pedagogy. *Teaching and Teacher Education: An International Journal of Research and Studies 27*(1), 174–186.

Brousseau, G. (1988). *Fragilité de la connaissance et fragilité du savoir*. Conference given at the CIRADE, January 22nd 1988 [VHS/color/2 cassettes]. Montreal, Canada: UQÀM/ CIRADE.

Bruce, C. D., Esmonde, I., Ross, J., Dookie, L., & Beatty, R. (2010). The effects of sustained classroom-embedded teacher professional learning on teacher efficacy and related student achievement. *Teaching and Teacher Education: An International Journal of Research and Studies, 26*(8), 1598–1608.

Cai, J., & Wang, T. (2010). Conceptions of effective mathematics teaching within a cultural context: Perspectives of teachers from China and the United States. *Journal of Mathematics Teacher Education, 13*(3), 265–287.

Chazan, D., & Ball, D. (1999). Beyond being told not to tell. *For the Learning of Mathematics, 19*(2), 2–10.

Cobb, P., & Jackson, K. (2011). Towards an empirically grounded theory of action for improving the quality of teaching at scale. *Mathematics Teacher Education and Development, 13*(1), 6–33.

Cobb, P., Perlwitz, M., & Underwood-Gregg, D. (1998). Individual construction, mathematical acculturation, and the classroom community. In M. Larochelle, N. Bednarz, & J. Garrison (Eds.), *Constructivism and education* (pp. 63–80). Cambridge: Cambridge University Press.

Cooney, T. J. (1988). The issue of reform: What have we learned from yesteryear? *Mathematics Teacher, 81*(5), 352–363.

Cooney, T. J. (1994). Teacher education as an exercise in adaptation. In D. B. Aichele, & A. F. Coxford (Eds.), *Professional development for teachers of mathematics: NCTM 1994 Yearbook* (pp. 9–22). Reston, VA: National Council of Teachers of Mathematics.

Davis, B. (1994). Mathematics teaching: Moving from telling to listening. *Journal of Curriculum and Supervision, 9*(3), 267–283.

Davis, B. (1996). *Teaching mathematics: Toward a sound alternative*. New York: Garland.

Davis, B. (1997). Listening for differences: An evolving conception of mathematics teaching. *Journal for Research in Mathematics Education, 28*(3), 355–376.

Davis, B. (2004). *Inventions of teaching: A genealogy. Mahwah*, NJ: Erlbaum.

Davis, A. (2008). Effective teaching: Some contemporary mythologies. *Philosophy of Mathematics Education, 23*. Retrieved June 24, 2011, from http://people.exeter.ac.uk/PErnest/pome23/index.htm

Davis, B., & Simmt, E. (2003). Understanding learning systems: Mathematics education and complexity science. *Journal for Research in Mathematics Education, 34*(2), 137–167.

Davis, B., & Sumara, D. (2003). Why aren't they getting this? Working through the regressive myths of constructivist pedagogy. *Teaching Education, 14*(2), 123–140.

Davis, B., & Sumara, D. (2010). The hard work of learning and the challenges of "good" teaching. *Education Canada, 50*(4). Retrieved June 24, 2011, from http://www.cea-ace.ca/education-canada/article/hard-work-learning-and-challenges%E2%80%9Cgood%E2%80%9D-teaching

Desimone, L., & Long, D. A. (2010). Teacher effects and the achievement gap: Do teacher and teaching quality influence the achievement gap between Black and White and high- and low-SES students in the early grades? *Teachers College Record, 112*(12), 3024–3073.

Dunne, J. (1993). *Back to the rough ground. Practical judgment and the lure of technique*. Notre Dame, IN: University of Notre Dame Press.

Edwards, D., & Mercer, N. (1987). *Common knowledge: The growth of understanding in the classroom*. London: Routledge.

Ertesvåg, S. K. (2011). Measuring authoritative teaching. *Teaching and Teacher Education: An International Journal of Research and Studies, 27*(1), 51–61.

Fauvel, J., & Gray, J. (1987). *The history of mathematics: A reader*. London/Milton Keynes: Macmillan Education/The Open University.

Felder, R. M., & Silverman, L. K. (1988). Learning and teaching styles in engineering education. *Engineering Education, 78*(7), 674–681. Retrieved June 21, 2011, from http://www4.ncsu.edu/unity/lockers/users/f/felder/public/Learning_Styles.html

Forman, E. A., & Ansell, E. (2001). The multiple voices of a mathematics classroom community. *Educational Studies in Mathematics, 46*, 115–142.

Forman, E. A., & Ansell, E. (2002). Orchestrating the multiple voices and inscriptions of a mathematics classroom. *Journal of the Learning Sciences, 11*(2&3), 251–274.

Forman, E. A., Larreamendy-Joerns, J., Stein, M. K., & Brown, C. A. (1998). "You're going to want to find out which and prove it": Collective argumentation in a mathematics classroom. *Learning and Instruction, 8*, 527–548.

Friesen, S. (2009). *What did you do in school today? Teaching effectiveness: A framework and rubric*. Toronto: Canadian Education Association. Retrieved June 24, 2011, from http://www.cea-ace.ca/publication/what-did-you-do-school-today-teaching-effectiveness-framework-and-rubric

Fritzen, A. (2011). Teaching as sheltering: A metaphorical analysis of sheltered instruction for English language learners. *Curriculum Inquiry, 41*(2), 185–211.

Hoong, L. Y., & Chick, H. (2007/2008). An insight into the 'balancing act' of teaching. *Mathematics Teacher Education and Development*, *9*, 51–65.

Hsieh, S.-W., Jang, Y.-R., Hwang, G.-J., & Chen, N.-S. (2011). Effects of teaching and learning styles on students' reflection levels for ubiquitous learning. *Computers and Education, 57*(1), 1194–1201.

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35*(2), 81–116.

Jaworski, B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education, 9* (2), 187–211.

Kieren, T. E. (1995, June). *Teaching mathematics (in-the-middle): Enactivist views on learning and teaching mathematics*. Paper presented at the Queens/Gage Canadian National Mathematics Leadership Conference, Queens University, Kingston, Canada.

Kieren, T. E., & Simmt, E. (2009). Brought forth in bringing forth: The inter-actions and products of a collective learning system. *Complicity: An International Journal of Complexity and Education, 6*(2), 20–27.

King, K. D. (2001). Conceptually-oriented mathematics teacher development: Improvisation as a metaphor. *For the Learning of Mathematics, 21*(3), 9–15.

Lobato, J., Clarke, D., & Ellis, A. B. (2005). Initiating and eliciting in teaching: A reformulation of telling. *Journal for Research in Mathematics Education, 36*(2), 101–136.

Loughran, J., & Russell, T. (2007). Beginning to understand teaching as a discipline. *Studying Teacher Education, 3*(2), 217–227.

Martin, L. C. (2008). Folding back and the dynamical growth of mathematical understanding: Elaborating the Pirie–Kieren Theory. *Journal of Mathematical Behavior*, *27*, 64–85.

Martin, L. C., & Towers, J. (2011). Improvisational understanding in the mathematics classroom. In R. Keith Sawyer (Ed.), *Structure and Improvisation in Creative Teaching* (pp. 252–278). New York: Cambridge University Press.

Mason, J. (2002). *Researching your own practice: The discipline of noticing*. London and New York: Routledge/Falmer.

Maturana, H. R. (1988). Reality: The search for objectivity or the quest for a compelling argument. *Irish Journal of Psychology, 9*(1), 25–82.

Maturana, H. R., & Varela, F. J. (1992). *The tree of knowledge: The biological roots of human understanding*. Boston, MA: Shambhala Publications Inc.

Nicoll-Senft, J. M., & Seider, S. N. (2010). Assessing the impact of the 4MAT teaching model across multiple disciplines in higher education. *College Teaching, 58*(1), 19–27.

O'Connor, M. C., & Michaels, S. (1993). Aligning academic task and participation status through revoicing: Analysis of a classroom discourse strategy. *Anthropology and Education Quarterly, 24*(4), 318–335.

Pesek, D., & Kirshner, D. (2000). Interference of instrumental instruction in subsequent relational learning. *Journal for Research in Mathematics Education, 31*(5), 524–540.

Pirie, S. E. B., & Kieren, T. E. (1994). Growth in mathematical understanding: How can we characterise it and how can we represent it? *Educational Studies in Mathematics, 26*(2/3), 165–190.

Proulx, J. (2003). Pratiques des futurs enseignants de mathématiques au secondaire sous l'angle des explications orales: Intentions sous-jacentes et influences. Masters' thesis, Université du Québec à Montréal, Montréal, Canada.

Proulx, J. (2007). (Enlarging) secondary-level mathematics teachers' mathematical knowledge: An investigation of professional development. Doctoral dissertation, University of Alberta, Alberta, Canada.

Proulx, J. (2010). Is "facilitator" the right word? And on what grounds? Some reflections and theorizations. *Complicity: An International Journal of Complexity and Education, 7*(2), 52–65.

Proulx, J., Simmt, E., & Towers, J. (2009). The enactivist theory of cognition and mathematics education research: Issues of the past, current questions and future directions. Research Forum. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *Proceedings of the 33rd annual meeting of the International Group for the Psychology of Mathematics Education, Vol. 1* (pp. 249–252). Thessaloniki, Greece: PME.

Potvin, P. (2004). *L'enseignement radical*. Montreal, QC: Guérin.

Puntambekar, S., Stylianou, A., & Goldstein, J. (2007). Comparing classroom enactments of an inquiry curriculum: Lessons learned from two teachers. *The Journal of the Learning Sciences, 16*(1), 81–130.

Ramsey, W., & Ransley, W. (1986). A method of analysis for determining dimensions of teaching style. *Teaching and Teacher Education: An International Journal of Research and Studies, 2*(1), 69–79.

Schaefer, K. M., & Zygmont, D. (2003). Analyzing the teaching style of Nursing faculty: Does it promote a student-centered or teacher-centered learning environment? *Nursing Education Perspectives, 24*(5), 238–245.

Schumacher, P., & Kennedy, K. T. (2008). Lessons learned concerning a student centered teaching style by university mathematics professors from secondary school educators. *Education, 129*(1), 102–109.

Siemon, D., Virgona, J., Lasso, M., Parsons, V., & Cathcart, J. (2004). Elaborating the teacher's role—Towards a professional language. In M. Johnsen Høines & A. B. Fuglestad (Eds.), *Proceedings of the 28th annual meeting of the International Group for the Psychology of Mathematics Education*, *Vol. 4* (pp. 193–200). Bergen, Norway: PME.

Smith, J. P. III (1996). Efficacy and teaching mathematics by telling: A challenge for reform. *Journal for Research in Mathematics Education, 27*(4), 387–402.

Sumara, D. J., & Davis, B. (1997). Enlarging the space of the possible: Complexity, complicity, and action-research practices. In T. R. Carson & D. J. Sumara (Eds.), *Action research as a living practice* (pp. 299–312). New York: Peter Lang.

Thompson, A. G., Philipp, R. A., Thompson, P. W., & Boyd, B. A. (1994). Calculational and conceptual orientations in teaching mathematics. In D. B. Aichele & A. F. Coxford (Eds.), *Professional development for teachers of mathematics: NCTM 1994 Yearbook* (pp. 79–92). Reston, VA: National Council of Teachers of Mathematics.

Towers, J. (1998). Teachers' interventions and the growth of students' mathematical understanding. Doctoral dissertation, University of British Columbia.

Towers, J. (2002). Blocking the growth of mathematical understanding: A challenge for teaching. *Mathematics Education Research Journal, 14*(2), 121–132.

Towers, J., & Martin, L. C. (2009). The emergence of a 'better' idea: Preservice teachers' growing understanding of mathematics for teaching. *For the Learning of Mathematics, 29*(3), 44–48.

Varela, F. J. (1999). *Ethical know-how. Action, wisdom and cognition*. Stanford, CA: Stanford University Press.

Varela, F. J., Thompson, E., & Rosch, E. (1991). *The embodied mind: Cognitive science and human experience*. Cambridge, MA: MIT Press.

Authors

Jo Towers, University of Calgary, Faculty of Education, EDT712, 2500 University Drive NW, Calgary, AB, T2N1N4, Canada. Email: towers@ucalgary.ca

Jérôme Proulx, Université du Québec à Montréal, Mathematics Department, Didactics section, C.P. 8888, Succ. Centre-Ville, Montréal, Qc, H3C 3P8, Canada.
Email: proulx.jerome@uqam.ca