A Lesson Based on the Use of Contexts: An Example of Effective Practice in Secondary School Mathematics

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The importance of using real-life contexts in teaching mathematics is emphasised in many policy and curriculum statements. The literature indicates using contexts to teach mathematics can be difficult and few detailed exemplars exist. This article describes the use of real-life contexts in one New Zealand Year 11 algebra lesson. Data included a video recording of one lesson and the teacher's reflections on the lesson. Analysis of the lesson revealed the importance for its success of the ways in which the learning tasks and their contexts were introduced, ongoing referral to the contexts, consolidation of prior mathematics learning, and teacher questioning. The lesson described illustrates how meaningful links to real-life contexts can be developed to promote mathematical understanding, how a balance between focusing on the mathematics and the context can be achieved, and that these require careful planning. The lesson example and its analysis indicate that awareness of the complexity of implementing context-based mathematics learning is important for those who promote or want to implement context-based mathematics teaching, including policy makers, teacher educators, and teachers.

Background

Teaching mathematics through context-based examples is endorsed by professional mathematics teaching bodies (e.g., National Council of Teachers of Mathematics, 2000) in official curriculum documents (e.g., Ministry of Education, 2007), and through curriculum exemplars (e.g., Australian Curriculum, Assessment and Reporting Authority, n.d.). Scrutiny of literature about the use of contexts in teaching mathematics reveals that teaching in real-world contexts can be problematic (e.g., Inoue, 2009; Verschaffel, Greer, & De Corte, 2000) and the productive use of contexts requires pedagogical skill (Doorman et al., 2007). Few examples of context-rich mathematics lessons have been documented to date. However, detailed descriptions of effective context-based lessons together with teacher commentary about the lesson have the potential to contribute to understanding the complexities of using real-life contexts within mathematical instruction.

A range of meanings for the term context exists in the mathematics education literature; in this paper we use the term to refer to real-life situations. The literature examining the use of word problems will be used to highlight issues that are pertinent to the discussion of the use of contexts.

Shulman (1986) argued for the exemplification of principles of good practice through dissemination of accounts of successful mathematics teaching practice. In order to illustrate effective use of contexts, this article reports on the analysis of a lesson in which students were required to develop and make sense of algebraic relationships that were found by exploring real-world contexts. We begin by describing literature most closely related to using contexts to help the students understand mathematical ideas. This is followed by information describing the New Zealand setting of the lesson and then its analysis. We conclude with a discussion of the implications for those involved in the provision of mathematics education of using context-based teaching.

Contexts

There are a variety of ways in which contexts can be used in mathematics instruction. One approach is to use "pure mathematical tasks that have been 'dressed up' in a real-world context that for their solution merely require that the students 'undress' these tasks and solve them" (Palm, 2009, p. 3). Many textbook problems exemplify this approach. Other tasks can require more extensive investigation by students such as those that more faithfully represent the mathematical problems people solve in situations outside school (Organisation for Economic Co-operation and Development, 1999).

Typically, teachers use links to contexts to motivate students and support the learning of mathematics content, rather than to develop the ability to explore real-world contexts through the use of mathematics (Gainsburg, 2008). Most commonly, problems involving contexts are presented as direct applications of mathematical techniques. In these cases the students merely need to follow the procedures developed in recent lessons (Llinares & Roig, 2008) rather than have to grapple with the realities of the context.

A study of teaching eighth grade in seven industrially developed countries found that the proportion of problems with real-world connections posed in mathematics classrooms varied between the countries from 9% to 42% (Hiebert et al., 2003). Mathematics teachers in the Netherlands made greater use of contexts than in other countries in this study (Hiebert et al., 2003). The Dutch advocates of the Realistic Mathematics Education [RME] approach to mathematics (e.g., Gravemeijer & Doorman, 1999) argue for extensive use of "experientially real" (p. 111) contexts as vehicles for the development of mathematics. The RME approach includes requiring students to grapple with contextual problems and in the process of doing so, creating mathematical tools for the solving of problems. Contextual problems are chosen carefully to match the learning needs of the students and to potentially enable students to create mathematical models that can then be used as objects to assist the development of mathematical thinking (Gravemeijer & Doorman, 1999; van den Heuvel-Panhuizen, 2003).

While mathematics educators continue to advocate context-based mathematics instruction, examples in textbooks and classrooms often employ a surface level approach in that the use of contexts may not be associated with a strong focus on the development of mathematical thinking (Doorman et al., 2007). Because context-based problems are most often framed using words, the literature identifying particular difficulties that students have in solving word problems is now discussed.

Greer (1993) tested 13 and 14-year-old students with context-based word problems and found that frequently no consideration of context was used when answering the questions. For example, over 90% of students attempted to solve the following problem by direct proportion.

> A girl is writing down names of animals that begin with the letter C. In one minute she writes down 9 names. About how many will she write in the next 3 minutes? (p. 245).

Similarly, Belgian 10 and 11-year-old students typically ignored the real-world considerations when solving word problems and used the numbers to calculate answers that were unrealistic (e.g., Verschaffel, De Corte, & Lasure, 1994). Such results have been replicated in many similar studies with pre-service teachers (e.g., Verschaffel, De Corte, & Borghart, 1997) and primary school students (e.g., Palm, 2008; Reusser & Stebler, 1997; Yoshida, Verschaffel, & De Corte, 1997).

Verschaffel et al. (2000) argued that the students ignoring the details of the context was a consequence of their past histories in the mathematics classroom.

> [S]tudents' responses to word problems that apparently disregard considerations of reality should be interpreted as showing that they are adhering to conventions learned and reinforced over a considerable period of time (p. 66).

In related work with university students, Inoue (2005, 2009) found that fewer than half of the student responses took real-life considerations into account when solving problems such as "John's best time to run 100 metres is 17 seconds. How long will it take him to run 1 kilometre?" (Inoue, 2005, p. 70). Questioning students who had calculated answers without reference to contextual factors revealed that some spontaneously indicated that they would answer differently in a real-world setting; however, a greater proportion required further prodding to recognise that their response may not be correct. Inoue (2009) concluded that actions that could assist students to incorporate the practicalities of the context when solving problems include discussing problems where the context must be taken into account to create realistic solutions, and discussing the assumptions that need to be made in specific situations before attempting to generate solutions.

In assessment items students may encounter word problems that require them to take account of particular realistic considerations, but penalise those students who take more general realistic considerations (Boaler, 1994; Cooper & Harries, 2002). English 11-year-old students were asked "There is a lift in the office block. The lift can carry up to 14 people. In the morning rush, 269 people want to go up in this lift. How many times must the lift go up? (Cooper & Harries, 2002, p. 7). In conventional testing the answer 20 would be the only answer seen as correct as the student has recognised that after division by 14, the fractional answer needs to be rounded up. Later in the same questionnaire the students were asked to comment on the answers: 19.21, 25 and 15 times. The question asked was to think about how each answer may have been calculated and to consider whether or not each answer was a feasible solution to the context problem. The answer of 25 could be valid if the lift was not completely full each time, and the answer of 15 recognises that some people used the stairs instead of the lift. When presented with the question in this manner, some students were prepared to consider a broader range of real-world considerations than would normally be rewarded in mathematics classes, and the researchers argue that word problems used in teaching and assessment should include emphasis on realistic considerations in general and not just on very specific mathematical considerations (Cooper & Harries, 2002). Palm (2008) conducted research with Swedish 11-year-old students and found that requiring solutions to word problems to be acted out increased the likelihood that students would use their real-world knowledge when solving the problems.

The literature regarding the difficulties associated with the use of word problems has largely been informed by research that has been conducted through purpose-designed tests rather than having been centred in classrooms. Next we consider the literature reporting students' and teachers' views about the classroom implementation of context-based teaching. This literature indicates that many students support the use of carefully chosen contexts in mathematics teaching, and that secondary school mathematics teachers find it difficult to develop suitable contexts.

The majority of the British secondary school students reported in Boaler's (2000) study stated they found mathematics classes boring and the content meaningless. In contrast to these experiences, when describing subjects they enjoyed, they commented on the meaningfulness and links of subject matter to their world. For students in the initial years in an Australian secondary school, Attard (2010) found that lessons that integrated the mathematics content with material from other subjects increased student engagement; however, some students wanted lessons focusing directly on mathematical content to be taught alongside the context-based lessons. A British study found that 13 and 14-year-olds listed the use of interesting contexts when asked to identify features of 'fun' lessons; however, they regarded some contexts used in mathematics lessons as unappealing and "did not see through fence designing or table manufacturing an opportunity for practising certain algebraic skills that are transferable to contexts that are personally relevant to them" (Nardi & Steward, 2003, p. 352).

Gainsburg (2008) found that 80% of American middle school and secondary teachers reported that they typically sourced "real world connections" (p. 201) in their mathematics teaching from their own ideas or experiences, and the teachers reported that many of the examples presented in textbooks were inadequate. Although the majority of the teachers in Gainsburg's study indicated that they used real-world connections at least weekly, many of the connections were brief and extended context-based activities were seldom used. When asked to explain why they didn't make greater use of real-world connections, the most common responses were that this approach was too time consuming and that resources and training were needed to assist them to make such connections. It is possible that the teachers' stated need for training may be related to feelings of lack of success with teaching mathematics using context-based problems.

In a study that attempted to provide such training, Canadian secondary school mathematics student teachers visited workplace sites to observe and interview staff, and were required to develop classroom activities based on this experience. The prospective teachers found it difficult to identify mathematics in the workplace and, when identified, to incorporate such mathematical ideas into teaching sequences at an appropriate level for their classes (Nicol, 2002) which may indicate that successful development of real-life contexts in mathematics lessons requires deep knowledge of curriculum content, practice and experience.

The Study

The New Zealand Ministry of Education's (2007) rationale for studying mathematics and statistics includes the statement "Mathematics and statistics have a broad range of practical applications in everyday life, in other learning areas, and in workplaces" (p. 26) which indicates an expectation that students will be able to apply the mathematics that they learn in context-based situations. Additionally, there is an expectation that mathematics teaching at all levels will be set in a "range of meaningful contexts" (Ministry of Education, 2007, fold-out pages). Year 11 is the first year in which New Zealand students are assessed towards national qualifications. At this level the assessment of mathematics is done through problems set in real-life or mathematics contexts (e.g., New Zealand Qualifications Authority, 2011). This is exemplified in a sample examination paper (New Zealand Qualifications Authority, 2011) that set graphs and relationships problems in the context of using a sausage sizzle to raise funds.

At the beginning of the 21st century the Ministry of Education undertook an initiative to enhance the teaching of mathematics in New Zealand schools, specifically focused on building teachers' knowledge of students' development of mathematical ideas, and using assessment information to further understanding of their students' mathematical progress (Ministry of Education, 2001). In secondary schools the initiative focused on the teaching of Years 9 and 10 mathematics (e.g., Harvey & Higgins, 2007). In each school one teacher was given a time allowance to lead the professional development of the mathematics-teaching colleagues in their school and these leaders were supported with training and external mentoring. The leadership role included running workshops, appraisal of teaching, and mentoring peers.

Teachers involved in the initiative reported a moderate increase in the use of real-life contexts in Year 9-11 classrooms and attributed this change in practice to the professional development (Harvey & Averill, 2009). The lesson described in this paper was drawn from a study aimed at investigating and reporting examples of effective mathematics teaching in senior secondary schools that took part in the initiative. Full ethical approval was granted for the study. This article focuses on describing the elements that appeared to lead to the successful use of context-based mathematics teaching in one Year 11 mathematics lesson. The lesson was video-recorded and teacher reflections were audio-recorded and transcribed.

Participants and method

The lesson was based in one all girls' secondary school with approximately 1300 students and serving a mid to low-income urban community. Craig (pseudonym), the assistant to the head of the mathematics department at the secondary school, understood the purpose of the wider investigation for which the data were collected. As an experienced teacher with responsibility for leading the *Secondary Numeracy Project* (Ministry of Education, n.d.) in his school, Craig volunteered to teach a lesson and was given several weeks' notice of the timing of the recording. He was videotaped teaching a class of 22 Year 11 students of average ability in mathematics. Effective teacher-student relationships appeared to be in place and the students seemed confident they could learn well in the teacher's class, happy to seek assistance and to offer answers.

The lesson was videotaped from the back of the room so that the evidence of student involvement and actions could be gathered. Immediately after the lesson Craig was audio taped as he reflected on the lesson with prompts from the interviewer. The lesson was viewed many times by the lead author to build understanding of the actions and purpose of the teacher. The audiotape of teacher reflections was transcribed and analysed. Themes that emerged from the lesson and teacher reflections are reported.

The videotaped lesson

Two contexts were explored in the lesson. The first context, the focus of the first part of the lesson analysis, led to finding relationships between the length of a Bailey bridge (one quickly assembled from prefabricated steel girders) and the number of triangles used to construct its sides. Craig used photographs he had taken in a nearby region, and compared that region's terrain and climate with those of the school, and used photographs of Bailey bridges to introduce the mathematical tasks (see Figure 1).

Figure 1. Photograph of Bailey bridge used to introduce the context

Local links were exploited in the second context by exploring a situation from within the school, that of carpeting a senior staff-member's office. The carpet consisted of plain carpet squares surrounding a square central feature of a rose motif. The motif could have been purchased in a range of sizes, and the task for the students was to calculate the number of plain tiles required for different dimensions of the motif. This context gave rise to a quadratic relationship.

Examples of effective practice during the lesson (introduction of the task and context, ongoing referral to the context, review of previous knowledge, questioning, consolidating and extending, and interactions with individuals) are discussed in turn below. Transcripts of parts of the lesson and interview are used to illustrate the features. Craig is denoted by C, and S indicates a contribution by a student. Sections in italics provide the authors' commentary on the lesson.

Introduction of the context and the task

Each context was introduced in a way that preserved the links to the real-life situation with a focus on a broad range of ideas rather than a cursory treatment of the context merely in order to introduce the mathematical ideas (see Figure 2). In each case, the task's definition was sufficiently open to give the students some control over how they went about the task. The classroom environment of positive relationships between the teacher and students and the seating arrangements encouraged constructive discussion between pairs of students.

The Bailey bridges context was introduced using photographs Craig had taken in a nearby region.

C: As you drive down the West coast, you cross a lot of bridges. Does anyone know why you cross a lot of bridges on the West coast?

S: A lot of rivers.

C: Great, there are a lot of rivers. Why are there a lot of rivers?

S: Because of the mountains.

C: Yes it's got to do with mountains. What is it to do with mountains?

S: Because the water comes from them.

C: Actually you're right. You know when we have nor-westers here and they are hot and dry, does anyone know what happens on the West Coast?

Craig discussed orographic rainfall, the frequency of floods and the need to use Bailey bridges to provide temporary access, after access is lost because of flooding. The focus moved to discussion of the construction of the sides of Bailey bridges which he simplified to being made up of congruent equilateral triangles with each side being 10 metres long. The mathematical task given was to create three different representations of the number of triangles required to construct the sides of the bridges for different lengths of span. The representations required were: a table; a graph; and an algebraic rule.

Figure 2. Craig's introduction of the context-based problems

Craig supported the students in their exploration of the tasks by circulating around the desks and privately questioning individuals to help them continue and extend their exploration to help solve the problems. Craig's introduction and development of the context (see Figure 2) illustrates the way that he encouraged, and built on, student contributions to set up the context-based investigations. Student work produced after the bridges context had been introduced and prior to class discussion showed that many students were able to explore the context algebraically (see Figure 3). The progress of students through this task varied and many students completed the table and graph but did not find the algebraic relationship.

Triangles required to construct one side of bridge when bridge length is

(a) 20 metres and (b) 30 metres

Table summarising relationship between length of bridge and number of triangles required.

Length of bridge (L) (metres) Number of triangles (T)

10 2

20 6

30 10

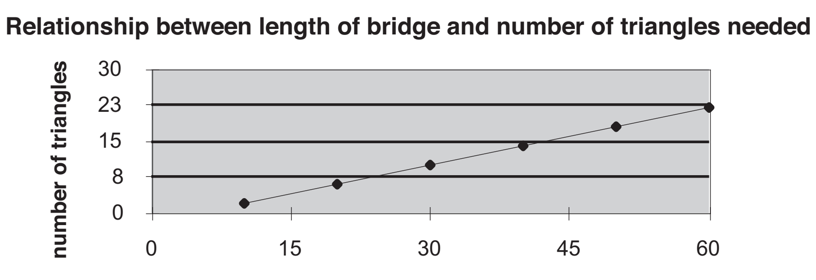
40

50

60 14

18

22



Relationship between length of bridge (L) and number of triangles (T) $T = \frac{4}{10}L - 2$

Figure 3. Replication of initial student answers to the Bailey bridge task developed from video footage

Ongoing referral to the context

Greer (1993) and Inoue (2009) are among others who have highlighted problems associated with students ignoring the context when dealing with mathematical tasks. Once Craig had introduced the contexts and mathematical tasks, a factor that appeared to contribute to the success of the lesson was his emphasis on referring back to the context when students were involved in solving the tasks, rather than focussing solely on the mathematical aspect of the tasks. The following extract (see Figure 4) shows how Craig introduced, and worked with the class to resolve, the mathematical complexities arising from the Bailey bridge context. Also apparent is the frequency with which he emphasised how the bridge context affected possible answers.

After 15 minutes of student independent work Craig asked two volunteers to come to the board: one recorded her table of results on the board, while the second provided Craig with points to plot to create the graph. He used this student work as a basis for discussing progress on the problem. Initially he dealt with bridges that spanned less than 10 metres, before asking the students to consider a bridge spanning a 25 metre gap.

C: We have to be careful with what we just did. I'm going to ask you something now. How many triangles are required for a bridge that is 25 metres long?

S: 8

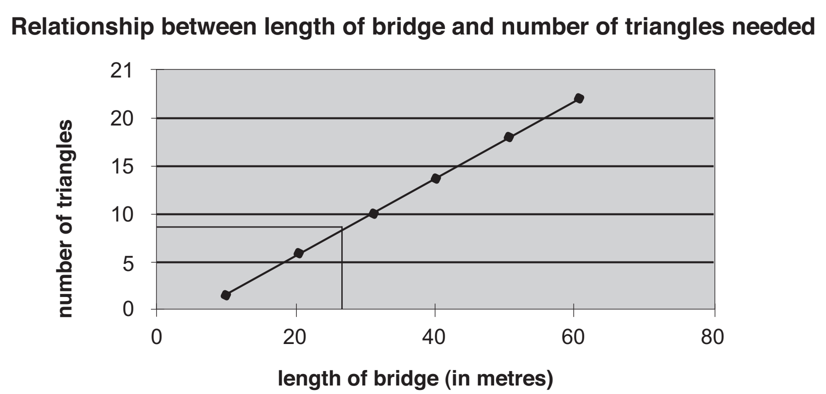
C: Good Teri. How did you come up with 8?

The student explained how she got the point off the graph.

C: Here's what Teri did and I want you to tell me whether practically if this is actually OK.

C: If we have a bridge that is 25 metres long. It means – and you went like this didn't you Teri – you need 8 triangles (Craig shows how reading off the graph gives a value of 8.)

Replication of Craig's graph showing how a 25 metre long bridge would appear to need 8 triangles to construct the sides:



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C: Show me on the bridge. If I have a bridge that is 25 metres long, we need 8 triangles. Show me on the bridge how I would arrange my 8 triangles. Show me on the bridge how Teri's 8 triangles can be organised.

S: Has to be an odd number.

C: Oh has to be an odd number. Good call. Why does it have to be an odd number?

Pause

S: Because otherwise it will be ending like this (gesture with arm indicating that the bridge would be incomplete at one end.)

C: Uh oh. Watch this please, 10 metre bridge, 20 metre bridge. This is how long it needed to be. But we forgot to put this triangle on the end. What is going to happen? Can you see that?

Replication of Craig's diagram showing an attempt to make a bridge using 4 triangles on each side:

Actually, although these points are lined up in a straight line, the in-between things can't be read off it, can they? In which case, actually, the table makes a lot of sense, doesn't it?

Building on this, Craig went on to show how to provide intervals for the domain, so that the number of triangles required could be read off directly (see Table 1).

Craig returned to a discussion of the equation of the line. Through questioning, the slope of the line was linked to the extra triangles required as the length of the bridge increases. Questioning was used again to find the y-intercept of the graph and to establish the fact that a y-intercept of -2 has no relevance to the context.

Figure 4. Development of the mathematical relationships with links to context

During discussion of the bridge example, Craig structured questions and required students to consider the answer to reveal that graphs had to be treated with care since not all values that could be read from the graph would be feasible in the context (e.g., non-whole number values). This enabled him to discuss the modification of the table to model the situation more accurately (Table 1). The ongoing reference back to the context and the preparedness to expose the complexities of the situation appeared to help give this teaching episode an authentic flavour.

Craig wove the context and the mathematics together throughout the lesson (see Figure 4). Through careful questioning he built on student contributions to identify aspects of the initial student answers that did not accurately portray the context. Investigating the number of triangles required when the bridge length was 25 metres illustrated that the algebraic relationships that had been developed were only valid when the bridge length was a multiple of the length of the sides of the triangle. Similarly, examining the values generated by the relationship when the bridge length was zero illustrated that the relationship was not valid when considering a bridge of length zero metres.

Table 1.
Modified table summarising relationship between length of bridge and number of triangles required

Length of bridge (L) (metres)	Number of triangles (T)
≤ 10	2
≤ 20	6
≤ 30	10
≤ 40	14
≤ 50	18
≤ 60	22

Reflecting on the interview after the lesson, Craig explained:

> I am now trying to do as much as possible to make it real so that the students have something to hook onto and to support the move from number to algebra and to generalisation … I think we've learnt here not to be too quick about jumping away from the context and jumping in to doing tables and rules but allowing them to maintain the context. The context actually gives all the clues as to how it fits.

Reviewing of previous mathematics knowledge

Through the lesson Craig took opportunities to review concepts from topics the class had met previously. These included: the concepts and calculation of the gradient and y-intercept of the relationship (see Figure 5); substitution of coordinates that satisfied the relationship to check that the equation was correct; and showing how to substitute values into an expression that included a fraction. During whole class teaching, Craig took voluntary contributions from many students and directly questioned others to check for student understanding. Engaging two students to help him in constructing the table, graph, and algebraic relationship on the board also served as a review of prior knowledge. Reviewing this knowledge provided support for those students who had found the work challenging and enabled them to make sense of the lesson.

C: With straight lines we have been talking about some other things, haven't we? What other things have we talked about?

S We need a y-intercept.

C: Yes we need a y-intercept. Brianna, where is the y-intercept?

S: Um . . . Um.

C: With this line here, where does it cut the y-axis? At the moment I haven't, have I? How am I going to get the y-intercept on this graph?

S: You need to extend the line.

C: You want me to take my ruler, and extend it out a bit further. There it goes, down there. And it's going to cut. Um. Weird? -2

C: By the way Stevie, you answered the question and so did several other people but I am going to pick on you for a moment. Stevie, -2. How did you come up with exactly -2?

S: On my graph each of my little squares is 2.

C: So you have read it straight off the graph.

S: Yep.

C: You've said there it is (gesture to graph). Has anyone come up with another way of coming up with -2?

S: Could it be from the first number that you started with?

C: Have a look at this. (points to the pair 10, 2 on the table) Think about this for a moment, You've got it Liana I can see your eyes light up. Where do we get - 2?

S: Because it goes up in fours.

C: Because it goes up in fours so how do I get -2 on the y –axis?

S. Because you went back 4.

C: Because I went backwards 4. So it's actually where we start.

If I have a zero bridge to make. Zero length; I need -2 triangles. Doesn't quite do it, does it?

C: So sometimes where we start on the y-axis actually has no relevance to the situation whatsoever. Can you see that? It's got absolutely no relevance to the y-intercept but it's going to help us. Let's have a think about how it helps us.

Figure 5. Lesson extract showing consolidation of the concept of y-intercept through questioning

Questioning

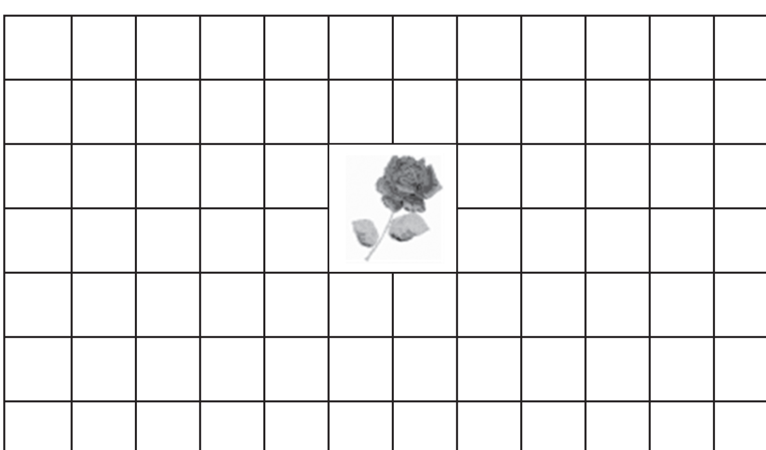
When working with students as individuals and when conducting discussion, Craig probed student understanding through questioning, and used the answers to develop the discussion. This use of questioning is illustrated in each of the lesson extracts (see Figure 5 in particular). Craig's purposeful use of questioning to invite student contributions and to inform him of how the students were thinking was made explicit in his reflection:

> In order for me to know what is going on I need their cues and I need to ask lots of questions … The lesson might go in lots of different directions based on how the students think about it rather than the way I think about it.

Consolidating and extending

In order to consolidate and extend the learning, Craig introduced a second context-based task (see Figure 6). This task again required the students to find and represent an algebraic relationship; however, in this case the context gave rise to a quadratic relationship. The process of creating a table through systematically calculating specific values was followed by drawing a graph, and attempting to find its equation. Students were required to consider the constraints of the situation in deciding the appropriate domain for the relationship.

In the last 20 minutes of the lesson Craig introduced the second context by showing photographs of a carpet in one of the school's offices which had recently been re-laid using carpet squares. The carpet featured a square inlay of a rose motif with dimensions two squares by two squares.



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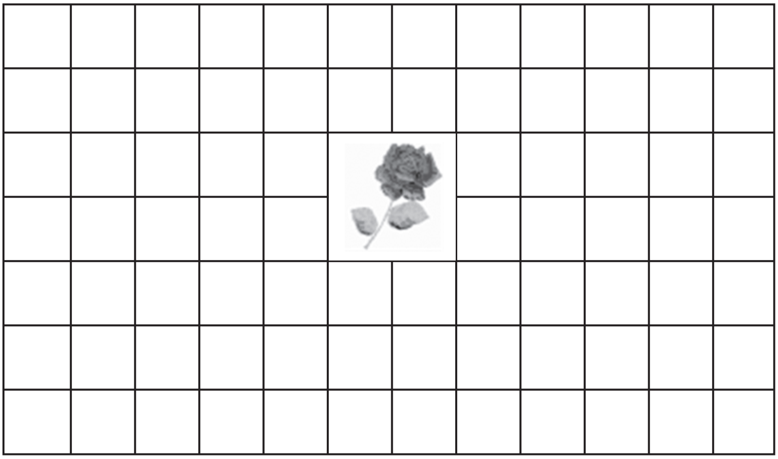

Craig informed the students that the motif could be obtained as any sized square. The task was to represent an algebraic relationship between the edge length of the rose motif and the number of plain tiles in three ways. Students who asked Craig to supply dimensions for the graph were asked to work those out for themselves by considering the information in the context.

The interactions that Craig had with the students in this time included:

- *re-explaining the task to get students started,*
- *assisting students to fill in their table of values*
- *challenging a student to find the equation of the curve, and*
- *reminding a student of the restrictions on the domain of the graph caused by the dimensions of the room.*

This is exemplified by the following interaction which started with Craig working with one student, but soon all four students in the group were participating:

C I've got a question for you. Does the graph carry on and on, and where should the graph stop? I think your graph has gone too far. Why has it gone too far?

S Because of the room.

C Yes, the room is only 7 squares wide so you can't have a rose that is 8 squares by 8 squares.

The first student showed understanding, another student appeared to show partial understanding, so Craig directed his next question to her:

C Did you understand what I just said?

S The room is only 84 squares so the rose can't go bigger than that

C The room is only 7 squares wide so we can't possibly have a graph that is 8 squares wide (hand gestures to support this).

At this point all four students showed understanding and Craig continued to the next part of the lesson.

Figure 6. Lesson extract showing consolidation and extending of task, and interactions with individuals

The carpet task consolidated the work done in the bridges task as it required the students to again show the algebraic relationship in three different ways. The students appeared to progress through this task at a greater rate than they did through the bridge task. The carpet context gave rise to a more demanding algebraic relationship than the linear relationship which arose in the bridges context, and hence use of this context extended student skills. Craig elaborated how both contexts consolidated material that had been taught in previous lessons:

> In previous lessons we have worked from number to algebra and to graphs to create patterns, and we have used graphs to see what happens. In the carpet example, the taking away squares had a meaning. It wasn't just that if you have x squared, it is a parabola, but the concept of the square related to real-life.

Interaction with individuals

During the time that students worked individually on the carpet task, Craig circulated and assisted. In addition to the silent observations that he made of student work, Craig had 13 different interactions with students in the 11 minutes that the class worked on the problem. Each interaction was tailored to the progress and needs of the individual student. In each interaction he was unhurried, gave his full attention to the student, and posed questions and prompts to engage the students in thinking. Craig's reflection indicates that Craig deliberately used the time when the class was working as individuals to give individually focused feedback and support to the range of students.

> … knowing conceptually where they're at and their ability to move, it's not a class anymore, it's a group of individual students and they are all at different places ... although today's lesson wasn't necessarily differentiated delivery, I can't just do one size fits all … the individual conversations that I had with individuals around the room gave me opportunities to do different things with the examples …

Craig reported the value of the professional development in relation to pedagogy that enabled identifying specific learning needs:

> I think the professional development has given us much greater understanding of the different needs of the individual girls. The graph work has enabled me to ask lots of different questions and problems in different ways around the room.

Discussion

The bridge and carpet contexts provide excellent examples of Inoue's (2009) conditions in that solving the problems requires use of the practical aspects of the problems' contexts. In the observed lesson, both of the contexts were introduced in an unhurried way. Craig's sharing of non-mathematical information about the contexts and use of photographs enabled an holistic treatment of the contexts.

The Bailey bridge context was introduced through questions and discussion of terrain and climate that could also be studied in other curriculum areas, an illustration of successful integration of content from other subjects to enhance the lesson as described by Attard (2010). Whereas this setting up of the lesson may have been described as time consuming by the participants in Gainsburg's (2008) study, in this case it may have contributed to the effectiveness of the lesson.

The lesson exhibited elements of teaching called for by Inoue (2005) in that links to the context were maintained throughout the teaching sequence and the relationships developed were tested against the context, and discussed as mathematical solutions that were only true under certain conditions. Craig's deliberate focus on the potential to read erroneous values from the graph may have assisted the students to be vigilant in taking real-world considerations into account when solving problems, thus avoiding difficulties noted by researchers (e.g., Greer, 1993; Verschaffel et al., 1994; Verschaffel et al., 1997) regarding the context merely being used to introduce the content and the final answer not necessarily relating to the richness of the context. While keeping the context under consideration, the teacher ensured that the key focus of teaching and learning mathematics (i.e., consolidating students' algebraic skills) was maintained throughout the lesson.

This weaving together of the context and the mathematics associated with it showed the potential to both support the learning of mathematics and to give insight into how mathematics can be used. Simplifications to the context were made to make the context more mathematically manageable and although the context appeared to be from the real world, it is likely that the approach to finding the number of triangles required differed from what would actually occur in a bridge-building situation in that it is unlikely that Bailey bridges are supplied in the form of prefabricated triangular sections. Rather than striving to use contexts where the mathematics done in the classroom is the same as that used in the real world, it may only be necessary for teachers to use contexts in such a way as to be mainly faithful to the context.

The lesson illustrates how Craig was able to use the context-based lesson to build on mathematical ideas that the class had met earlier. The increased mathematical complexity of the second context enabled students to consolidate and extend skills developed during work on the first context. Craig's careful questioning during class discussion and his work with individuals enabled him to build understanding of the progress of the students, which he used to inform his teaching.

The presentation and discussion of this lesson serves as one example of effective practice. The rich episodes may serve as models of context-based mathematical problems for other practitioners. It is possible that the classroom learning environment, established teacher-student relationships, passion for the subject, and depth of knowledge which enabled Craig to develop this lesson based on these contexts, may be key elements in the confident and successful implementation of this lesson.

Conclusions and recommendations

Given that teaching mathematics through context-based examples is endorsed by many professional mathematics teaching and policy bodies as well as by students, and the challenges teachers report regarding teaching mathematics using contexts, it is essential that effective context-based mathematics teaching is explored and described. Difficulties for teachers and students have been associated both with context-based teaching approaches and use of word problems. The videotaped lesson illustrates that weaving together of the mathematics and the context can lead to purposeful mathematics teaching. It is unlikely that students in studies conducted by Nardi and Stewart (2003) would have nominated the context of Bailey bridges as engaging; however, the lesson by Craig illustrates that a context which may seem to be very dry in the eyes of teenagers can be used as an interesting and engaging lesson when executed by a skilled and passionate teacher.

This article adds to the literature by providing analysis of an example of the use of contexts to support effective teaching that highlights key features that appeared to contribute to its success through:

- introduction of the context and the task;
- ongoing referral to the real-life context;
- timely reminders of previous mathematical knowledge necessary for the task;
- adept questioning;
- consolidation and extension of mathematical ideas; and
- effective teacher-student interactions.

To assist with curriculum implementation and to develop students' perceptions of the relevance of mathematics to everyday life, it is recommended that further lessons illustrating the productive use of contexts be documented. Furthermore, the lesson and teacher reflection indicate that teachers can be encouraged and assisted to develop context-based mathematics teaching through professional development. Implications of this research for pre and in-service education and resource development include ensuring teachers possess a bank of tasks linked to contexts known to be realistic, purposeful, of high interest and effective in supporting students' mathematical learning and understand ways of maximising the effectiveness of such tasks when implementing them. Further research is required to ascertain whether this, or similar lessons, can be replicated by other teachers with the same level of success. However, documenting this lesson provides a model teachers and teacher educators can use towards developing expertise in context-based mathematics teaching.

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