

A Specialist Professional Experience Learning Community for Primary Pre-service Teachers Focussed on Mathematical Problem Solving

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Received: 28 October 2015 Accepted: 23 February 2017

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Problem solving has been identified as an important approach to learning and teaching mathematics, yet many primary pre-service teachers (PSTs) struggle to implement it during their professional experience. In this paper, we report the experiences of a group of four primary PSTs who, in an additional professional experience placement, formed a learning community for teaching mathematical problem-solving lessons to a composite Year 5/6 class. The PSTs visited the school each fortnight over a school term to observe an experienced teacher teaching problem-solving lessons. They then co-taught the same class using a problem-solving approach and observed their peers co-teaching. We report the reflections of these PSTs and evaluate various aspects of the professional experience learning community. The results indicate that membership of the learning community gave the PSTs the opportunity to think more deeply about problem-solving lessons through the lens of a teacher and it assisted the development of their knowledge, skills and confidence in teaching such lessons. The results also indicate that having had this experience, PSTs are more likely to test and develop their skills in teaching problem-solving lessons in future professional experience placements.

Introduction

Research studies of pre-service teachers (PSTs) in primary mathematics have identified low confidence levels among PSTs for teaching mathematics (Hamlett, 2009), mathematics anxiety (Rayner, Pitsolantis, & Osana, 2009), and generally negative attitudes to mathematics (Young-Loveridge, Bicknell, & Mills, 2012). Poor content knowledge (Livy & Vale, 2011) and limited pedagogical content knowledge (Marshman & Porter, 2013) have also been reported. Consequently, PSTs' mathematics lessons tend to emphasise rote learning which can lead to a 'shallow teaching syndrome' (Stacey, 2003) where the focus is on procedural efficiency at the expense of conceptual understanding.

A problem-solving approach to teaching mathematics has been recognised by a growing body of research as a means of developing students' conceptual understanding (e.g. Francisco & Maher, 2005). It is considered to be a core practice for helping students to make sense of mathematics and learn how to solve authentic problems (Anthony et al., 2015). It is an expectation of the Australian Curriculum that mathematics be taught through problem solving; however, in their own experience at school, most PSTs have been taught through a traditional, transmissive approach (Frid & Sparrow, 2009).

In this study we developed a specialist mathematics professional experience based on Wenger's notion of a community of practice (Wenger, 1998) to investigate its impact on PSTs' teaching of problem-solving lessons. This professional experience was for four primary PSTs as

their initial placement, the aim being to support them to use a problem-solving approach during their initial teaching of mathematics. Based on experiences reported by these PSTs, we discuss how they learnt from an experienced teacher and how they learnt with and from each other.

Teaching mathematics through problem solving

The process of mathematical problem solving is at the centre of mathematical thinking and learning (Stacey, 2002). A mathematical problem is defined by Schoenfeld (1985) as a question for which the solution path is not known to the solver. There can be multiple strategies for arriving at the solution to a mathematical problem, some strategies being more efficient than others.

There is broad consensus amongst mathematics educators that problem solving assists students to understand mathematics, gives them a more positive attitude towards the discipline, promotes flexibility and creativity, and prepares students to apply their mathematics to solve unfamiliar problems encountered in the workplace (AAMT and AIG, 2014). For these reasons, problem solving has been included as a 'proficiency' or 'process' at the heart of the Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2013). The Australian curriculum describes the process of problem solving as follows:

Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify that their answers are reasonable.

Teaching mathematics through problem solving requires particular pedagogical skills on the part of the teacher. The teacher needs to choose a suitable non-routine problem for the class (one that enables all students to make at least some progress towards a solution); understand the range of strategies that students have at their disposal; plan questions that elicit these strategies; anticipate student errors and follow up on students' responses in ways that encourage deeper thinking and make connections between mathematical concepts (Sullivan, 2011). The teacher also needs to be proficient with structuring small groups and class discussion so that students are given time to reflect on the answers of their peers and engage each other by asking questions and inviting each other to explain their thinking (Nelson, 2001; Bailey & Taylor, 2015). These teaching practices are considered to be "high-leverage" practices because when they are successfully enacted, they develop high-level thinking, reasoning and skills (Forzani, 2014; Hlas & Hlas, 2012).

A problem-solving approach to teaching mathematics presents a major challenge for many PSTs in primary education. It has been suggested that teachers' own knowledge and confidence in mathematics is an important factor determining whether they adopt a problem-solving approach to teaching the subject matter (Anderson, 2003). This is because teachers require mathematical insight to be able to choose or create a suitable problem-solving context and flexibly adapt to situations and solution strategies as they develop during the lesson. They also need a deep understanding of students' strategies in order to conduct fruitful mathematical conversations that help students connect a newly acquired concept to concepts they currently hold (Guberman & Gorev, 2015).

A classroom culture needs to be developed in which students expect to be able to learn from their peers, have a willingness to persist and are given opportunities to reason (Sullivan & Davidson, 2014). Liljedahl (2016) describes this as a 'thinking classroom'. In a 'thinking

classroom', individuals think and construct knowledge collectively through classroom activity and discussion. Liljedahl found that if a classroom has a culture of individual work and direct instruction, the majority of the class is unable to persist with problem solving.

Theoretical framework

Social theories of learning that draw on models of communities of practice are commonplace in mathematics education research. Lerman (2000) described the growing interest in social learning theories as a "social turn", or "... the emergence into the mathematics education research community of theories that see meaning, thinking, and reasoning as products of social activity" (p. 23). Social perspectives on learning to teach focus on how PSTs develop knowledge and understanding of teaching through their increasing participation in socially situated practices (Goos, 2014). Le Cornu & Ewing (2008) describe a similar kind of approach to professional experience for PSTs which they call a 'learning community'. As the term suggests, a learning community is a cohesive, mutually supportive group of people with a common interest, in which the focus is on their collective knowledge, not just an individual PST's learning.

A learning community encourages PSTs to develop "a commitment to reciprocity and reciprocal learning relationships and a deepening participatory process ... where they learn to value the learning of others as much as their own" (Le Cornu & Ewing, 2008, p. 1803). Dufour and Eaker (1998) characterised a professional experience learning community as one in which there is collaborative learning, shared vision, reflective conversations and a focus on inquiry and experimentation. Similarly, Ponte et al. (2009) describe a group of people "involved in some kind of activity that learn together and, more importantly, learn from each other" (p. 197). Hence a professional experience program that is positioned as a learning community involves co-teaching, shared risk-taking and collaborative meaning-making through professional dialogue (Le Cornu & Ewing, 2008).

Learning communities have their origins in Wenger's (1998) notion of a community of practice (CoP). In a CoP, learning is situated (Lerman, 2001) and occurs through co-participation in the activities of the group (Leiken, 2008). Wenger describes three dimensions of practice: mutual engagement, joint enterprise and shared repertoire. A CoP is formed when people come together to focus on an issue or concern of common interest. This *mutual engagement* suggests that the members possess a level of knowledge and understanding of the issue that allows them to learn with and from each other. Learning through interacting in a *joint enterprise* allows members to share information and develop their expertise by engaging in joint activities and discussions. It is through these interactions that members build relationships with each other and gradually form a community around the domain (Gray, 2004). Over time, the joint pursuit of an enterprise by the community members helps to acquire a *shared repertoire* or collection of experiences and reflections. As Jaworksi (2014, p. 6) has noted, "The words "mutual", "joint" and "shared" together emphasise the communicative nature of a community of practice".

Bailey and Taylor (2015) argued that "experiencing and reflecting upon a problem-solving approach is an important step towards learning about ambitious mathematics teaching" (p. 121). Gaining this experience and reflecting upon it could be achieved through PSTs' participation in a learning community focused on the teaching of problem-solving lessons. In this study, a small group of PSTs were invited to form such a professional learning community. The activities of the community involved PSTs observing the problem-solving lessons of an experienced teacher, preparing and co-teaching problem-solving lessons with a peer, giving and receiving feedback, and writing a reflective journal. These activities went beyond what they had learnt and experienced in their university classes.

In our study, Wenger's three dimensions are applied as follows: *mutual engagement* refers to how the PSTs used their prior understanding and experience of learning mathematics themselves through problem-solving activities. *Joint enterprise* refers to how the PSTs worked together towards a common goal, namely to develop their teaching expertise for problem-solving lessons. It includes the notion of each individual PST's mutual accountability to the group through peer observation, co-planning and co-teaching. *Shared repertoire* refers to the outcome of the learning community: their common language and what they learnt through reflecting on their planning and teaching of problem-solving lessons.

Our research focuses on the advantages that could be gained by PSTs beginning their teaching of problem solving within a professional learning community. We draw upon PSTs' reports of their experiences to answer the following research questions, each relating to one of Wenger's three dimensions:

1. What prior learning and experiences supported the PSTs' mutual engagement in learning to teach problem-solving lessons? (*mutual engagement*)
2. Through their joint enterprise in the activities of the learning community and their shared reflections, what did the PSTs learn about teaching problem-solving lessons? (*joint enterprise*)
3. What aspects of participating in the learning community did the PSTs intend to adopt in their future teaching of mathematics? (*shared repertoire*)

Method

Context and participants

Our research took place in the first half of the academic year. We focused on a group of four PSTs as they first learned how to teach problem-solving lessons in a primary mathematics classroom. In Semester 1 of the previous year, the PSTs satisfactorily completed an introductory numeracy unit, EDUC258 (Mathematics in Schools). This unit adopts a socio-constructivist approach to learning and teaching mathematics by promoting problem solving to engage students and help them learn fundamental mathematical concepts. Tutorial activities in EDUC258 provide regular opportunities for PSTs to collaborate in small groups as they solve rich tasks, reflect on their own mathematical learning, and discuss how the tasks could be used in the classroom.

At the time of the study, the PSTs were enrolled in the third year of their four-year double degree program. They were all competent mathematicians, having successfully completed a calculus-based mathematics course in secondary school and they had all achieved a merit grade in EDUC258. After obtaining the necessary ethics approvals from the university, the participants were recruited via an advertisement on the university Moodle site calling for PSTs who were confident mathematicians and who wished to build on the knowledge and skills they had learned in EDUC258. Volunteers were required to apply via email and include a brief explanation as to why they wanted to participate. Four PSTs, all female, responded and all were accepted into the study. None of the PSTs had previously undertaken any professional experience prior to their participation in this study so this was their first classroom teaching. Their activities in this specialist mathematics professional experience, which was additional to their regular placements, were not formally assessed.

The PSTs worked with an experienced primary school teacher who was teaching an enrichment program for high-achieving mathematics students. The class of approximately 20

students was a combined Year 5/6 group (aged 10-11) that met weekly for one hour. The lessons focussed on the Challenge Stage of the Mathematics Challenge for Young Australians (Australian Mathematics Trust, 2016). The Mathematics Challenge provides teachers with lesson materials and extension activities. It aims to encourage and foster student interest in mathematics through solving interesting and relevant mathematical problems.

Activities of the learning community

In designing activities for the learning community, we drew on Wenger's notions of mutual engagement, joint enterprise and shared repertoire. We planned for mutual engagement of PSTs in the learning community by building on their prior learning from EDUC258 through opportunities to observe a class teacher implement a problem-solving approach. The joint enterprise of the learning community focussed on the PSTs' co-planning and co-teaching, observing and providing feedback on each other's co-taught problem-solving lessons. In doing so, we intended the PSTs to build relationships with each other that would develop their expertise in teaching such lessons. By providing opportunities for discussion amongst the PSTs immediately following each lesson with subsequent personal reflection, we anticipated that they would acquire a shared repertoire that might help them make sense of their experiences and clarify how they intended to teach mathematics.

The PSTs and the first author made six fortnightly visits to the school during Term 2. For the first two visits, they observed the teacher teach a problem-solving lesson. As they observed these lessons, the PSTs were encouraged to refer to the dimensions of the Quality Teaching Framework (NSW Department of Education and Training, 2008). These focus on *intellectual quality* (deep understanding, problematic knowledge, higher-order thinking, substantive communication), *quality learning environment* (engagement, social support, self-regulation), and *significance* (knowledge integration, connectedness). While students were working in small groups, the PSTs moved around the classroom to engage with them and introduce themselves. Immediately following each lesson, the teacher facilitated a discussion with the PSTs about her aims for the lesson, the ways that she had structured the activities, and the student learning which had taken place. The PSTs contributed to this discussion by asking questions and sharing their observations. The discussions lasted about 15 minutes.

Each PST chose a partner and over the remaining four visits the two pairs of PSTs alternated between co-planning and co-teaching the lesson during one visit and observing their peers the next. Immediately following the class, the teacher facilitated a 15-minute discussion on the PSTs' co-taught lesson and encouraged all of the PSTs to contribute their ideas. Following this discussion, the PSTs individually wrote a reflective journal, each entry being about 500 words in length.

Data sources and analysis

The aim of this research study was to investigate the PSTs' perspective on their experiences as they learned how to use problem-solving activities for teaching mathematics within a professional experience learning community. The data for the study include the PSTs' reflective journals which they wrote up after joint reflection on each school visit; their individual written responses to a questionnaire completed in the week following the completion of the school visits; and their responses to a semi-structured interview conducted about eighteen months later at the completion of their undergraduate study. The questions asked and the PSTs' responses referred to the specialist mathematics professional experience as "the program".

In their reflective journals (Figure 1) the PSTs reflected on the purpose of the lesson, the activities used, and what they had learned about teaching a mathematics problem-solving

lesson. In the questionnaire (Figure 2) they summarised their learning from the program, commented on each of the various aspects of the program and indicated which ones they would adopt in their own teaching of mathematics. Three of the four PSTs were interviewed individually. They commented on the benefits and drawbacks of the program, their professional experiences following the program, and their intention to teach problem-lessons in the future (Figure 3). The phrase “problem-solving” was not used in any of the questions asked. This was because we did not want to direct the PSTs’ responses. We wanted to investigate the extent to which the problem-solving aspects of the lessons (as opposed to aspects relating to teaching in general) were noticed and commented upon as part of the shared repertoire of the learning community.

The three semi-structured interviews were audio-recorded and transcribed and typically lasted for between 15 and 20 minutes each.

Reflective journal instructions

Following each school visit, I would like you to write a short reflection on your experience and keep these in a reflective journal.

As a rough guide, each entry should be about 500 words, but there is no strict word limit.

For every journal entry, you should try to address some/all of the following:

- What were the learning outcomes for the lesson?
- What kinds of activities were the children involved in during the lesson?
- Did you think the lesson outcomes were achieved? Why/why not?
- What did you discover about *how mathematics can be taught effectively* from this lesson?
- What did you discover about *how students best learn mathematics* from this lesson?

Is there anything from this lesson that you will take into your own classroom practice? If so, what is this and why do you think it is something you want to include in your own teaching?

Figure 1. Instructions given for the writing of a reflective journal

The questionnaire

1. What did you learn from participating in this program?
2. Please comment on the following aspects of the program:
 - i. Observing lessons taught by the class teacher
 - ii. Observing lessons taught by fellow pre-service teachers and providing feedback to them
 - iii. Working with a partner to plan and teach lessons
 - iv. The feedback you received on your lessons from fellow pre-service teachers
 - v. Writing up a personal reflection on your lessons.
3. What aspects of the program do you intend to adopt in your teaching of mathematics?

Figure 2. Questionnaire items

The semi-structured interview

1. Looking back, what do you think were the benefits of participating in the program?
2. Were there any unintended negative consequences?
3. Were you able to implement any of the *Working Mathematically* strategies from the project in your other pracs? If so, how?
4. Do you think your participation in the project helped you in your later professional experience placements? If so, how?
5. What aspects of the program do you intend to adopt in your future teaching of mathematics?
6. Any further comments?

Figure 3. Questions forming the basis of the semi-structured interview

An initial reading of the data revealed many commonalities in the responses of the PSTs. This was not surprising because, from the beginning of the specialist professional experience placement, they were mutually engaged in learning to teach problem-solving lessons. Because our research questions concerned their collective learning, responses are attributed to the community as a whole rather than the individual members of it.

We used a descriptive case study design (Yin, 2003) based on qualitative content analysis (Kohlbacher, 2006). The methods of analysis are described in Table 1. Initially we jointly read through all of the PSTs' written reflections, questionnaire responses, and interview transcripts and used a deductive method of coding whereby ideas or comments expressed by the PSTs were classified according to *a priori* concepts—Wenger's three dimensions of a CoP.

Disagreement over the classification of an idea or comment into one of the three categories was rare. When we disagreed, we discussed the idea in relation to Wenger's theory until agreement was reached.

Together, we examined the ideas and comments placed in the *shared repertoire* category to determine what it was that the PSTs were learning through reflecting on their co-planning and co-teaching of problem-solving lessons. The comments were tabulated and further categorised using open coding to identify common themes and determine the frequency of each theme, frequencies being indicative of the relative importance of a theme to the PSTs' learning. When each of us suggested a different theme for the coding of a comment or idea, the two themes suggested were combined under a more general theme. For example, the PST comment "it is important to ask the students questions rather than just tell them the best possible strategy" was categorised as "question students" by one researcher and "facilitate discussion" by the other. These two themes were combined into the category "Promote discussion of strategies". After the codes and their frequencies were finalised, they were grouped into two subsets: those related to teaching in general and those related more specifically to teaching mathematical problem-solving lessons.

Table 1.

The data sources and analysis methods used in relation to each research question

Research Question	Sources of Data	Analysis of Data
What prior learning and experiences supported the PSTs' mutual engagement in learning to teach problem-solving lessons?	Reflective journals Questionnaire Interview	Content analysis against Wenger's dimensions of practice
Through their joint enterprise in the activities of the learning community and their shared reflections, what did the PSTs learn about teaching problem-solving lessons?	Reflective journals Questionnaire	Content analysis against Wenger's dimensions of practice
What aspects of participating in the learning community did the PSTs intend to adopt in their future teaching of mathematics?	Reflective journals Questionnaire Interview	Content analysis against Wenger's dimensions of practice followed by open coding of themes Frequency data Descriptive examples

Results and discussion

Mutual engagement

Mutual engagement concerns how members of the CoP use their prior learning of the focus issue to learn with and from each other. Prior learning acts as a source of coherence in a CoP (Wenger, 1998) that supports the collaborative engagement of participants in the activities of the community. In discussing the mutual engagement of the PSTs in our study, we consider their prior learning about teaching mathematics through problem-solving lessons.

In common with the findings of Frid and Sparrow (2009), none of the PSTs in our study reported any first-hand experience of problem-solving lessons in mathematics when they were at school. As one PST wrote in her questionnaire, "Teaching problem-solving strategies was always a mystery to me, as I was not really taught this in my own school experience." Another PST wrote in her reflective journal:

Prior to my involvement with this program, I was always fearful of teaching maths and especially nervous with teaching problem solving. This is because I was never taught explicit problem solving strategies and did not enjoy maths in school.

The comment about not enjoying maths at school is concerning, though it is common among primary PSTs (Young-Loveridge, Bicknell & Mills, 2012). The PSTs in our study reported how the activities in EDUC258 were markedly different to the rote learning activities they experienced at school. They also expressed some apprehension about how they might successfully implement problem-solving activities in their own lessons. For example, one PST wrote in response to the questionnaire:

... after doing 258, I think that was the pivotal changing point but this really was the first time I actually thought about how I implement what we've learnt in the classroom. So I did intend to do it, but this was really okay now I know what I'm doing. I remember lots of people doing 258 were like 'Oh it sounds so amazing, but how do we actually do it'. So having some of those ideas from 258 and then seeing what was done in this classroom was really useful.

As noted by Bailey and Taylor (2015), participating in problem-solving activities and reflecting on the experience is an important aspect of developing PSTs' positive dispositions towards teaching through problem solving. They argued that this is a first step and recognised a next step to be PSTs' enactment of a problem-solving approach in the classroom. Our study confirms this but shows that there is a crucial intermediate step. In being a learner in a problem-solving lesson, PSTs gain first-hand experience of the benefits of problem solving and its potential as a productive approach to teaching mathematics. However, they have seen problem-solving lessons principally through the lens of a learner, rather than a teacher, so they may not have thought deeply about how to teach such lessons. We contend that PSTs need to observe an experienced teacher teaching problem-solving in the classroom and reflect on what they have observed before they are ready to teach such lessons themselves. Our study shows that this readiness can be more easily achieved with the support of their peers.

Shifting from the position of a learner to take on the role of a teacher in such lessons is a considerable challenge. The PSTs in our study could not envisage teaching a problem-solving lesson without first seeing it modelled, not only in their tutorials, but also in an actual classroom setting. A PST remarked in her interview:

I feel it's completely different when you learn the theory and then when you implement it in the classroom so that was really good to kick start. And especially with the maths side of it, being a maths program, I thought it was really good to see how to structure a maths lesson because I personally understand maths but teaching it was another thing and trying to find those strategies that worked.

The school visits of the learning community took place after the PSTs had studied EDUC258. Consequently, when PSTs visited the school they were able to build on both their theoretical knowledge of constructivism and their tutorial experiences of problem-solving activities. They could link their prior learning about problem-solving lessons to their classroom observations and reflections to explore how such lessons could be enacted with a group of school children. As one PST explained in her questionnaire:

This program provided me with my first observation of a maths lesson in a primary school of my degree. It gave me the opportunity to see the theoretical content of mathematics education in action.

We also believe it was important that the co-teaching was not formally assessed. This meant that PSTs were able to take pedagogical risks without fear of receiving an unsatisfactory professional experience report. PSTs therefore regarded the learning community visits as an opportunity to try-out some of the ideas they had learned about in their university studies, as the following interview comment shows:

Because it was my first experience in the classroom it was good to have that 'trial' before I went to my actual prac placements. So that was really helpful, seeing how the classroom worked and what we needed to do when we taught.

It was significant that these problem-solving lessons were the PSTs' very first professional experience because the learning community activities validated the theory which they had recently learned and this provided a reference point for their future classroom practice. Problem solving was legitimised as a useful approach to learning and teaching mathematics. For example, as one PST wrote in her questionnaire, "The program has shown me that collaborative discussion and learning for the students as well as pre-service teachers is very beneficial to learning." This suggests that early exposure to observing and reflecting on the problem-solving lessons of an expert teacher followed by peer-supported planning and teaching might serve as formative experiences in shaping PSTs' views about the value of problem-solving in learning and teaching mathematics.

Joint enterprise

The joint enterprise of a CoP relates to interactions among participants in shared activities and discussions. For Wenger (1998), negotiating the joint enterprise encourages a sense of mutual accountability among members that allows them to learn with and from each other. In the present study, the joint enterprise of the CoP arose mainly in opportunities for PSTs to engage in co-planning and co-teaching, peer observation, and peer feedback for problem-solving lessons.

Co-planning with a peer

The first activity for PSTs associated with co-teaching involved jointly planning the lesson with a partner. PSTs had very limited experience writing lesson plans in their university studies and this was the first time they had been asked to prepare a lesson that they would actually teach. PSTs described the process of planning a problem-solving lesson where students would be active participants in the lesson rather than passive recipients of the teachers' knowledge as "challenging" and "stressful". During the interview, one PST explained why lesson planning was so challenging:

We found it very difficult to create a lesson, especially trying to find activities that would be suitably challenging to the students. We also struggled with lesson organisation, trying to find which divisibility rules would be best introduced in which order, and what activities should be implemented into the lesson to scaffold these rules rather than just spoon-feeding the rules to the students.

Even though the task of planning a problem-solving lesson was challenging, the additional support from a peer was seen as "supportive" and "helpful" because it allowed PSTs to collaborate and share their thinking. One of the PSTs mentioned the value of peer support for lesson planning in her interview:

It was really beneficial to work with other students when planning a lesson because that was quite a daunting thing. You know, how you're going to sequence a lesson and just coming up with ideas to have a lesson that allowed the students to think about what they were doing rather than just writing down answers. So having other people to work with and generating ideas for lessons was really beneficial.

The above quotes also highlight how, right from the first lessons they taught, the PSTs were thinking carefully about how to plan student-centred lesson activities. That was despite the fact they recognised that planning a problem-solving lesson was far more challenging than would be required for a more traditional lesson. One PST responded in the questionnaire: "The program also allowed me to experience first-hand the effort and thought required to prepare a lesson, especially a lesson on mathematical problem solving". We suggest that PSTs adopted a constructivist stance in their planning because they had sufficient pre-requisite knowledge and skills from their participation in EDUC258 and because they had observed and reflected on successful problem-solving lessons taught by an expert teacher. The two problem-solving lessons they had observed from the classroom teacher set the tone for the PSTs' own lessons that followed.

We also suggest that a different level of commitment is required from PSTs when they collaboratively plan a lesson. When each person feels a sense of responsibility to their partner, they strive to make a positive contribution to the planning process. Such 'reciprocal learning' (Le Cornu & Ewing, 2008), which is a defining feature of learning communities, can often result in a better quality lesson. For example, one PST wrote in her reflective journal about the way she and her partner went about planning their first lesson:

We found the time and researched possible activities and information about the topic we were going to teach. We then talked our way through the lesson plan by actually acting it out as we would teach it; this way it felt more natural for when we would actually teach the lesson. Our first draft however was too simple for the class we were teaching, which was a fear we had whilst constructing the lesson, but after revising it and making necessary changes we were happy with the final product and I believe it made it better through the actual deliverance of the lesson.

The above quotation demonstrates another benefit gained from peer lesson planning: the ways that PSTs were able to critique their work and anticipate potential flaws so that they could revise their lesson. In a similar way, another PST reflected in her questionnaire on co-planning and noted how "having two people thinking of ideas meant we could often pull each other up on what would work and what wouldn't, which might not have been possible if generating the lessons by myself". Here too, we surmise that the earlier observations of the expert teacher played some part in raising PSTs' awareness of key features of problem-solving lessons in mathematics which facilitated their ability to critically reflect on their lesson plans.

Co-teaching with a peer

Working alongside a peer was also beneficial for PSTs while teaching their lessons. Co-teaching made the PSTs "less anxious" which "made my first experience much easier and more enjoyable". In practical terms, co-teaching provided support for each PST because they could rely on their partner to assist in managing the class and ensuring the lesson proceeded as planned. One PST spoke about this in her interview: "When we were teaching the lesson it was good to have that extra support to help you if we forgot the lesson structure and also to help monitor the students' understanding and concentration". Another PST made a similar point in the questionnaire: "During the lesson, if one of us lost our place, it was handy to have the other to fall back on if required. Overall, it was extremely beneficial, and I believe it resulted in well-structured, coherent lessons".

The impact of co-teaching went beyond the actual lesson. The PSTs believed that peer support helped them overcome their anxious feelings and ensure that their lessons were of a higher quality. This, in turn, gave them greater confidence to experiment further in subsequent lessons. One PST reflected on the value of peer support in her journal:

This lesson was an excellent first lesson, giving me great confidence in my ability to teach effectively. This first lesson has shown how an excellent lesson can be developed and implemented if you are creative in your teaching strategies, both in the planning and teaching stages. This lesson has served as a great encouragement, and I can't wait to teach again in a couple of weeks.

Peer observation

Opportunities for observing peers' lessons were another important feature of the learning community. PSTs regularly commented on the value of watching their peers teach because it was a "great way of seeing different teaching strategies". Peer observation also helped PSTs identify effective teaching techniques that they could incorporate into their own lessons. In her questionnaire, a PST wrote, "One thing that was really beneficial was being able to see their unique teaching style and see how I could adopt some of their own methods into my own teaching practices". PSTs also reported that their peers' teaching practices provided a benchmark that allowed the observers to identify their own teaching strengths and weaknesses, as the following interview comment shows:

Observing lessons of other students helped me see strengths and weaknesses more objectively and helped me understand the current capabilities of myself and my peers. It also helped me understand my own strengths and weaknesses and I felt it helped me improve my own teaching.

Interestingly, PSTs did not make such comments when they reflected on classroom teacher's lessons. In those instances, they discussed the teacher's pedagogical approach and identified specific features of teaching a problem-solving lesson they wanted to employ in their own lessons. However, PSTs did not make any comparisons between the teacher's methods and their own classroom practice. Perhaps this is because the teacher's lessons occurred before the PSTs had taught, but there are other responses from PSTs that could indicate an alternate reason. These comments relate to their recognition that the classroom teacher was at a different stage in her career to them. An interview comment from a PST hints at this difference:

They [peers] have some different ways of teaching. So she's [the teacher] got her set way because she's had all that experience. She also seemed a lot calmer.

We suggest that "all that experience" set the class teacher apart from the expectations PSTs had of themselves so they found it more realistic to make judgements about their own teaching skills in comparison to their peers. Benchmarking against peers was appropriate because "I know it was their first time teaching as well - so we were all just as nervous" and "It was a lot easier to relate to my own teaching skills and ideas since we were all in the same stage of our teaching experience".

Observing their peers' lessons provided a different kind of experience for PSTs—one that was just as valuable as watching an experienced teacher in action. Noting how much they could potentially learn from a colleague at the "same stage of our teaching experience", PSTs gained insights into their own teaching practice that they did not find when observing the expert teacher. The following questionnaire response illustrates this point:

It was good to observe these [peer] lessons to identify what areas I need to develop in or what I could also use since it proved effective during their lessons. It was a great opportunity to see what worked when teaching the lesson outcomes, what the students reacted best to in terms of

giving instructions and information, and it helped me to pinpoint what things I could do to improve my teaching.

Another important difference between the teacher's and PSTs' lessons was how the former were, not unexpectedly, of a much higher quality than the latter. So, lessons taught by peers provided more opportunities to identify aspects of the lesson that did not go well and to learn from them. Such opportunities were less commonly reported in the PSTs' reflections on the teacher's lessons. In her reflective journal, a PST wrote:

I really liked being able to observe how other student teachers taught. I think it was a great way of seeing different strategies, as well as highlighting to me problems that can occur in a classroom that I had not thought of. Furthermore, it highlighted to me things I sometimes did when teaching and how this may not be the most appropriate strategy to adopt.

The comment highlights the value of peer observation as a means of improving PSTs' pedagogy and it suggests that while PSTs can learn a great deal from observing an experienced teacher, there is also much to be gained by observing one's peers. Our results indicate that even PSTs who have very limited classroom experience have a lot to offer in support of their peers' learning.

Peer feedback

Giving and receiving peer feedback was promoted by encouraging PSTs to make observation notes using the Quality Teaching Framework (NSW Department of Education and Training, 2008). Some PSTs used elements of the framework as sub-headings to organise their notes, while others did not directly use the language of the framework though they did discuss many of its features.

There were relatively few comments from PSTs about the value of giving feedback to peers. One questionnaire response noted that providing feedback was beneficial because clarifying one's thoughts about what to write aided the process of reflection which could assist the giver as well as the receiver: "Furthermore, by providing them with feedback on what worked really well and what they could have improved on, it made me further reflect on my own pedagogy and how I could use my knowledge of what they had done to improve my future lessons".

PSTs commonly mentioned the benefits of receiving feedback from peers. They noted how written feedback was particularly useful because "I can always refer back to them". Feedback was sometimes consistent with PSTs' own self-reflections as it "matched a lot of the things I thought I, or both of us, could have done better". More often, however, the feedback prompted PSTs to consider alternative actions and viewpoints because "often they [peers] picked up on things that I never would have thought of or picked up on". Also, feedback could illuminate aspects of their teaching which PSTs had not noticed because they were so caught up in delivering the lesson: "It [peer feedback] gave me ideas about what worked and what didn't, especially when I had missed these myself. It helped me to develop my teaching". A PST made a similar point in her interview:

But it was also really helpful that the other students gave their opinions as well because sometimes it's really hard to know where you're at, especially when you're in the moment when you're teaching and you're trying to remember what you have to say and what sorts of things you have to do next. But, when you're on the outside, watching, observing someone else, it's much better and you'd be able to get another view or another perspective on it.

The above comment illustrates the challenge for beginning teachers who concentrate so much on their own actions that they fail to take account of whether or not students are learning (Star & Strickland, 2008). In her interview, a PST remarked that she had learned a great deal from

observing peers lessons, “especially because you actually asked us to write feedback, otherwise I don’t think I would have thought about it that deeply”. We suggest that opportunities for PSTs not only to observe their peers’ lessons but also to give written feedback on them could develop their ability to notice key features of their own and each other’s lessons. As they had done when discussing the value of observing peers’ lessons, as opposed to those of the teacher, PSTs noted the value of receiving feedback from a colleague at a similar point in their development: “Getting feedback from my peers was very informative as they are in the same position as I am and know what sort of things to expect or that I would have been thinking and feeling”. This comment underlies the importance of collaboration and feedback, even among novices. Since they are usually at a similar level of competence, the PSTs who provide feedback can offer useful perspectives on teaching to their peers. Importantly, in writing up the feedback for their peers, PSTs can also hone their skills in learning to notice.

Shared repertoire

The shared repertoire of the CoP is the participants’ experiences and reflections concerning their common endeavour, namely their learning to teach mathematical problem-solving lessons. As PSTs who were preparing to become generalist primary teachers, they were also interested in the broad business of teaching. Table 2 shows the various themes that group members wrote about in their reflective journals and questionnaires, together with the frequency with which these themes occurred. We report relative frequencies (percentages) as these give some measure of the perceived importance of each theme.

Table 2.
The frequency with which themes were addressed by the PSTs in their reflection journals and the Questionnaire

Theme	Frequency	%
<i>Teaching mathematical problem-solving lessons</i>		
Remind students of Newman's prompts	28	13%
Promote discussion of strategies	26	12%
Encourage collaborative problem solving	16	7%
Ask students for their reasoning	14	6%
Activate prior mathematical knowledge	11	5%
Explain the mathematics clearly	8	4%
Choose rich tasks	7	3%
Clarify the question	6	3%
Encourage students to generate their own strategies	6	3%
Demonstrate a strategy	6	3%
Write the question and student responses on the board	4	2%
Ask students to record strategies	3	1%
Help students make connections	2	1%
<i>Teaching in general</i>		
Structure and time the lesson well	23	10%
Provide scaffolding when needed	9	4%
Get all students involved	8	4%
Make the purpose of the lesson clear	8	4%
Ensure every student is listening or working	6	3%
Plan for interactivity	5	2%
Emphasise important points	5	2%
Know your students	4	2%
Speak slowly and clearly	4	2%
Plan for differentiation	4	2%
Put students in small groups	4	2%
Be patient and respectful of students	2	1%
Give praise	2	1%
Rehearse the lesson	2	1%

The first 13 themes in Table 2 (61% of the reflections) relate specifically to teaching mathematical problem-solving lessons and the tasks used, while the remaining reflections (39%) were on themes that relate more generally to the practice of teaching. In a similar study of secondary mathematics PSTs' reflections on problem-solving lessons (Cavanagh & McMaster, 2015), we found that only 49% of their reflections related to teaching methods and tasks. This could be partially explained by the secondary PSTs placing a greater emphasis on classroom management than the primary PSTs (18% for the secondary PSTs compared to 6% for the primary PSTs).

The most frequent theme of a general nature was the structure and timing of the lesson (10%). Our earlier work (Cavanagh & McMaster, 2015) found a similar percentage for secondary PSTs' reflections on this theme. Although the structuring and timing of a lesson is important to all teaching subjects, it is particularly important in a mathematics lesson that the essential mathematical generalisations are reached and reinforced at the completion of the lesson.

In relation to teaching a problem-solving lesson, the primary PSTs placed greatest importance on the use of Newman's prompts (13%). Newman's prompts (see White, 2005) are questions the teacher asks to prompt students to read and comprehend the question, think of a strategy, talk through the strategy, record the solution process and check it. Above all, the PSTs emphasised Newman's prompts to comprehend the question by breaking it down and putting it in their own words.

Almost as important as Newman's prompts was the theme of promoting the discussion of strategies (12%). These were the two dominant themes of PSTs' reflections right from the first day when they observed the experienced teacher's lesson:

The learning outcomes for the lesson were to learn how to strategise when solving mathematical problems. Also, to interpret and simplify the question by restating it into their own words and, in doing so, identify the key words (what you need to find out and what you need to know before finding the answer).

The themes of encouraging students to solve problems collaboratively (16%), questioning students' reasoning (14%) and encouraging students to generate their own strategies (6%) are all closely related to the theme of sharing strategies. In the questionnaire completed at the completion of the program, all the PSTs restated the importance of class discussion and also mentioned the value of using of Newman's prompts to break down the problem, for example:

By breaking this problem solving process down it one, assists the students in there process of thinking and two, helps the teacher to see where, if anywhere, the students are having trouble in their problem solving abilities.

In her final interview, one PST said she modified Newman's prompts when teaching mathematics in a subsequent school experience:

I sort of extended it a bit. I did like a very similar but longer process: visualise the problem, understand the question, clarify it (change it into your own words), simplify it (pull out which operations you're going to use and what numbers you're working with), strategies, solve, and check. So I extended it, but it was good having that as a start-up.

PSTs' future intentions

The learning community was designed primarily so that PSTs could learn how to teach problem-solving lessons in mathematics; however, the participants also identified other benefits that they intended to implement. These related mainly to the impact of peer observation and the value of self-reflection as aids to improve their classroom practice. One PST wrote in her questionnaire that peer feedback was "really useful ... and I will definitely be taking all the feedback I received on board in my future teaching". Another wrote that her reflective diary entries "will always be there so I can refer back to them and base later lessons on them so to improve my preparation and deliverance of lessons".

PSTs reported that their experiences had demonstrated the value of noticing and reflecting and they commented about how they intended to adopt these practices in the future. As one PST wrote in her questionnaire, "This program has really made me think about my teaching ... it has emphasised the importance of collaboration and reflection in teaching and I hope to be able to include these aspects into my future teaching". Another PST's questionnaire response

noted how the practice of keeping personal reflections on teaching “really made [me] think about how I could improve my teaching. It has really helped me with my planning and implementation of lessons on my current prac as I have a foundation to improve on”. And another PST wrote that the act of reflecting on lessons was “of great use to me as it was really beneficial to see how effective the lessons actually were. This is something I want to continue in my own practice”.

In the interview conducted at the end of their undergraduate studies, PSTs looked back over all of their professional experience placements and looked forward to their future teaching careers. They all made similar comments about the value of peer observation and personal reflection. One PST commented about her reflective practice, noting that it was “really helpful in just sitting down and taking a moment to think about what just happened and what was effective and what wasn’t effective. Even for myself. So I was much more diligent with reflection on prac”. Another PST discussed her intention to adopt peer observations of her lessons:

... it’s something I’d definitely like in the future – having someone else just sit in and watch every now and then to see if you could do something else. Because even in a lesson when you think it went really well, it’s still good to hear another perspective.

Conclusion

In our study we adapted a pedagogy of collaborative learning to primary PSTs learning how to teach mathematics problem-solving lessons. Our results demonstrate the value of PSTs firstly becoming familiar with the theoretical basis of constructivism and attempting problem-solving activities themselves in tutorials. These activities are enhanced by observing and reflecting on the work of an experienced teacher teaching problem-solving lessons with a class that is accustomed to such lessons. This opportunity enabled the PSTs to notice the most salient aspects of a problem-solving lesson and its impact on students’ learning of mathematics. However, this is a small-scale study of four volunteers so there is a need for future research studies to investigate a learning community for a larger group of PSTs. Also, our participants were all high-achieving students so future studies could explore whether similar results are possible with a more heterogeneous group.

The main implication from our study is that PSTs are best placed to teach problem-solving lessons if they have experienced them as learners, observed an experienced teacher teaching problem-solving lessons, and are then supported by peers in a learning community. We recommend that mathematics teacher education courses adopt this structure in their professional experience programs. With these opportunities in the initial stages of a professional experience program, PSTs are more likely to develop pedagogical practices that support student learning through problem-solving, and have the confidence to implement a problem-solving approach in their future teaching of mathematics.

Acknowledgement

The authors acknowledge the detailed feedback and valuable suggestions provided by the reviewers.

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