Learning about teaching through research and vice versa: Towards developing methods in graduate coursework

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Received: 7 March 2017 Accepted: 23 January 2018

Abstract: The research on methods used in graduate mathematics education courses is limited, however existing groundwork suggests that curriculum should provide students with experiences that align with the practices of mathematics education researchers. At the same time, calls to connect mathematics education research and classroom practice have been articulated both within and outside the literature on the preparation of mathematics education researchers. This study describes a process that we call learning about teaching through research and vice versa (LTR). Specifically, the process involved mathematics education graduate students (who already hold teacher certification) engaging in a mathematical task, reading a research paper about the same mathematical task, and then completing an assignment based on viewing video data from a school classroom where middle school students engaged with the same task. Phenomenography was used to analyse written survey data and report that graduate students experienced the process as teachers, researchers, and teacher-researchers. The results indicate that the implemented methodology 1) offered students an opportunity to experience practices similar to those in which mathematics education researchers engage while pursuing scholarly inquiries, and 2) provided a setting where students learned about teaching and mathematics education research. Finally, the results support the claim that the LTR methodology acts as an example where research and practice enhanced one another and thus connect research and practice.

Keywords teacher learning · professional development · graduate coursework in mathematics education

The disconnect between mathematics education research and classroom practices

The lack of influence of research on classroom practice has been discussed widely and appears in several contexts within mathematics education research. For instance, Bishop (1998) noted researchers tend not to focus on teaching approaches that support mathematics learning. He raised concerns about ‘‘researchers' difficulties of relating ideas from research with the practice of teaching and learning mathematics’’ (p. 30). Similarly, Boaler, Ball, and Even(2003), while discussing the preparation of future mathematics education researchers, expressed concerns of the greater community that “researchers focus on questions that are not crucial to those working in practice and produce recommendations that are not useable” (p. 496). Similarly, in teacher education the lack of influence of research on practice becomes a major dilemma. Johnson (2010) found that practicum/field experiences have proven to be the most influential factor in pre-service teachers' future teaching, yet there is often a mismatch between what is discussed and promoted in course work and what students observe in practicum (Beswick & Muir, 2013). Discussion of a disconnect between educational research and classroom practice can also be found in the literature on teacher-researchers. For example, Jarowski (1998), in the discussion about her collaborative work with teachers, cites Hargeaves's observation that the gap between research and practice "betrays the fatal flaw in educational research" (p. 9).

Although some of these citations date from over a decade ago, surprisingly little has changed, as demonstrated by the literature on teaching interventions, and Schoenfeld's (2014) work on "powerful classrooms." These bodies of emerging research typically draw claims about their worth by discussing the disconnect between research and practice (e.g. Schoenfeld, 2014; Stylianides & Stylianides, 2013). The need for investigating ways of connecting research and practice is clearly laid out in the mathematics education literature, and there is clear demand for the research community to direct attention towards methods of bridging research and practice.

Here, we aim to weave together a discussion about research and practice by examining a methodology/process used in a graduate mathematics education course that incorporates and extends elements that have shown promise in connecting research and classroom practice. In particular, we describe a method that involved graduate students 1) engaging with a mathematics task, 2) reading a theoretical paper about the task, 3) watching raw video of grade six students engaging with the task, 4) identifying events within the video that exemplify the constructs from the theoretical paper and finally, 5) choosing events that best exemplify the constructs within the theoretical paper and crafting written arguments that convince others that the chosen video evidences the construct. We have come to call this process *learning about teaching through research* (LTR) and vice versa. We used phenomenography to analyse written survey data and report graduate students' experiences with the method/process. Specifically, we examine the question: *In which ways did graduate students experience learning about teaching through research and vice versa?* We claim that the obtained results indicate that our approach connected research and classroom practice, as graduate student participants developed their teaching and research practices through the same process.

Towards connecting research and classroom practice

Recommendations and compelling arguments that lay the groundwork towards bridging the gap between research and classroom practice may be found in several areas of mathematics education research literature. For instance, Schoenfeld's (2014) first fundamental assumption in his research-and-practice dialectic is that "Research and practice can and should live in productive synergy, with each enhancing the other" (p. 404). He writes that, in his work, including that on problem solving, frequently theory suggested the implementation of ideas, and the teaching experience nourished his intuition for theoretical explorations. Specifically, Schoenfeld (2014) writes that his theoretical work often "suggested avenues for the improvement of practice" (p. 408).

Research studies focused on teacher education through the use of video have achieved promising results in addressing the gap between theory and practice (e.g., Blomberg, Renkl, Sherin, Borko, & Seidel, 2013; Lin, 2005; Taylor, 2002). Video has allowed teacher education instructors to provide their classes with common events to discuss as the technology allows instructors to identify and share classrooms processes more aligned with current research. Unfortunately, there is often a mismatch between what pre-service teachers observe or experience in their field/practicum placements and what is being suggested in the university course work. However, Beswick and Muir (2013) point out that one of the challenges in the further implementation of video in teacher education courses will be in helping pre-service teachers "reflect deeply on their own and others' teaching practice" (p. 50). In other words, learning is related to reflection and video becomes a valuable tool in connecting research and practice.

Literature that discusses connecting research and practice typically includes notions of teaching being more formalised and also offers ways forward. For instance, in Jarowski's (1998) collaborative work, one of the teachers said "it seemed to me that any teacher is constantly engaged in research. Anything that you do, if you try to learn from it, that's research. It's just that what we're trying to do here is more formalised research" (p. 6). Mason and Davis' (2013) notions of knowing, with respect to teachers, have a similar formalised tone that becomes apparent when they state that "the responsible teacher has a theoretical frame by means of which to justify his or her choices of actions" (p. 191). Suggestions about moving towards connecting research and classroom practice can be found in discussions that compare the ways teachers communicate their ideas and how ideas from "formalized research" are communicated in research journals. For instance, Lester and Wiliam (2002) citing Schwandt (1995) speak about how the lack of "relevance of most educational research for teachers and other practitioners can be attributed to how members of these communities communicate their ideas" (p. 493), meaning that it can be difficult for teachers to read research papers, make sense of the ideas, and relate them to their own context. Indeed, it is not surprising that teachers may be experiencing difficulty in reading research papers, as reading research papers is challenging even for graduate students in mathematics education (Nardi, 2015). Lester and Wiliam (2002) assert that to attend to the research and classroom practice divide, we [researchers] should

> promote a renewal of a sense of purpose for our research activity that seems to be disappearing, namely, a concern for making real, positive, lasting changes in what goes on in classrooms. We suggest that such changes will occur only when we become more aware of and concerned with sharing of meanings across researchers and practitioners (p. 496).

Teacher learning, reflection and moving towards conducting research

Teacher learning is often studied in the context of teacher knowledge and often leads to ideas about reflection in the context of professional development and formalized mathematics education research. Shulman (1986) defines knowledge for teaching as "the most useful forms of representation of [topics], the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others" (Shulman, 1986, p. 9). He further states that, because there is no one single most powerful form of representing the subject matter taught, "the teacher must have at hand a veritable armamentarium of alternative forms of representation, some of which derive from research whereas others originate in the wisdom of practice" (Shulman, 1986, p. 9).

Hill, Ball, and Schilling (2008) build on Shulman's work by re-conceptualizing his ideas as Mathematical Knowledge for Teaching – a dual category consisting of subject matter knowledge and pedagogical content knowledge (PCK). These, in turn, are subdivided into several knowledge domains, one of which Hill et al. (2008) identify as knowledge of content and students (KCS), i.e. "content knowledge intertwined with knowledge of how students think about, know, or learn this particular content" (p. 375), more specifically, the "conceptions and the preconceptions" students bring with them to the study of mathematics, and the ability of teachers to recognise these (Hill et al., 2008, p. 375).

Park and Oliver (2008), attempting to re-conceptualise pedagogical content knowledge (PCK) as it relates to science teaching, found that teachers develop PCK through "reflection-in-action and reflection-on-action" (p. 261). Parker and Oliver draw on Schon's (1983, 1987) work to define knowledge-in-action as a response to "an unexpectedly challenging moment" in a "teaching circumstance" and knowledge-on-action as a re-organization of knowledge after a teaching action has occurred as a consequence of teachers' reflection. Park and Oliver (2008) state that teachers' reflections serve as a major vehicle for improving both pre-service and in-service teachers' PCK (p. 281).

In mathematics education research specifically, Mason and Davis (2013) consider knowledge-in-action when discussing their notion of in-the-moment-pedagogy. Mason and Davis (2013) write that "it is one thing to notice an absence of something from a learner but quite another thing to have a sensible pedagogical action come to mind when needed" (p. 183). They place an emphasis on "what comes to mind moment by moment when they [teachers] are planning or leading a lesson" (p. 182). These ideas can be traced back to Mason's work on the discipline of noticing (Mason, 2002), where there is a direct relation to reflecting, as well as connections made between research and practice.

The importance of reflecting and learning from experience is apparent in Mason's (2002) claim that

> the only strategy, the only action that human beings have access to in order to learn from experience is to become consciously aware of recent actions that proved fruitful and then to imagine themselves having this action come to mind in some similar situation in the future (p. 192).

He further explains that

> professional noticing is what we do when we watch someone else acting professionally (teaching a lesson, working with a client, leading a workshop, delivering a lecture or training session) and become aware of something that they do (a task they set, a pattern of speech they employ, a gesture they use, a question they ask) which we think we could use ourselves. Perhaps when reading a book or article [or completing an assignment], something is described or suggested, and we can see ourselves "doing that" in our own way in our own context (p.30).

It follows that learning occurs when one watches or reads about a strategy or concept, and then expresses one's intention to use the strategy or a concept in one's own context or practice.

Mason's (2002) ideas about learning and reflection, specifically his work on the discipline of noticing, also draw connections between research and practice. In particular, he describes how research articles can be used to enhance teaching and, specifically, how professional development can occur through "reading professional journals, and in following up research articles" (p. 140). Mason (2002) also makes direct claims that the discipline of noticing "can straddle the two worlds of professional development and research" (p. 149). He writes that "the Discipline of Noticing is conceived as a practice which can be increasingly precise and disciplined, turning professional development into research if that is desired" (p. 211). He talks about increased discipline in terms of being explicit about research questions, perspectives, the objects of study, analysis of data, and validation of the research claims. Mason (2002) argues that in professional development "there may be no need to convince others as long as one is personally satisfied" (p. 151), whereas research is a systematic inquiry and demands

> explicit attention to context including what other authors have said about the particular question or related questions, method employed, some form of validation or cross-checking, and some form of reporting of the results of that enquiry to other people so that it can be criticized, debated, challenged, and perhaps taken up by others (p. 151).

Lester and Wiliam (2002) present similar ideas when they discuss the differences between teachers and researchers in terms of personal convictions. Specifically, they discuss knowledge validation and talk about how teachers “often rely on personal judgments” (p. 492). This is starkly different from researchers’ practice of situating arguments in existing literature, employing established research methods or making arguments by analyzing data through conceptual or theoretical perspectives.

The connections between research and practice are clearly laid out in Mason’s work. In addition to suggesting that professional learning can occur through reading research papers, he also offers ways to evidence teacher learning and the learning of future mathematics education researchers through graduate student participants’ explicit expression of their intentions to use a strategy or concept in their own context.

Preparing future mathematics education researchers

The literature on preparing future mathematics education researchers, which would include research on teaching methods within post-graduate programs in mathematics education, is limited (Boaler et al., 2003; Nardi, 2015). In one of the few pieces that focus on the education of graduate students in mathematics education, Boaler et al. (2003) question the “teaching curriculum” of those programs and courses that present research knowledge and practices of research separately. Customarily, graduate students read about research and are told about the practices of researchers instead of experiencing them. In their chapter entitled "Preparing mathematics education research for disciplined inquiry: Learning from, in and for practice", Boaler et al. claim that researchers need to develop “attitudes that include being sceptical, being open to surprise, trying to prove one's ideas wrong and considering alternatives” (p. 493) and that the practices of mathematics researchers should be embedded throughout a program that prepares mathematics education researchers. Namely, these practices include “reading, formulating a research question, using data carefully to make and ground claims, moving from the particular to the general, considering mathematics, and communicating research findings” (Boaler et al., 2003, p. 497). The authors identify reading and formulating questions as practices of disciplined enquiry, and raise concerns regarding whether graduate students are reading for the “gist of an argument” (Boaler et al., 2003, p. 499) or to “examine closely what the author is doing” (Boaler et al., 2003, p. 499). Specifically, when mathematics education researchers read mathematics education research papers, they strive to make sense of the complexity involved in the research. For example, authors need to provide convincing arguments and ask questions that are situated in and framed by related literature. Reading mathematics education papers is an important task researchers engage in, and Nardi (2016) has answered Boaler et al.’s (2003) call to embed the practice of reading researcher papers in graduate coursework. She describes an approach that focuses on the analysis of the mathematics education research literature, which involves searching for relevant materials, critical reading, writing a literature review, and presenting or discussing research findings (p. 142). However, literature that investigates the embedding of the other practices is still lacking.

One practice Boaler et al. (2003) focus on is analysing data. They emphasise the importance of the decisions mathematics education researchers make when it comes to identifying which pieces of data should be analysed in greater depth and “which are less crucial” (p. 502). They clarify that mathematics education researchers must also consider mathematics which “involves sensibilities about what is important mathematically, the role of term, for example” (p. 509). They suggest that graduate students should be provided with more opportunities to experience “ways data may be explored to extract their complexity and of the assumptions behind such work” (p.505). Boaler et al. do not provide designs for a curriculum that focuses graduate student learning "in, from and for practice"; however, they do a commendable job of satisfying their claim that their aim is to lay the ground for such future work.

Prominent researchers have made calls for academics to direct efforts towards the preparation of mathematics education researchers, and it is this call to which we are responding. In this paper, we are presenting results of a study that aims to add to the "relatively novel and exciting area of research" (Nardi, 2015, p.152) on preparing mathematics education researchers. The study was designed to embed specific practices of mathematics education researchers such as analysing data to identify events, making decisions about which events are more crucial to analyse, crafting convincing written arguments to support claims, and reading research papers to make sense of what the author is trying to do.

The study

In this study, we investigated how graduate students experienced the process of learning about teaching through research and vice versa. The project aimed at developing a teaching methodology for training novice researchers in reading mathematics education research literature. Specifically, the research was guided by the question: *In what ways did graduate students experience learning about teaching through research and vice versa?*

For the purpose of the study, we chose a research article written by Mamolo, Ruttenberg-Rosen and Whiteley on "Developing a network of and for geometric reasoning" (2015). The article was chosen because it described attempts to create a theoretical model for restructuring advanced mathematical tasks for learners of varied ages, mathematical maturity, and strengths. As such, Mamolo et al.'s (2015) work contained the development of a theoretical framework necessary to support the proposed model. The framework combined several constructs such as scaffolding, key developmental understandings, and conceptual blending, and thus lent itself to the study of several theoretical ideas that graduate students could explore. Our approach to working with this task and the article had four phases: I) Experiencing the experimental treatment described in the research paper; II) Reading and discussing the research paper based on this experimental treatment; III) Watching video to identify and justify instances of constructs defined by the authors of the article in the conceptual/theoretical framework of their study; and IV) Crafting written arguments to demonstrate the identification at least one instance each of scaffolding, key development understanding, and conceptual blending in the video data.

In Phase I, graduate students engaged in a hands-on investigation of the optimisation problem outlined in the Mamolo et al. (2015) article: Given a sheet of paper, construct an open box with optimal volume with respect to the surface area of the box. Graduate students were invited to engage in developing a solution to this problem collaboratively and through experimentation. In Phase II, graduate students read and then discussed in class the Mamolo et al. (2015) article. When reading the article, the graduate students were asked to focus on the development of a theoretical framework as a foundation of the model proposed by the authors. More specifically, graduate students were asked to consider three theoretical constructs or

> theoretical underpinnings that inform [authors'] understanding of the mathematical content, pedagogical approaches, and cognitive processes of and for mathematics learning. These considerations formed the basis of [their] thinking and analyses of networked learning experiences (Mamolo et al., 2015, p. 486).

Mamolo et al. (2015) adopted the constructs under consideration in this study from the following sources: key developmental understandings (KDU's) (Simon, 2006), conceptual blending (Turner & Fauconnier, 2002), and scaffolding interventions (Anghileri, 2006). In Phase III, graduate students were to view video-recordings of middle school students engaged in the same learning activity the graduate students worked on in Phase I and read about in Phase II of this study. When viewing the videos, the graduate students' objective was to notice and record the "enacted instances" of the constructs (KDU, conceptual blending, and scaffolding) identified and integrated into the theoretical framework.

The process concluded with Phase IV when graduate students chose amongst the instances they identified as the most representative examples and crafted arguments to convince their peers that they had identified at least one instance each of KDU, conceptual blending, and scaffolding in the video data. The graduate students submitted these arguments and completed an online survey about their experiences. The scope of this paper does not permit a detailed discussion of the observations, but rather focuses on two qualitative questions from the survey, namely: "What is the purpose of mathematics education research?" and "How did your participation in this project influence your future reading of mathematics education [research] papers? (i.e., What have you learned? What are you left thinking about? Wondering? Asking? What might you do differently when you read research in the future?)"

Participants

The six study participants, Selena, Marc, Emma, Chloe, Nora, and Claire, were recruited from a pool of graduate students in a course entitled "Research in Mathematics Education." The course was taught by the second author. At the time of the study, all graduate student participants were enrolled either in a Masters of Education or a Masters of Mathematics for Teachers degree program. It is worth noting that the Masters of Mathematics for Teachers degree program is offered through the Mathematics Department. Students in both degree programs can specialize in mathematics education by concurrently completing a Diploma in Mathematics Education. All study participants were novices in the sense that none of them had conducted any research on their own and for some this was their first experience engaging with mathematics education research.

For Marc and Selena, this was their very first graduate level mathematics education course. Marc was in his first year of the Masters of Education program. He had no prior experience in reading research papers or engaging in research; however, he had more than three years of experience teaching high school mathematics. His reason for enrolling in the graduate program was to learn more about the teaching and learning of mathematics. Selena had no prior research experience. She worked in school administration and curriculum planning, but held credentials that would allow her to teach all school grade levels. Selena enrolled in the program because she "missed the academic atmosphere" and also believed that it would further her career, as she aspired to become a school administrator.

Emma was in the Masters of Mathematics for Teachers degree program and most of her courses were mathematics content focused. This was her second course that had education content. Emma had been a high school mathematics teacher for over two years. She was in the middle of her coursework and was looking forward to writing a research survey paper in partial fulfilment of her Diploma requirements. She undertook graduate studies for professional development.

The other graduate student participants (Chloe, Nora, and Claire) were finishing the last of their course work in a Masters of Education program. They had all completed two graduate courses in mathematics education prior to enrolling in the course in which the study took place. Chloe is certified to teach in all school grades. She had some teaching experience and significant experience as a research assistant. When asked why she decided to undertake graduate studies, Chloe wrote that it might help her become a better teacher and that she wanted to gain knowledge of the latest research in education. She also indicated that she read no mathematics education papers in her undergraduate degree in education. Nora was also at the end of her studies. The "Research in Mathematics Education" course was the last of the seven courses in her program, and she was about to begin working on her major research project. Nora had more than two years of experience teaching college level mathematics even though her teaching qualifications were in the primary/junior and intermediate/senior divisions. Her reason for taking on graduate studies was to further develop her knowledge of the field [mathematics education]. Claire was qualified to teach in the intermediate/senior division and had taught high school mathematics for over two years. Claire's reasons for undertaking graduate studies were to further her knowledge of teaching and learning.

Data Analysis

We employed a phenomenographic approach in data analysis. Phenomenography is a qualitative research method that investigates how people either conceptualise or experience a phenomenon. Phenomenography is rooted in empirical needs as opposed to theoretical needs (Åkerlind, 2012) and finds its history in the work that investigates the ways in which students conceptualise learning (Marton, 1976; Marton & Säljö, 1997; Säljö, 1975). The research approach is focused on developing qualitatively different categories of ways of experiencing a phenomenon from the collected data by employing iterative procedures. It is unlike phenomenology in that it is not about "find[ing] the singular essence, but the variation and the architecture of this variation in terms of the different aspects that define the phenomena" (Marton & Booth, 1997, p. 117). It is worth noting that phenomenography has a long history in higher education and has been used in many studies (Entwistle, 1997).

We began to analyse the data by independently reading the survey responses to "get a feel" for the data and to identify different trends. In the phenomenographic tradition, we looked for similarities and differences in the way participants responded to the survey question under consideration. We then compared our preliminary findings. On the first reading of the results, two categories emerged relating to the ways in which graduate student participants experienced the approach. While answering one particular survey question ("How did your participation in this project influence your future reading of mathematics education [research] papers?"), the study participants made comments from what seemed to be two distinct points of view: that of a teacher, and that of a researcher. Graduate student participants' responses were classified as researcher responses if they contained research-specific language and terms/ideas such as: as a researcher, theoretical underpinnings, collecting data, filtering data, research questions, etc. Graduate student participants' responses were classified as teacher responses if they contained teaching-specific language and terms/ideas such as: as a teacher, in my classroom, learning activities, teaching strategies, applying to classroom situations, etc. Typical responses are organised in the table below (See Figure 1).

		Ideas
Researcher responses	As a researcher…	Theoretical underpinnings… Collecting data… Filtering data…
Teacher responses	As a teacher… In my classroom…	Learning activities… Teaching strategies… Applications to classroom situations…

Figure 1. Examples used to represent ideas.

Marton and Booth (1997) write that phenomenographic data analysis is an iterative process of refinement of categories until consistency is achieved. With that in mind, further examination of data revealed some difficulties in categorising particular comments as teacher comments or researcher comments. In answering the survey question, some of the graduate student participants used research-related language and ideas and teacher-related language and ideas simultaneously in the same phrase. To refine our categories, we created new categories with particular defining qualities, as outlined below.

Findings

Graduate student participants experienced the process of using classroom videos in making sense of mathematics education literature in three overarching ways: (1) as a teacher, (2) as a researcher, and (3) as a teacher-researcher. These categories all involve graduate student participants' descriptions of what they noticed throughout the process, but they have distinct features. We determined that graduate student participants experienced the process as teachers if their comments focused on classroom teaching and learning from a personal point of view and did not include a direct reference to research literature. We determined that they experienced the process as researchers if their comments focused on what they noticed about conducting research and did not include a direct reference to teaching practice. Finally, we determined that they experienced the process as teacher-researchers if their comments included direct references to both research and teaching.

In our analysis, we illustrate each category with direct quotes that show typical graduate student participants' responses within that category. These categories may have emerged from the experiences of one participant, a few participants, or all six participants. Identifying the number of comments within each category or determining how many students made comments that fit within that category is not included in our analysis since phenomenography addresses the range of experiences and does not attempt to determine the most common experiences.

Category 1: Graduate student participants experiencing the process as teachers

Statements that fell under this category had a sense of graduate student participants being personally convinced of certain factors that influence student learning and their teaching practice. The comments had a sense of personal satisfaction or personal knowledge and did not include discussion of specifics from research papers. One example is Chloe's comment: "I know that working collaboratively in groups and then sharing information between groups allowed students to excel in their understanding." This comment indicates a personal sense of knowledge, and there is no direct reference to any literature. Moreover, the paper that Chloe read does not specifically use the terms "collaboration" or "excel in understanding," so it is reasonable to assume her description is based on her classroom experience. Chloe does not reference any other literature or identify characteristics of what it means to excel in understanding, or describe how this was demonstrated in the video data. All graduate student participants' comments in this category shared these characteristics, that is, the language and concepts were not explicitly discussed in Mamolo et al. (2015) and the comments were primarily experiential and anecdotal, with no discussion of defining qualities or other relevant literature, and there was no detail regarding how these ideas were evidenced in the video data.

Another statement that we placed in this category also comes from Chloe: "I did not anticipate how students would conceptualise the problem. I would not have assumed students would go so far as to compare the ratio of the height to the base length." Again, this comment does not refer to any particular theoretical construct from the paper, is not situated in research in general, and does not contain a reason as to why Chloe did not think students would conceptualise the mathematical task in terms of a "ratio of the height to the base length." All comments that demonstrated graduate student participants experiencing the process as teachers involved the word "I," indicating a sense of personal belief or conviction not rooted in Mamolo et al. (2015) or other literature.

Category 2: Graduate student participants experiencing the process as researchers

This category includes graduate student participants' remarks regarding experiences that led them to gain insights about conducting research. These statements do not suggest an intention to apply these insights in classroom situations. More specifically, these comments identified or reflected on methods for data collection, data analysis, the process of writing a research paper, or general questions on existing and future research.

Some of the comments were more general and spoke about the process and time devoted to crafting a research paper. Graduate student participants noted that they "learned about the efforts that are put into producing a mathematics education paper" (Emma) and realised that "a great deal of data is created from just a few minutes [of video]. The data is then filtered by the researcher's theoretical framework and research question(s)" (Chloe).

Others spoke directly of methods used for data analysis and what types of strategies they had learned: "I have learned how to analyse and track data in an easier fashion. By this, I mean that by using the time stamps as a starting point, I was able to determine what pieces were more relevant than others.… In the future, I may want to read up on different ways to analyse data, which will give me more practice at examining data.… I think I will also view the data more than once, to really see if I am able to pin point the same instances I saw the first time or if I find something different" (Nora).

There were also comments that focused on questions, both the generation of research questions and questions about existing research. There were comments that tested the limitations of the theoretical paper by providing specific details identifying potential concerns or suggesting future directions, as in Nora's case: "I am left wondering how this type of study would differ from another class (e.g. a different grade level)." Similarly, Chloe also suggested further directions of research by saying:

> I am left wondering to what extent collaboration promotes a better understanding. I would like to see the same activity given to students working individually with no collaboration and then compare the two studies and then see how collaboration has an impact on student understanding.

All of the comments in this category are different from those in Category 1 because they do not convey a sense of personal conviction or knowing but do include ideas and reflections on their current experiences with analysis methods, how these may inform their future research, and suggestions for directions in further related research.

Category 3: Graduate student participants experiencing the process as teacher-researchers

Comments in this category spoke directly to constructs from the theoretical paper selected for this project or other research and related them to classrooms or graduate student participants' personal teaching experiences. These ideas could not be classified under experiencing-the-process-as-a-teacher because they contained ideas from research, and they could not be classified under experiencing-the-process-as-a-researcher because they described classroom work or experience. Comments in this category focused on what the paper or research/researchers have to offer classroom practice, what classroom practice has to offer graduate student participants' reading of research papers, or specifically addressed the overlap/connection between research and practice and how classroom practice and research could enhance one another.

Graduate student participants' comments outlined multiple ways that research can enhance teaching practice. For example, Selena said that

> [Engaging in the process] has made me aware that sometimes the most relevant teaching strategies and applications of the paper may be hidden within the theoretical framework, as was the case with the KDU's, conceptual blending, and scaffolding.

Other comments spoke more generally about how mathematics education research can or should influence classrooms. For example, Chloe said that "teachers have the backbone of a community of critical thinkers [researchers] providing them the support they need to meet the needs of students." She explained: "Simply teaching based on what 'feels good' is like flying a plane with no dashboard instruments." Graduate student participants indicated that they felt that the purpose of mathematics education research was to help teachers address the concerns of students by saying things like "research is a great way to give us (teachers) tips" (Nora), and "all education should be founded on research findings…. It [mathematics teaching] needs to be guided by research, which ideally analyzes and compares various methods, strategies, ways of learning, student development" (Selena). Emma was very explicit when she said the purpose of research is "to support educators in their practice." There were also ideas about research provoking change by "develop[ing] better methods of teaching which ultimately improve our education system and the learning of our students" (Marc).

Graduate student participants spoke about what classrooms have to offer research. For instance, graduate student participants described how videos from classrooms (classroom practice) influenced their reading of the paper, as it helped them understand the concepts from the paper. Emma illustrates this by saying:

> I believe that I now have a better understanding of these underpinnings and will use them more thoroughly and meaningfully while teaching mathematics… after watching the videos, I believe that I will be more conscious of re-voicing, conceptual blending, scaffolding and KDUs in my own teaching.

Graduate student participants pointed to how practice can inform research through discussing the limitations of the research presented in the paper they read. For example, Marc said that "seeing students attempt to develop their visual/spatial reasoning in this exercise illustrates many difficulties the authors either failed to see or neglect to mention." Marc's comment questions the Mamolo et al. (2015) paper, but does not specify examples to support his critique, giving it a more personal tone. Marc spoke about how the videos influenced his reading of the paper and what classroom practice has to offer theoretical work when he said "putting concepts/theoretical models in practice shines a light on the real-life strengths/weakness of the actual model." Again, Marc's comments lack evidence to support his comments.

More complex statements spoke directly to graduate student participants' insights about the importance of connecting research and practice. For example, Claire commented on what the process offered both her future classroom practice and future reading of research papers. She stated that,

> when I read research in the future, I will think of the concepts in terms of student learning in my own classroom in order to develop connections between the research and the classroom, as well as to develop a better understanding of the research.

Similarly, Selena spoke of how the process helped her "learn to make connections between a theoretical framework, a piece of research, and the classroom application of the concept. In the future when reading mathematics education [research] papers I will be able to better bridge them with concrete classroom practices." Selena further explained her ideas by speaking about how research can influence beyond the classroom by talking about complex relations between research and practice. Specifically, she said that "educators, curriculum writers and policy makers can then be informed by the research and hopefully make the most effective decisions to help students learn. I see it as a sort of network (teachers, researchers, the ministry), connected, interdependent, informing one another and supporting one another." These and other comments falling under this category are different from the previous two categories, as they speak to both classroom practices and mathematics education research.

Discussion

While a mathematics education researcher could have perhaps anticipated the categories that emerged, the significance of our findings lay in the related literature and the background of our research participants.

In terms of background of the participants, it is useful to discuss Chloe's specific case, as Chloe's comments were used to exemplify the ways of experiencing LTR methodology as a teacher. While her comments (See Category 1: Graduate student participants experiencing the process as teachers) do exemplify personal conviction and personal judgment, this can be explained through her history. At the time of the study, she was completing a course-based Masters degree in Education and "Research in Mathematics Education" was only her third graduate course in mathematics education. To become a mathematics education researcher, she would have to complete another 4-year program and, in comparison to an established mathematics education researcher, she had very limited exposure to mathematics education research. According to her reflections, Chloe had no experience reading research papers during her undergraduate studies in Education. However, she believes that she should not be "rely[ing] on personal judgments" (Lester & Wiliam, 2002, p. 492) by explicitly stating that basing teaching on what "feels good"/(personal convictions) and not grounding teaching in research is like "flying a plane with no dashboard instruments." She extends her personal opinions about collaboration and understanding (that are found in the first category) by recommending a future study that could give her insight about how collaboration is related to understanding. This indicates that she is starting to consider alternatives (something mathematics education researchers do, according to Boaler et al., 2003) and wanting to make judgments based on "studies." It is not unreasonable that Chloe, like many teachers, may "rely on personal judgments" (Lester & Wiliam, 2002, p. 492) because she has not come into contact with research papers that specifically discuss collaboration as it relates to understanding or she is still learning how to read research papers. This is just where Chloe is in her learning (and perhaps where she should be as a Masters student), somewhere on a continuum between teacher and researcher, because some of her comments did appear in the last two categories. It is also important to note that all six participants had comments in the last two categories, thus none of the graduate student participants experienced the process only as teachers. All the graduate student participants are somewhere on the continuum between mathematics education researcher and teacher.

Below we highlight the significance of the results of graduate student participants experiencing the process as teachers, as researchers, and as teacher-researchers in relation to ideas in existing research about 1) the practices of mathematics education researchers, 2) learning from experience as researcher and teacher, and 3) connecting research and classroom practices.

Practices of mathematics education researchers

Practices and attitudes typical of mathematics education researchers, as outlined by Boaler et al. (2003), can be found in several of the excerpts provided in the Findings section. For instance, our data has clear examples of the development of attitudes that align with "being sceptical… and considering alternatives," attitudes of researchers that were identified by Boaler et al. (2003). Specifically, Marc is being sceptical of and starting to consider alternatives when he talked about possible limitations in Mamolo et al.'s (2015) work. Similarly, Nora and Chloe give clear indications that they are beginning to consider alternatives and develop questions when they talk about future studies that can be conducted to address the relationship between the task and grade level and the relationship between collaboration and understanding. This suggests that the LTR methodology presents students with an opportunity to consider alternatives and begin to develop research questions.

Along with evidencing the development of research questions and alternatives, the comments graduate student participants made are also indicative of developing other practices of mathematics education researchers. In particular, our data speaks to ways of engaging students in reading papers using methods similar to those of mathematics education researchers. For instance, Emma points to reading the article in more involved ways when she asserts that said she learned about the extensive efforts required to craft a research manuscript. She clearly has read the article in ways that align with "[examining] closely what the author is doing" (Boaler et al., 2003) and building an awareness of the culminating efforts involved when developing a convincing argument that is situated in and framed by related literature.

The practice of analysing data like a mathematics education researcher was encouraged by the design of the process, and was experienced by graduate student participants as described in our data. By asking students to choose events in the videos that best exemplified concepts from the research paper, the assignment design led students through the process that mathematics education researchers engage in when identifying which pieces of data should be analysed in greater depth and "which are less crucial" (Boaler et al., 2003, p. 502). Graduate student participants' experiences of developing this practice can be seen in Nora's comment: "I think I will also view the data more than once, to really see if I am able to pin point the same instances I saw the first time or if I find something different." Interpreting the findings through the prism of Boaler et al.'s ideas about practices and attitudes of mathematics education researchers leads us to the conclusion that the approach did, in fact, afford the graduate student participants an opportunity to experience practices and consequently develop the attitudes of mathematics education researchers.

Learning from experience

Framing learning through the tenets of "teachers' knowledge," and through Mason's (2002) idea that learning from experience occurs when we can envision ourselves enacting that strategy in our own context, leads to compelling evidence of the graduate students in this study learning about their own teaching, while also learning about the practices of mathematics education research. In particular, graduate student participants' comments of their intentions to use concepts from the paper in their own classrooms indicate learning about teaching, while comments about using strategies from the assignment in their own research indicate learning about the practices of mathematics education research.

For instance, Emma has learned about teaching from this experience because she clearly describes her plans to use concepts from the paper in her own context (i.e., her own teaching). Emma explained that she now has a better understanding of theoretical underpinnings and will be using them more meaningfully. She clarified by saying that, "after watching the videos, I believe that I will be more conscious of re-voicing, conceptual blending, scaffolding and KDUs in my *own* [emphasis added] teaching." These comments are clearly in line with Mason's ideas about learning and the professional development of teachers.

Chloe indicates she has also increased her awareness for future teaching, and in particular broadened her KCS (Hill et al., 2008), when she reflected about her preconceptions about not anticipating that students would compare the ratio of the height to the base length. The process has given her opportunities to identify the importance of this mathematical relationship and broaden her conceptions of what students will bring with them and how students may think about the task from the research paper.

Learning about research practices is also seen through the excerpts provided in the results. For example, Nora indicates that she envisions herself using similar data analysis strategies offered in the assignment when analysing her own video data by describing how she has identified particular data analysis techniques that have allowed her to track critical events within the data and plans on using them in future research contexts. Viewing the results through a lens grounded in Mason's work (2002) leads us to conclude that graduate student participants are learning about teaching, *and* learning about the practices of mathematics education researchers. Viewing the results through Hill et al.'s (2008) idea of KCS also indicates that the process is enhancing teacher practices.

Connecting research and practice

Arguments about how the LTR methodology connects research and practice can be made in relation to a) how graduate student participants enhanced (learned about) teaching through experiencing the practices of mathematics education researchers and enhanced (learned about) research through classroom practices (video from classrooms), and b) recommendations in the literature in how to connect research and practice.

There was clear evidence that methodology employed in the graduate course facilitated both learning about teaching and learning about the practices of mathematics education researchers. It is reasonable to say research and practice were bridged as graduate student participants learned about teaching through their experiences with the practices of mathematics education researchers and vice versa.

In fact, some of the comments describe explicitly how the process has offered graduate student participants strategies that will enhance both research and practice. For instance, Claire talks about concrete examples/strategies in her own classroom and how using concrete examples will help her in reading research papers. This comment exemplifies the kinds of productive synergy between research and classroom practices that student participants experienced when learning about teaching through research and vice versa.

There are two ways that the LTR enacts Lester and Wiliam's (2002) suggestion that to connect research we need a

> renewal of a sense of purpose for our research activity that seems to be disappearing, namely, a concern for making real, positive, lasting changes in what goes on in classrooms.

> We suggest that such changes will occur only when we become more aware of and concerned with sharing of meanings across researchers and practitioners (p. 496).

The LTR methodology 1) supports teachers' learning to read and make use of researchers' ideas and 2) educates future mathematics education researchers to view the purposes of mathematics education research as informing and supporting classroom practice.

Graduate student participants (all of whom have teaching certifications) commented (e.g., Claire, Emma, and Selena) about plans to use concepts from the paper in their own teaching practice, which clearly indicates that the LTR method supports teachers in making use of research ideas. Noteworthy is Selena's insight into the connection between teaching practice and theoretical framework: "sometimes the most relevant teaching strategies and applications of the paper may be hidden within the theoretical framework." Her comments indicate that the LTR methodology afforded her an opportunity to learn how to read research papers as a teacher, meaning that she read about theoretical constructs for the purpose of finding teaching strategies to be applied in her classroom. The significance of these findings is that they indicate how we can use methods such as the LTR to support the "sharing of meanings across researchers and practitioners" through research papers. In Selena's specific case, she recognized the shared place of the constructs within the theoretical framework of the paper she read and within her teaching practice.

The comments graduate student participants made clearly indicate how they all conceptualize the purpose of mathematics education research is to support classroom teaching. Some spoke directly about helping teachers address students' needs, and others inferred long-lasting change by talking about complex support networks between policy makers, ministries of education, research, and teaching. The LTR methodology has considerable potential in the preparation of future mathematics education researchers, as well as addressing the need for establishing research-enhanced classroom teaching practices.

Conclusion

Investigators in the field claim that embedding the practices of mathematics education researchers in graduate coursework shows promise; however, examples of this are thoroughly lacking. Over the course of this study, we involved graduate students in activities that mathematics education researchers practice when engaged in scholarly pursuits. Specifically, graduate students engaged in reading research literature, developing understanding of concepts at the foundation of a selected conceptual/theoretical framework, identifying critical events in video data, making decisions about which events are more crucial, and finally constructing convincing arguments to lay claims. Our findings show that, when course work involves detailed study of theoretical underpinnings through analysing classroom data and crafting convincing arguments to lay claims about constructs within a theoretical framework, graduate students demonstrate learning about both teaching practices and mathematics education research practices.

Furthermore, graduate students begin to think about the affordances that research has to offer classroom practice and vice versa, and the synergy between them. Our findings confirm that the LTR does address concerns about connecting research and teaching practices and that the LTR methodology being studied here is one that exemplifies Schoenfeld's idea that "research and practice can and should live in productive synergy, with each enhancing the other"(p. 404).

We hope that through reading this paper, you have envisioned how you can use and develop the LTR methodology in your context, as we are assuming that you, the reader, are a mathematics education researcher. Our desire is for the LTR methodology to become part of the existing research on methods in graduate level courses, providing an example which encourages others to pursue research about methods in graduate work. We hope that one day teachers, graduate students, and mathematics education researchers will be reading this paper and envisioning how they can use its elements in their own contexts.

References

Åkerlind, G. (2012). Variation and commonality in phenomenographic research methods. *Higher Education Research and Development*, *31*, 115–127. doi:10.1080/07294360.2011.642845

Anghileri, J. (2006). Scaffolding practices that enhance mathematics learning. *Journal of Mathematics Teacher Education, 9*(1), 33–52. doi:10.1007/s10857-006-9005-9

Beswick, K., & Muir, T. (2013). Making Connections: Lessons on the Use of Video in Pre-Service Teacher Education. *Mathematics Teacher Education and Development*, *15*(2), 27-51.

Bishop, A. J. (1998). Research, effectiveness, and the practitioners' world. In A. Sierpinska & J. Kilpatrick (Eds.), *Mathematics Education as a Research Domain: A Research for Identity* (an ICMI study) (pp. 33–45). London: Kluwer.

Boaler, J., Ball, D. L., & Even, R. (2003). Preparing mathematics education researchers for disciplined inquiry: Learning from, in, and for practice. In A. Bishop & J. Kilpatrick (Eds.), *International handbook of mathematics education* (pp. 491–521). Dordrecht, Netherlands: Kluwer.

Blomberg, G., Renkl., A., Sherin, M. G., Borko, H., & Seidel, T. (2013). Five research-based heuristics for using video in pre-service teacher education. *Journal of Educational Research*, *5*(1), 90-114.

Entwistle, N. (1997). Introduction: Phenomenography in higher education. *Higher Education Research and Development*, *16*, 127–134. doi:10.1080/0729436970160202

Hill, H., Ball, D., & Schilling, S. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, *39*(4), 372–400.

Jaworski, B. (1998). Mathematics teacher research: Process, practice and the development of teaching. *Journal of Mathematics Teacher Education,* 1(1), 3–31.

Johnson, D. (2010). Learning to teach: The influence of a university-school partnership project on pre-service elementary teachers' efficacy for literacy instruction. *Reading Horizons*, *50*(1), 23-48.

Lester, F. K., & Wiliam, D. (2002). On the purpose of mathematics education research: Making productive contributions to policy and practice. *Handbook of international research in mathematics education*, 489-506.

Lin, P. (2005). Using research-based video-cases to help pre-service teachers conceptualize a contemporary view of mathematics teaching. *International Journal of Science and Mathematics Education*, *3*, 351–377.

Mamolo, A., Ruttenberg- Rozen, R., & Whiteley, W. (2015). Developing a network of and for geometric reasoning. *ZDM Mathematics Education*, *47*, 483–496. doi: 10.1007/s11858-014-0654-3

Marton, F. (1976). What does it take to learn? Some implications of an alternative view of learning. In N. J. Entwistle (Ed.), *Strategies for research and development in higher education*, (pp. 32–42). Amsterdam: Swets and Zeitlinger.

Marton, F., & Säljö, R. (1997). Approaches to learning. In F. Marton, D. Hounsell, and N. Entwistle (Eds.), *The experience of learning* (pp. 39–59). Edinburgh, Scotland: Scottish Academic Press.

Marton, F., & Booth, S. (1997). *Learning and awareness*. Mahwah, NJ: Lawrence Erlbaum.

Mason, J. (2002). *Researching your own practice: The discipline of noticing*. Routledge.

Mason, J., & Davis, B. (2013). The importance of teachers' mathematical awareness for in-the-moment pedagogy. *Canadian Journal of Science, Mathematics and Technology Education*, *13*(2), 182-197.

Nardi, E. (2015). "Not like a big gap, something we could handle": Facilitating shifts in paradigm in the supervision of mathematics graduates upon entry into mathematics education. *International Journal of Research in Undergraduate Mathematics Education*, *1*(1), 135-156.

Park, S., & Oliver, J. (2008). Revisiting the conceptualisation of pedagogical content knowledge (PCK): PCK as a conceptual tool to understand teachers as professionals. *Research in Science Education*, *38*(3), 261-284.

Säljö, R. (1975). *Qualitative differences in learning as a function of the learner's conception of the task*. Gothenburg: Acta Universitatis Gothoburgensis.
Schoenfeld, A. (2014). What makes for powerful classrooms, and how can we support teachers in creating them? A story of research and practice, productively intertwined. *Educational Researcher*, *43*(8), 404-412.
Schon, D. A. (1983). *The reflective practitioner*. New York: Basic Books.
Schon, D. A. (1987). *Educating the reflective practitioner: Toward a new design for teaching and learning in the professions*. San Francisco: Jossey-Bass.
Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher*, *15*(2), 4-14.
Simon, M. (2006). Key developmental understandings in mathematics: a direction for investigating and establishing learning goals. *Mathematical Thinking and Learning, 8*(4), 359–371.
Stylianides, A. J., & Stylianides, G. J. (2013). Seeking research-grounded solutions to problems of practice: classroom-based interventions in mathematics education. *ZDM*, *45*(3), 333-341.
Schwandt, T. A. (1995, November). Justifying claims in evaluation: Something old, something new? Paper prepared as a response to the panel, "Justifying Claims in Evaluation," at the joint meeting of the American Evaluation Association and the first International Evaluation Conference, Vancouver, British Columbia.
Taylor, P. (2002). Implementing the standards: Keys to establishing positive professional inertia in preservice mathematics teachers. *School Science and Mathematics*, *102*(3), 137–142.
Turner, M., and Fauconnier, G. (2002). *The way we think: conceptual blending and the mind's hidden complexities*. New York: Basic Books.

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