Informing Programmatic-Level Conversations on Mathematics Preservice Teachers' Problem-Solving Performance and Experiences

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Problem solving is an important aspect of the mathematics classroom and teachers should work to promote problem solving in their classrooms. The purpose of this explanatory mixed-methods study was to examine mathematics preservice teachers' (PSTs) problem-solving performance and connect it with their K-12 and tertiary education problem-solving experiences. The present study is part of a research agenda involving the development and uses of the Problem-Solving (PSM) series. Research conducted in the present study was done within the context of a mathematics teacher education program evaluation and made connections between mathematics content and education faculties. PSTs from one mathematics teacher education program completed one problem solving measure from the PSM series. PSTs were also representatively sampled to participate in a structured interview investigating their problem-solving experiences. Based upon results from this study, we drew the conclusion that PSTs in this teacher education program need more problem-solving experiences in their education to prepare them for their future classrooms.

Keywords · Mathematics teacher education · content knowledge ·preservice teachers · problem solving · assessment

Introduction

Effective mathematics teaching includes the presentation of tasks that require high levels of cognitive thinking and problem solving (National Council of Teachers of Mathematics [NCTM], 2000, 2014). "Problem solving is an important way of doing, learning, and teaching mathematics" (Chapman, 2005, p. 225) giving students opportunities to use varied solution strategies and to solidify and extend what they know, as well as stimulate mathematics learning (NCTM, 2000). In addition, problem solving in the mathematics classroom engages students in a way that low-level skills emphasised in a test-driven curriculum cannot (Lesh & Zawojewski, 2007). It is for these reasons that problem solving is now recommended in standards and curricula around the world (e.g. Common Core State Standards Initiative [CCSSI], 2010; NCTM, 2000). Most countries involved in international comparison studies such as Trends in Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) include problem solving or reference to engaging students in solving real-world problems in their primary and secondary schooling. Examples include Australia (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2014); United States of America (CCSSI, 2010); Japan (Higher Education Bureau, Ministry of Education, Culture, Sports, Science and Technology, 2012); Cyprus (Xenofontos & Andrews, 2012) as well as the TIMMS education policy (Mullis, Martin, Goh, & Colter, 2016). Of the 56 countries in the TIMSS study in 2015, 37 (66%) explicitly stated problem solving in their secondary mathematics standards (Mullis et al., 2016). Some countries, Canada, Iran, and Jordan for example, mention problem solving as specifically being related to real-world exploration. Other countries, such as Chinese Taipei, Ireland, and Singapore, (Mullis et al., 2016), describe problem solving as a skill or process that students need to develop within the mathematics curriculum. In the United States of America, problem solving is mentioned in both the content and practice (process) standards, and there is mention of connections to problem solving within real-world contexts (CCSSI, 2010). According to the Australian Curriculum, problem solving is one of the four mathematical proficiencies during students' time in primary and secondary school (ACARA, 2014). From these documents it is clear that it is recommended that students should be experiencing problem solving in their primary and secondary education. Because problem solving is such an important aspect of mathematics teaching, it may be concluded that both in-service teachers and pre-service teachers (PSTs) need to be good problem solvers related to the mathematics content taught in their classroom (Chapman, 2005; Mullis et al., 2016). The purpose of this study is to explore mathematics PSTs' problem-solving performance as well as PSTs' problem-solving experiences during their K-12 and tertiary education coursework.

Literature Related to Problems and Problem Solving

Definitions and Frames

For the purpose of this study, problem solving is defined as, "the process of interpreting a situation mathematically, which usually involves several cycles of expressing, testing, and revising mathematical interpretations" (Lesh & Zawojewski, 2007, p. 782). In contrast, an exercise is a task meant to promote students' proficiency with a known procedure (Kilpatrick, Swafford, & Findell, 2001). Students can only engage in problem solving when they are given a problem, not an exercise, to complete (Schoenfeld, 2011). While the term 'problem' is used routinely in classrooms to refer broadly to all mathematics tasks, problem solving requires engagement with more than proficiency, which is described in the next paragraph.

In this paper, problems are framed using two distinct literature bodies. The first framing of a problem is that it is a task for which the solution strategy is not known in advance, requires critical thinking, and can be solved in more than one way (NCTM, 2014; Schoenfeld, 2011; Yee & Bostic, 2014). Second, and because this study involves word problems, the frame used for word problems is that the tasks are open, realistic, and complex (e.g. Bostic, Pape, & Jacobbe, 2016; Matney, Jackson, & Bostic, 2013; Verschaffel, De Corte, Lasure, Van Vaerenbergh, Bogaerts, & Ratinckx, 1999). Open word problems allow for multiple, viable strategies. Realistic word problems draw upon elements that encourage problem solvers to use their experiential knowledge and connect mathematics learned in and outside of the mathematics classroom. Complex word problems encourage problem solvers to persist and mathematically reason because the solution pathway and solution are not necessarily evident at first glance. These frames for problems and word problems share similarities and ground the present study in the problem-solving literature. A key reason for using both frames is that taken collectively they clarify the use of word problems within the present study. For instance, proving the square root of two is irrational may be a problem for some PSTs; however, it is not a word problem. The present study focuses on examining PSTs' problem-solving outcomes with word problems within the context of curricular standards they will use as teachers.

PSTs' Beliefs About Word Problem Solving

The focus of this study is PSTs' outcomes from word problem solving, which falls under a broad literature on problem solving. Thus, we draw upon literature related to studies that have investigated PSTs' beliefs about problem solving that included word problems as a focus in their study. Some used word problems as their context for study. For example Son and Lee (2016) found that 46 out of 96 elementary PSTs (48%) viewed problem-solving instruction as something to deliver after direct instruction, which might include steps to carry out a series of mathematical procedures. Xenofontos and Andrews (2012) compared PSTs' beliefs about problem solving and problem-solving instruction in

England and Cyprus using a broader lens of problem solving that included but was not limited to word problems. Results suggested that PSTs, especially in England, thought of problem solving as an activity done after learning about a new concept. Both studies framed problem solving as a process that a student cannot complete without first being given explicit instruction on how to solve the problem. This is not how the literature defines problem solving or characterises problem-solving instruction.

Other studies confirm that PSTs may have little understanding of the nature of problem solving as defined in the literature. Chapman (2005) found that approximately 83% of the PSTs in her sample initially thought word problems were verbal exercises. Whereas a verbal exercise is written in words and not mathematical symbols, it is still designed to promote efficiency with a known procedure and does not have the elements of problem solving described earlier. Chapman (2005) hypothesised that these beliefs may be due to the PSTs' lack of exposure to problem solving when drawing on their experiences in the K-12 classroom. However, Chapman was unable to confirm that lack of problem-solving experiences in K-12 classrooms led to these beliefs about verbal exercises. It could also have been that PSTs inferred that problems and exercises were largely similar in nature as a result of the focus on mathematical procedures rather than a metaprocess. Based on that hypothesis, this present study included interview questions designed to elicit descriptions from PSTs of their K-12 and tertiary education problem-solving experiences.

Misconceptions about what word problems and problem solving are might stem from the types of instruction PSTs receive both in the K-12 and post-secondary levels. Many mathematics content courses use direct instruction with an emphasis on memorisation of procedures and definitions (Association of Mathematics Teacher Educators [AMTE], 2017). Blömeke (2012) surveyed 8,000 PSTs in 15 countries during their final years of university coursework to learn more about mathematics PSTs' teaching programs. Blömeke (2012) found that there was a 75% likelihood that PSTs would experience most of their mathematics content courses as lectures – focusing on procedural competence. With a majority of time being spent on learning procedures and completing exercises, little time is allowed for PSTs to engage in problem-solving experiences, including engagement with word problems. Furthermore, PSTs agree that they need to improve their problem-solving skills (Son and Lee, 2016). PSTs need more opportunities to engage in problem solving at the university level if teacher educators hope to foster positive and appropriate views of problems and problem-solving instruction.

Program Implications

With the importance of problem solving clearly emphasised in both prior research (e.g. Lesh & Zawojewski, 2007; NCTM, 2000, 2014) and within K-12 mathematics curriculum standards worldwide (Mullis et al., 2016), it makes sense that teachers, themselves, should be effective problem solvers. If PSTs are expected to design and lead instruction focused on these standards, then it follows they should be able to solve problems related to the content standards. The *Standards for Excellence in Teaching Mathematics in Australian Schools* state, "Excellent teachers of mathematics have a sound, coherent knowledge of the mathematics appropriate to the student level they teach" (Australian Association of Mathematics Teachers [AAMT], 2006, p. 2). Consequently, PSTs will be required to teach their students how to problem solve within the content they learn as part of the K-12 curriculum and to "exemplify the mathematical thinking that will be expected of their students" (AMTE, 2017). Chapman suggested that during their mathematics teacher education program PSTs should have explicit instruction focusing on what problem solving is and how to implement it in their future classrooms (Chapman, 2005). Thus, PSTs must be competent problem solvers if they intend to help their future students learn how to problem solve (AMTE, 2017; Chapman, 2005; Son & Lee, 2016).

To combat PSTs' misconceptions about word problems and problem solving, researchers worldwide have begun to study their mathematics education programs. Sowder (2007) discussed the importance of problem solving in mathematics teacher preparation, but she also focused on the important role that university level mathematics teacher education programs play in teacher preparation. "No reform of mathematics education is possible unless it begins with revitalisation of undergraduate mathematics in both curriculum and teaching style" (Sowder, 2007, p. 204). The way undergraduate mathematics content and mathematics education courses are taught affects PSTs' beliefs about mathematics (Sowder, 2007; Xenofontos & Andrews, 2012). Changes may be needed in mathematics education and mathematics content courses that PSTs complete, specifically to include more problem solving, which in turn may help PSTs improve their problem-solving performance (AMTE, 2017).

Ingram and Linsell (2014) conducted a programmatic study on students at the University of Otago. Results pointed to mathematics content as an area of weakness for primary PSTs in New Zealand. To ameliorate this perceived weakness among primary PSTs, the education faculty added additional mathematics courses for PSTs who failed their required foundational mathematics content knowledge exam (Ingram & Linsell, 2014). Follow-up investigations after adding the mathematics courses showed that they were helpful in preparing the primary PSTs. Countries like South Korea and South Africa have also evaluated their teacher education programs in light of other countries (Kwon & Ju, 2012; Visser, Posthuma, & Van der Walt, 2015). It was concluded that mathematics content and pedagogical content knowledge were equally important in PSTs' training (Kwon & Ju, 2012; Visser et al., 2015). Moreover, both mathematics content and mathematics education courses should provide PSTs with problem-solving experiences.

Findings from studies that focused on teacher education programs described how changes in programs were determined as a result of their findings. For example, Son and Lee (2016) suggested that studies should use interviews with PSTs during the beginning and end of their program to better understand their views of problems and problem solving. The present research will tie the results back to its program as a means to inform and foster positive changes for PSTs' mathematics teacher education experiences, which may spur similar work at other mathematics teacher preparation programs.

Aim of the Study

The present mixed-methods study extends research on PSTs' problem-solving performance and programmatic implications. This study offers a narrative about one mathematics teacher education program exploring PSTs' use of word problems using a validated and reliable measure. There is a noticeable lack of research drawing upon quantitative instruments that have robust validity arguments, used with K-12 or tertiary level students (Bostic, Krupa, Carney, & Shih, 2019) or with teachers (Bostic, Lesseig, Sherman, & Boston, 2019). This study is explicit in using a quantitative tool that has an established, robust validity argument, in an effort to effectively build upon the literature. The quantitative instruments used in this study were validated from studies with K-12 students, and the present study extends a research agenda around these instruments to inform the work of teacher education programs. These instruments are unique in that they include mathematics curricular standards that PSTs will teach in their future classrooms in the United States of America. Consequently, this study provides a unique lens on middle grade and secondary PSTs' performance in solving word problems related to the mathematics content they will teach in the future and by comparing to their performance on the same problem-solving test given to middle school students (i.e., grades 6, 7, and 8).

As the present study is concerned with future middle grades and secondary teachers, we conjectured that there may be differences between PSTs who are preparing to teach students in grades 4-9 (age 10-16) and those preparing to teach secondary students (age 14-19). We also wondered how a mathematics teacher education program might influence PSTs' performance on the measure connected to mathematics content they might teach. Many research studies use results from PSTs near the end of their program or the beginning as a single cohort. Özgen & Alkan (2012) compared early and end-of-program PSTs' success and this performance was tied to three problems, but it is still uncertain what growth during a PSTs' program might look like. We conjectured that students' problem-solving performance will grow during their four-year program and aimed to explore how, if at all, PSTs' problem-solving performance changes from their first year to their final year. Related to this conjecture, we wanted to contextualise those changes by investigating PSTs' K-12 and tertiary education experiences with word problems. Additionally, we aimed to compare PSTs to middle school students on performance on a middle school problem-solving assessment. Further, we compared first and fourth year PSTs performance as well as middle and secondary PSTs performance.

Method

This study used an explanatory mixed-methods approach (Creswell, 2012). Quantitative data consisted of performance on the problem-solving measures (PSMs), that are described later in this section. Qualitative data consisted of PSTs' responses during an interview following completion of the PSMs. The mixed methods approach was used to better understand the narrative of PSTs' problem-solving performance and to provide a unique lens that builds upon past research on PSTs' problem-solving outcomes. There were four research questions for this study, and three were related to the quantitative data.

- (RQ1) How do PSTs perform on problem-solving measures related to grade-level mathematics content they intend to teach?
- (RQ2) Are there any differences between middle grades and secondary PSTs' problem-solving performance?
- (RQ3) Are there any differences between first-year and fourth-year PSTs' problem-solving performance?

In addition, qualitative data were used to determine potential influences on the PSTs performances and relate to the fourth research question:

- (RQ4) What K-12 and tertiary education experiences may have influenced PSTs' problem-solving performance?

Context of the Study

The participants were PSTs who attended a university in the midwestern United States of America at the time of the study. Language might differ across contexts, thus, terms in this study have been clarified. For ease of understanding, *mathematics education* courses refer to specific, program required courses taught in the College of Education and *mathematics content courses* refer to selected, program-specific courses taught in the College of Arts and Sciences that lead to fulfilling licensure requirements.

PSTs may choose to seek a teaching license that allows them to teach middle grades students (grades 4-9; ages 10-16) or secondary students (grades 7-12; age 14-19). The middle-grades program is called Middle Childhood Education (MCE) and the secondary program is called Adolescent and Young Adult (AYA), which reflects the licensure language. MCE students are licensed to teach two content areas whereas AYA students are licensed for only one content area. All MCE PSTs that participated in the study were on track to earn a license in mathematics and an additional content area. MCE PSTs are required to take five mathematics content courses including calculus, statistics for teachers, algebra for teachers, geometry for teachers, and a mathematics seminar. MCE PSTs take three mathematics education courses: one first year-level mathematics education course focuses on number and pre-algebra concepts, then two mathematics education methods courses are completed during third and fourth years, respectively. All AYA PSTs were eligible for certification to teach mathematics in grades 7-12 (ages 14-19). The AYA mathematics PSTs complete a total of three mathematics education courses: one course focuses on number and algebra concepts during the second year, then two methods courses during third and fourth-years, respectively. They also take eleven mathematics content courses, which includes three calculus courses, discrete mathematics, linear algebra, modern algebra, modern geometry, probability and statistics, history of mathematics, and two elective mathematics courses. Both AYA and MCE PSTs take 10 general education courses. Courses at this university are 15 weeks long. MCE and AYA mathematics education and mathematics coursework meets current program accreditation standards as the university received a program renewal recently. For both programs, all mathematics content starts with a five-credit hour Calculus course; all other mathematics content or mathematics education courses are three credit hours.

Participants

The participants of this study were 123 undergraduate PSTs studying to be mathematics educators (Table 1).

Table 1
Participants in the study

	MCE (Middle School) Grades 4-9	AYA (Secondary) Grades 7-12
Year in the program		
First Year (First year in program)	56	26
Fourth year (Final year in program)	28	13
Total	84	39

Two cohorts of PSTs were involved in the study (Table 1). One cohort involved first-year PSTs in their first semester of the first year of their mathematics education program. 56 of these first year PSTs were middle school (MCE) PSTs and 26 were secondary (AYA) PSTs. The second cohort involved fourth-year PSTs in their first semester of their fourth and final year. Fourth-year PSTs have completed all of their mathematics content and education coursework. Twenty eight of these PSTs were middle school PSTs and 13 were secondary PSTs.
The PSTs were required to complete the PSMs as part of their typical coursework during their mathematics education program (see Table 2 for further description). Participation in the study had no bearing on their grade for the course and PSTs were asked to volunteer their results for the research study. The study was reviewed by the Human Subjects Review Board and deemed ethically appropriate. All names are pseudonyms, to protect the PSTs' identities.

Eight PSTs were purposefully selected for interviews using representative sampling. Four MCE PSTs, which included a pair of first-year PSTs and pair of fourth-year PSTs, were interviewed. Similarly, four AYA PSTs, which included a pair of first-years and a pair of fourth-years, were interviewed. We hypothesised no differences between genders based on results from Chapman (2005) hence this study representatively sampled based on year in the program. All PSTs interviewed were successful at completing their mathematics content and education courses. Respondents described their mathematics ability, when compared to other mathematics education peers, as either average or above average. By taking a purposeful sample of both AYA and MCE PSTs, any differences in problem-solving experiences across the two programs might be illuminated. Selecting both first-year and fourth-year PSTs allowed us to examine PSTs' problem-solving experiences at the beginning and end of their respective programs, allowing for a cross-sectional analysis. Eight PSTs were chosen because it provided enough interview data for making cohort comparisons (i.e., first-year and fourth-years) as well as programmatic comparisons (i.e., MCE and AYA). This sample size also offered sufficient data for in-depth qualitative analysis; more respondents' data would have been unwieldy for the study and required a more topical examination of the data (Creswell, 2012).

Data Collection and Instrumentation: Quantitative

The PSMs were developed to assess problem-solving performance of sixth, seventh, and eighth-grade students (for more information consult Bostic & Sondergeld, 2015, 2018; Bostic, Sondergeld, Folger, & Kruse, 2017). The PSMs were designed to function as both summative and formative assessments, and were intended to complement other data (e.g., tests and classroom observations) about students' problem solving. Substantial research was conducted to validate the results and interpretations of the PSMs. Each test was 15 or 19 questions in length and addressed mathematics content within one grade-level, which was aligned with Common Core State Standards for Mathematics (CCSSI, 2011). Each PSM was composed of a minimum three items from each of the five content domains described in grades six through eight of Common Core State Standards for Mathematics (i.e., Geometry, Statistics and Probability, Ratio and Proportions, Number Sense, and Expressions and Equations). Each item was a word problem. Items were developed with the same frameworks for problems and problem solving discussed earlier (Bostic & Sondergeld, 2015, 2018; Bostic et al., 2017). That is, the solution strategy was not known in advance to the problem solver, each item required complex thinking, and each item could be solved in more than one way. Additionally, each item had to adhere to the second framework that word problems should be open, realistic, and complex. Pilot studies during PSM development indicated that PSTs found the items required complex thinking, solved them in more than one way, and the solution and solution pathway were not readily apparent to them. Additionally, PSTs expressed that items drew upon realistic elements. The number of PSTs that completed each PSM is found in table 2. A sample question from each test is provided in Figure 1. Previous research indicated that whereas dichotomous and partial credit scoring of the PSMs had no impact on respondents' overall scores (Bostic, 2011; Sondergeld, Stone, Kruse, Bostic, & Matney, 2020), much can be learned from examining respondents' strategy use on the PSMs. Learning more about respondents' strategy use is valuable but it increases the scoring time by five-fold compared to dichotomous scoring.

Table 2
Participants completing the Problem-solving measures

	MCE	AYA
Test		
PSM 6	40	0
PSM 7	18	25
PSM 8	26	14

PSM6. #1:
Ruth is planning to serve ice cream sundaes to guests at her birthday party. She purchased 3 flavours of ice cream: vanilla, chocolate, and strawberry, 2 different sauces: chocolate and caramel, and 4 different toppings: bananas, nuts, sprinkles, and whipped cream. How many different types of sundaes can be made if every guest selects only one ice cream flavour, one type of sauce, and one topping?

PSM7. #1:
A water tower contains 16,880 gallons of water. Each day half of the water in the tank is used and not replaced. This process continues for multiple days. How many gallons of water are in the tower at the end of the fourth day?

PSM8. #1.
A land developer owns a piece of land measuring 4×10^2 acres. He wants to make sections of land for houses measuring 0.64 acre. How many sections can he make from the piece of land?

Figure 1. Sample PSM items

The validity argument supporting appropriate interpretations is fairly robust and addresses the five sources of validity evidence referenced by *The Standards for Educational and Psychological Testing* (American Educational Research Association, American Psychological Association, & National Council on Measurement in Education [AERA, APA, & NCME], 2014). A validity argument is a presentation of evidence related to a data collection instrument's results and interpretations (AERA et al., 2014; Bostic, Krupa, Carney, & Shih, 2019). The five sources of validity are test content, response processes, relations to other variables, internal consistency, and relations to other variables/bias. Evidence for all five sources is not necessarily warranted; however, stronger arguments provide evidence for multiple sources. To test for content validity, all items were reviewed by mathematics teachers and mathematicians and mathematics educators that held a PhD or EdD. Results from cognitive interviews indicated that middle school students and PSTs solved the problems in anticipated ways, which provided evidence for response processes. Rasch results supported that the PSMs are unidimensional in nature. Studies of bias and relationships to other variables conveyed that there was no bias towards a particular group (e.g., male or female, white or non-white). Internal consistency for the PSM6, PSM7, and PSM8 was measured using Rasch reliabilities and found to be .97, .98, and .98, which is considered excellent (DeVellis, 2012). Reliability, while it does not confirm validity, is a key component of a sound quantitative measure (Bostic et al., 2019; Creswell, 2012).

For this study, it was hypothesised that PSTs' mean scores would be greater than the middle school students' mean scores. Therefore, this study used the middle school means as a lower bound for the expectation of how the PSTs to score on the PSMs. For the purposes of our study, "successful problem-solving performance" was defined for undergraduate students' performance on the PSMs using feedback from experts. The PSTs took the PSMs at the end of the semester and were given 90 minutes to complete the assessment. The first author administered the PSMs. The students were allowed to use a pencil and paper on the exam and were not permitted to use resources such as calculators, notes, or the internet, which followed the same instruction as the middle-school children. All PSTs completed the PSM in a quiet room.

Data Collection and Instrumentation: Qualitative

One-on-one interviews were audio recorded. PSTs were all asked the same five structured questions in the same order (see figure 2). Following the explanatory mixed-methods approach (Creswell, 2012), these interview questions were developed to explain PSTs' performance on the PSMs and to extend prior research. In this sense, the questions were designed to elicit PSTs' problem-solving experiences from their K-12 and tertiary education. The interview questions are shown in Figure 2. The questions were asked in sequential order.

1. Do you think you are good at problem solving? Why or why not?
2. What were your problem-solving experiences like in your K-12 education?
3. What were your problem-solving experiences like at the university level?
4. How has your undergraduate education affected your understanding of problem solving?
5. What did you learn about problem solving from this assessment?

Figure 2. Interview Questions

Data Analysis: Quantitative

PSMs were scored dichotomously as correct or incorrect. The raw score was recorded out of 15 (PSM6) or 19 (PSM7 and PSM8) points. For RQ1, a one-sample *t*-test was performed to compare how PSTs did on each PSM as compared to the known population mean scores of middle school students who had taken the same PSM. For RQ2, a one-way ANOVA was applied to determine any significant differences between MCE and AYA students. For RQ3, a one-way ANOVA test was utilised to explore any significant difference between first-year and fourth-year PSTs' who took the PSMs. All quantitative analyses used a two-tailed interpretation.

Data Analysis: Qualitative

Inductive analysis (Hatch, 2002) was employed by the researchers to analyse qualitative data with the aim of generating a theme to contextualise the narrative of PSTs' problem-solving performance. First, the interviews were transcribed then read in entirety. Second, notes were made in the margins of transcripts about common ideas and recurring statements. For example, some common ideas were excessive use of exercises in high school, the use of word problems, an increase in problem solving at the university level, and a lack of confidence in problem solving. Third, the transcripts were read repeatedly to identify possible categories, which were aligned with the notes made in the first step. Multiple readings of the data drew out various categories and some were eliminated if there was insufficient evidence to support them. At the fourth step, the data were coded based on the resulting categories to ensure there was sufficient evidence to support them and any counterevidence was identified. In the fifth step, a common theme emerged from the given categories, which was supported by sufficient evidence, meaning numerous quotations on the same idea, and had little counterevidence, meaning one or no quotations contrary to the theme.

Results

Quantitative

Overall, there were three comparisons based on student performance on the PSMs. First, RQ1: How do PSTs perform on problem-solving measures related to grade-level mathematics content they intend to teach? Descriptive statistics are displayed in Table 3. Mean and standard deviation represent those from middle school students collected during prior work. Means and standard deviations (X_{MID}, SD_{MID}) serve as a comparison (i.e., lower bound) for our participants' scores (N_{PST}, X_{PST}, SD_{PST}).

Table 3
Descriptive statistics for PSM scores

PSM	N_{MID}	X_{MID} (%)	SD_{MID} (%)	N_{PST}	X_{PST} (%)	SD_{PST} (%)
PSM6	137	38.0	20.7	40	76.9	12.8
PSM7	654	25.7	44.0	43	66.6	17.7
PSM8	384	20.7	41.0	40	53.8	16.5

Research Question 1: PSTs' results on the PSM6, PSM7, and PSM8 were 77%, 67%, and 54%, respectively. Thus, PSTs were not able to correctly respond all of the grade-level mathematics items on the PSMs, which represented mathematics content they are expected to teach. The mean scores of the PSTs and the middle school students were compared using *t*-tests. On all three PSMs, there was a significant difference PSTs' and mean population scores. PSTs had statistically significant higher scores than sixth-grade students, $t(40)=19.194$, $p<.001$, seventh-grade students, $t(43)=14.995$ $p<.001$, and eighth-grade students, $t(40)=12.893$, $p<.001$. Taking a broad perspective, PSTs surpassed the average score of Grades 6, 7, and 8 students. PSTs answered approximately 65% of PSM items correctly whereas grades 6, 7, and 8 students answered 28% of the items correctly. In sum, PSTs had higher scores than middle school students.

Research Question 2: Next, assumptions for ANOVA were met, which allowed investigating RQ2. ANOVA results showed no significant difference between MCE and AYA students, $F(1,121)=.045$, $p=.833$. The conclusion from this result is that MCE and AYA students had on average, statistically similar problem-solving performance on the PSMs.

Research Question 3: Finally, ANOVA was used to investigate differences between first-year and fourth-year PSTs' performance on the measures. Again, assumptions for ANOVA were met. Results revealed as expected, fourth-year PSTs outperformed first-year PSTs on average, $F(1, 121)=10.471$, $p=.002$, $\eta^2=.08$. This is a medium effect size (Cohen, 1988) and can be interpreted as 8% of the variance in PSTs' scores may be attributed to year in their respective programs. Practically speaking, the average difference between fourth-year and first-year PSTs' scores is two correct responses to PSM items. Our conclusion is that fourth-year PSTs, across both MCE and AYA programs, had statistically higher PSM scores than their first-year peers.

Qualitative

Research Question 4: One theme emerged after analysing the PSTs' interview data: PSTs felt their K-12 and tertiary education experiences included a lack of problem-solving experiences such that most did not see problems like those on the PSMs. PSTs routinely and unanimously expressed that their lack of problem-solving experiences hindered their problem-solving performance. First, evidence from their K-12 experiences is presented, followed by undergraduate experiences, to support this theme.

Alli, a secondary first year PST, was clear when she said, "I feel like I didn't really get any experience with problem solving when I was in high school." Similarly, Andy, a middle school first year PST, expressed:

> I would say [there was] no [problem solving] in high school. It was like, here's something on the board. Here is a worksheet; go for it. There was no group work and no talking. We didn't have to get a deep understanding of what we were doing. It was just, here's this, here's the formula, put the number in the formula to figure out the answer.

Cindy, a middle school fourth year PST, shared a similar experience, "I don't really think I had very much problem solving [in K-12]. It was more, here's the content in a lecture form, especially in math class they used a lecture of the content, go home and do these [exercises]." Liz, a middle school fourth year PST, commented, "I felt like I just memorised the steps and the procedure to get the answer." This sentiment was similar to that of Cory, a secondary first year PST, who claimed, "I did not do a lot of problems [in high school], just a lot of exercises. There was never anything that made you think, 'How do I apply what I am learning to solve this problem?' It was always just doing the procedure." Amanda, a secondary fourth year PST, said, "[When problem solving] I focus on my frustration of not being able to solve this problem or riddle or whatever, and that kind of hampers on my ability to solve it." Amanda communicated how she struggled to find the initial solution to the PSM tasks because she struggled to perceive a viable solution strategy. This focus on exercises during K-12 instruction, instead of problems, gave PSTs a lack of problem-solving experiences to draw upon during PSM administration. Taken collectively, these experiences among the PSTs emphasise a K-12 school curriculum that centred on procedural efficiency and a lack of problem solving. Students who were engaged in this type of instruction sought to memorise procedures that only worked for specific types of tasks. The lack of problem-solving experiences the PSTs had in their education affected their problem-solving performance on the PSMs as PSTs.

When reflecting on experiences in their undergraduate courses, PSTs identified a lack of problem solving in their mathematics content courses at the university level. The general impression of university-level mathematics content courses from all interviewed participants was that the emphasis was on being able to correctly and quickly execute procedures. Amanda said, "I think the math courses, not the math education course, the math courses that I took were completely all [focused on building] procedural fluency." Amanda and others were asked to clarify what they meant by procedural fluency. Peter, a secondary fourth year PST, shared, "Once you get into higher math classes, it becomes more abstract. But it is not necessarily problem solving of a situation, it is more like do this [exercise]." Cory explained that in his mathematics education course, "I have done lots of problem solving in [my mathematics education] classes. I feel like whenever we are doing something, [the instructor challenges us to] take a step back and think of other ways to solve the problem." Cindy shared similar experiences about her mathematics education courses stating, "In the pre-math methods [mathematics education] course, we did more problem solving [than content courses]." Peter's comment about undergraduate mathematics content courses suggests that he had mathematics tasks to consider, but there was not necessarily problem solving as defined earlier inherent in those tasks, because he knew the strategy to employ. This is important because PSTs like Peter, enrolled in the AYA program take eight courses at or above the 3000 level and MCE PSTs take three courses at or above the 3000 level. Courses at the 3000 level and higher are typically reserved for students in the third or fourth year of their program.

These PSTs experienced a wide variety of mathematics content courses but still feel they are not getting much exposure to problem solving. While many shared that their mathematics education courses involved more problem solving than mathematics content courses, they agreed there was little problem solving in their mathematics content courses. When PSTs had opportunities to engage in problem solving in their mathematics content courses, they were not as frequent as in their mathematics education courses. This lack of problem-solving experiences in their undergraduate mathematics content courses gave PSTs little to draw on while working through the PSM tasks.

In summary, PSTs felt their K-12 and tertiary education experiences lacked problem-solving experiences so that most did not see problems like those on the PSMs. They were not prepared to problem solve with content they might teach in future classrooms. This theme from the qualitative data helps to explain PSTs' performance on the PSMs. The PSMs assess knowledge of middle grades mathematics content, which should be familiar to the PSTs based on their performance in mathematics courses. Moreover, these university students should be able to demonstrate mastery of content from K-12 education. It may be that PSTs know the procedures associated with mathematics content, but they lack the knowledge of how to utilise those procedures within the frame of problem solving. This idea comes from the fact that PSTs should know the middle grades content but have had little exposure to utilizing the content in the aspect of problem solving, as shared during the interviews.

Discussion

As indicated by the findings, PSTs unanimously agreed that they had little to no problem-solving experiences in their K-12 education, and little in their undergraduate mathematics content coursework. We discuss the findings in relation to problem solving and programmatic changes.

Problem-Solving Performance

The paucity of problem-solving during K-12 coursework left first-year PSTs with limited experiences to draw upon while problem solving, which they claimed had an impact on their problem-solving performance. The PSTs conveyed frustration about problem solving during PSM administration; they often could not find an entry point or identify a possible solution strategy. Fortunately, fourth-year PSTs had better problem-solving performance than their first-year peers. This seems to suggest that as PSTs take more coursework, they experience growth in their ability to use mathematics content while problem solving. It is plausible that there is an ameliorating effect at play when PSTs take more university-level mathematics education courses. PSTs are exploring the same content they learned in K-12 but through a pedagogical lens during their mathematics education coursework. This study cannot equivocally state whether greater mathematics content, mathematics education, or both types of courses have a bigger impact on students' problem-solving outcomes; thus, this area needs further exploration. The mathematics education coursework may have a stronger focus on conceptual understanding and involve more problem-solving experiences than their K-12 mathematics instruction, as PSTs communicated during their interviews. This pedagogical lens has a key focus on developing mathematical proficiency, which includes conceptual and procedural understanding, adaptive reasoning, strategic knowledge, and a productive disposition (Kilpatrick et al., 2001). Engaging PSTs in problem-solving experiences has power to generate more schematic connections that might lead to deeper mathematical understanding. Furthermore, these experiences in mathematics education coursework might have led to the increased performance by the fourth-year PSTs. Mathematics content courses are highlighted here because those PSTs who were interviewed specifically mentioned that they had more problem-solving experiences in mathematics education courses compared to their university mathematics content courses.

Previous studies (i.e. Chapman, 2005; Özgen & Alkan, 2012; Son & Lee, 2016) identified that PSTs often struggle to understand the concepts of problems, problem solving, and problem-solving instruction. Results from the present study illustrate that the sample of PSTs consistently performed better than the norms for middle school students' performance on the PSMs.

PSTs communicated a lack of K-12 and tertiary education problem-solving experiences, which they believe impacted their struggles with problem solving on the PSMs. Engaging PSTs in problem-solving is within mathematics teacher educators' scope of work. More problem-solving experiences during undergraduate mathematics content and education coursework may help students solve new and challenging problems (AMTE, 2017). Being able to draw on their past solution strategies and analogous problem situations has potential to promote problem-solving success during new problem situations (Polyá, 1945/1973; Schoenfeld, 2011), which may be found during the undergraduate coursework.

Concluding Comments

Previous studies (i.e. Chapman, 2005; Son & Lee, 2016; Xenofonotos & Andrews, 2012) investigated PSTs only during one year of a teacher education program. This study aimed to build on their work by drawing comparisons between (a) PSTs and middle school students and (b) first- and fourth-year PSTs to evaluate the impact of university coursework on PSTs problem-solving performance. This study extends previous literature because we were able to show that mathematics education courses, which focus on problem solving, improved PSTs' problem-solving performance to some degree. An implication from this study is that time focused on problem-solving experiences during university coursework is important to PSTs' growth as future teachers who are expected to promote problem solving during future in-class instruction.

The purposeful, representative sample of students in this study included both MCE and AYA PSTs, which is unlike other studies using convenience sampling while examining either elementary or secondary PSTs (i.e. Chapman, 2005; Özgen & Alkan, 2012; Son & Lee, 2016). This peer-reviewed study adds to those scholarly discussions about growth during a university preservice teacher education program with data from first and final years in the program, something not found elsewhere in the literature. The results of this study also provide a different nuance on previous research because preparation in the MCE program is different from elementary education programs that were cited in the literature review. Most elementary education programs had one or two mathematics education courses and no more than two mathematics content courses. As described earlier, this institution's MCE program includes two mathematics education courses and five mathematics content courses. PSTs in the MCE and AYA programs, unlike elementary programs, are trained more heavily in mathematics content and thus are expected to perform more highly on problem solving tasks. This study sheds light on the assumption that MCE and AYA PSTs are proficient problem solvers. With little research done on MCE PSTs, this study provides a snapshot of the current state of PSTs' problem-solving performance at one institution.

Limitations and Future Directions

All data were collected from one university. It is unknown whether PSTs' means and standard deviations on the PSMs are representative of the greater PST population; however, this is something to take up in future research. In summary, this study does not aim to generalise our conclusions to the entire population of PSTs, and instead provide results that have potential to inform the field of mathematics teacher educators. It is necessary to scale up the present study to other institutions within and outside of the USA to explore how programs may impact students' problem solving. Additionally, greater understanding of how coursework within various program and cultural norms specific to PSTs may shed light on other related factors contributing to PSTs' problem-solving performance.

A future direction for this study is to track students' progress in revised MCE and AYA programs. MCE and AYA programs have changed since the study's findings were shared with administrators and faculty at the university. Both offer an additional first year-level mathematics education course focusing on major aspects related to mathematics education. AYA students take two additional mathematics education courses focusing on Number and Algebra one semester and Geometry in the following semester. Thus, the authors intend to continue investigating how results from the present study might be different with the addition of new mathematics education courses that have a problem-solving focus. A second direction is to conduct a rigorous study of problem-solving performance for teaching related to the PSMs. Presently, a study is being conducted to create cut scores related to students' performance on the PSMs (e.g., above average, average, and below-average performance). A third direction for future research is to capture PSTs' confidence to teach problem solving. Noted earlier, Amanda felt frustrated when she was unsuccessful at solving PSM problems. There may be an affective component, like confidence, to engage in problem-solving teaching and learning, that mediates or influences PSTs' performance on the PSM.

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Referencing

American Educational Research Association, American Psychological Association, & National Council on Measurement in Education. (2014). *Standards for educational and psychological testing.* Washington, D.C.: American Educational Research Association.

Association of Mathematics Teacher Educators. (2017). *Standards for Preparing Teachers of Mathematics.* Available online at amte.net/standards.

Australian Association of Mathematics Teachers (AAMT). (2006). *Standards for excellence in teaching mathematics in Australian schools.* Retrieved from https://www.aamt.edu.au/Better-teaching/Standards

Australian Curriculum, Assessment, and Reporting Authority. (2014). *Mathematics F-10 Curriculum.* Retrieved from: https://www.australiancurriculum.edu.au/resources/mathematicsproficiencies/portfolios/problem-solving/

Ball, D., Ferrini-Mundy, J., Kilpatrick, J., Milgram, J., Schmid, W., & Schaar, R. (2005). Reaching for common ground in K-12 mathematics education. *American Mathematical Society, 52*, 1055-1058.

Blömeke, S. (2012). Content, professional preparation, and teaching methods: How diverse is teacher education across countries? *Comparative Education Review, 56*(4), 684-714.

Bostic, J. (2011). *The effects of teaching mathematics through problem-solving contexts on sixth-grade students' problem-solving performance and representation use* (Unpublished doctoral dissertation). University of Florida, Gainesville, FL.

Bostic, J., Krupa, E., Carney, M., & Shih, J. (2019). Reflecting on the past and thinking ahead in the measurement of students' outcomes. In J. Bostic, E. Krupa, & J. Shih (Eds.), *Quantitative measures of mathematical knowledge: Researching instruments and perspectives* (pp. 205-229). New York, NY: Routledge.

Bostic, J., Lesseig, K., Sherman, M., & Boston, M. (2019). Classroom observation and mathematics education research. *Journal of Mathematics Teacher Education. , 1-27, https://*doi.org/10.1007/s10857-019-09445-0

Bostic, J., Pape, S., & Jacobbe, T. (2016). Encouraging sixth-grade students' problem-solving performance by teaching through problem solving. *Investigations in Mathematics Learning, 8*(3), 30-58.

Bostic, J., & Sondergeld, T. (2018). In D. Thompson, M. Burton, A. Cusi, & D. Wright (Eds.), *Validating and vertically equating problem-solving measures. Classroom Assessment in Mathematics: Perspectives from Around the Globe* (pp. 139-155). Cham, Switzerland: Springer.

Bostic, J., & Sondergeld, T. (2015). Measuring sixth-grade students' problem solving: Validating an instrument addressing the mathematics Common Core. *School Science and Mathematics Journal, 115, 281-291.*

Bostic, J., Sondergeld, T., Folger, T. & Kruse, L. (2017). PSM7 and PSM8: Validating two problem-solving measures. *Journal of Applied Measurement, 18*(2), 151-162.

Chapman, O. (2005). Constructing pedagogical knowledge of problem solving: Preservice mathematics teachers. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education* (pp.225-232). Melbourne, Australia: Psychology of Mathematics Education.

Cohen, J. (1988) *Statistical power analysis for the behavioral sciences* (2nd ed.). Hillsdale, NJ: Erlbaum.

Common Core State Standards Initiative. (2010). *Common Core State Standards for Mathematics*. Retrieved from http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

Council for the Accreditation of Educator Preparation (CAEP). (2013). *2013 CAEP Standards: Excellence in educator preparation*. Retrieved from http://caepnet.org/standards/introduction

Creswell, J. (2012). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research* (4th ed.). Upper Saddle River, NJ: Pearson/Merrill Prentice Hall.

DeVellis, R. F. (2012). *Scale development: Theory and applications.* Los Angeles, CA: SAGE.

Hatch, A. (2002). *Doing qualitative research in education settings.* Albany, NY: State University of New York Press.

Higher Education Bureau, Ministry of Education, Culture, Sports, Science and Technology. (2012). *Higher education in Japan.* Retrieved from http://www.mext.go.jp/en/policy/education/highered/title03/detail03/__icsFiles/afieldfile/2012/06/19/1302653_1.pdf

Ingram, N., & Linsell C. (2014). Foundation content knowledge: Preservice teachers' attainment and affect. In J. Anderson, M. Cavanagh, & A. Prescott (Eds.), *Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia* (pp. 718–721). Sydney, Australia: Mathematics Education Research Group of Australia.

Ingvarson, L.; Ellio, A., Kleinhenz, E.; & McKenzie, P. (2006). Teacher Education Accreditation: A Review of National and International Trends and Practices. Retrieved from http://research.acer.edu.au/teacher_education/1

Kilpatrick, J., Swafford, J., and Findell, B. (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.

Kwon, O., & Ju, M. (2012). Standards for professionalization of mathematics teachers: policy, curricula, and national teacher employment test in Korea. *ZDM Mathematics Education, 44,* 211-222.

Lesh, R., & Zawojewski, J. (2007). Problem solving and modeling. In F.K. Lester (Ed.) *Second Handbook of Research on Mathematics Teaching and Learning: A project of the National Council of Teachers of Mathematics*. (pp. 763-803). Charlotte, NC: Information Age.

Matney, G., Jackson, J., & Bostic, J. (2013). Connecting instruction, minute contextual experiences, and a realistic assessment of proportional reasoning. *Investigations in Mathematics Learning*, *6*, 41-68.

Mullis, I. V. S., Martin, M. O., Goh, S., & Cotter, K. (Eds.) (2016). *TIMSS 2015 Encyclopedia: Education Policy and Curriculum in Mathematics and Science.* Retrieved from http://timssandpirls.bc.edu/timss2015/encyclopedia/

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author

National Council of Teachers of Mathematics. (2009). *Focus in high school mathematics: Reasoning and sensemaking.* Reston, VA: Author.

National Council for Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: Author

Özgen, K. & Alkan, H. (2012). The relationship between secondary school pre-service mathematics teachers' skills in problem solving dimensions and their learning style characteristics. *Educational Sciences: Theory & Practice, 12(2)*, 1173-1181.

Pólya, G. (1945/2004). *How to solve it: A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.

Schoenfeld, A. H. (2011). *How we think: A theory of goal-oriented decision making and its educational applications*. New York, NY: Routledge.

Son, J. & Lee, M. (2016). Preservice teachers' conception of effective problem-solving instruction and their problem solving. In M. Wood, E. Turner, M. Civil, & J. Eli (Eds.), *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 829-836). Tuscon, AZ: University of Arizona.

Sondergeld, T., Stone, G., Kruse, L., Bostic, J., & Matney, G. (2020). *Evaluating Dichotomous and Partial-Credit Scoring within a Constructed-Response Assessment: Is More Information Always Psychometrically Better?* Paper presented at annual meeting of the annual meeting of the American Education Research Association. San Francisco, CA.

Sowder, J. (2007) The mathematical education and development of teachers. In F.K. Lester *Second Handbook of Research on Mathematics Teaching and Learning: A project of the National Council of Teachers of Mathematics*. (pp. 157-223). Charlotte, NC: Information Age.

Verschaffel, L., De Corte, E., Lasure, S., Van Vaerenbergh, G., Bogaerts, H., & Ratinckx, E. (1999). Learning to solve mathematical application problems: A design experiment with fifth graders. *Mathematical Thinking and Learning, 1*, 195-229.

Visser, A., Posthuma, A., & Van der Walt, M. (2015) International comparisons of foundation phase number domain mathematics knowledge and practice standards. *South African Journal of Education, 35(1)*, 1-13.

Xenofontos, C., & Andrews, P. (2012). Prospective teachers' beliefs about problem-solving: Cypriot and English cultural constructions. *Research in Mathematics Education, 14*(1), 69-85.

Yee, S., & Bostic, J. (2014). Developing a contextualization of students' mathematical problem solving. *Journal for Mathematical Behavior*, *36,* 1-19.

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