Linking theory and practice: A case study of a co-teaching situation between a mathematics teacher educator and a primary classroom teacher

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In recent years there has been growing concern as to how to bridge the gap between the theory pre-service teachers engage with as part of their learning in their tertiary classrooms and the profession. To enable pre-service teachers to make stronger connections with the profession, a mathematics teacher educator worked collaboratively with a practicing teacher by co-teaching one cohort of pre-service teachers studying primary mathematics education. In this paper, we present two snapshots of the co-teaching experience and a framework that was used to describe how the co-teaching partnership helped the pre-service teachers to elicit mathematical thinking make connections between theory and practice, when engaged in mathematical discourse.

Keywords · preservice teachers · co-teaching · mathematical discourse · primary · theory and practice

Introduction

Most pre-service teachers’ (PSTs) experiences occur in university classrooms (tutorials) or during practicum in primary school classrooms. When considering teacher preparation and learning, courses should assist PSTs to develop a deep understanding of the mathematics they will teach, be grounded in research, and assist with the transition of learning during coursework to teaching in real classrooms (Kulm, 2008). These experiences should also provide PSTs with current images of teaching that determine their understanding of mathematics education, including how they might approach their mathematical teaching by linking practice and theory. There is agreement that knowledge for teaching mathematics includes a combination of theoretical and practical knowledge developed at university and in schools (practicum experiences) (Allen, Ambrosetti, & Turner, 2013; Novotná, 2009).

Currently there is an Australian initiative to improve the quality of pre-service teacher education including the knowledge for teaching mathematics (Teacher Education Ministerial Advisory Group [TEMAG], 2015) and universities need to consider how they might respond to these recommendations. The university where this study took place chose to invite practicing teachers to participate in teaching courses at both undergraduate honours level and masters level. The purpose of this initiative was to assist with building relationships between universities and schools, ensuring graduating teachers are classroom ready (TEMAG, 2015) and equipped with a diverse range of skills for teaching, including the specialised knowledge for teaching primary mathematics. As little has been written about co-teacher experiences in mathematics teacher education within university settings, this study will contribute to the research literature in this field.

Co-teaching is when two professionals such as a teacher and special education teacher work together to deliver instruction (Friend, 2008). Graziano and Navarrete (2012) identified several benefits of co-teaching including opportunities to vary content presentation, individualise instruction, scaffold learning experiences, and monitor pre-service teachers' understanding. They also suggest that co-teaching can promote equitable learning opportunities for all PSTs.

While a number of previous studies have documented situations where mathematics teacher educators (MTEs) have worked with practicing teachers to reflect upon and enhance classroom practices (e.g., Geiger, Muir, & Lamb, 2015; Goos & Geiger, 2006; Muir & Beswick, 2007), less common are examples of practicing teachers working alongside MTEs in their tertiary classrooms. In the study reported in this paper, a co-teaching initiative was enacted to enable a cohort of PSTs to make stronger connections with the profession. The following research questions underpinned our study:

> How does co-teaching provide opportunities to elicit pre-service teachers' mathematical thinking?
>
> How can a co-teaching situation help PSTs to make connections between the theory and practice of mathematics teaching?

Other strategies to help PSTs make connections between theory and practice have included the provision of video footage of mathematics teaching for them to view and critically analyse (e.g., Beswick & Muir, 2013), implementing lessons or co-teaching in classrooms (e.g., Anthony, Hunter, Anderson et al., 2015a; Cavanagh & Garvey, 2012; Perkins, 2015), role-playing and rehearsal (e.g., Grossman, Hammerness, & McDonald, 2009; Muir, Allen, Rayner, & Cleland, 2013), the use of representations of practice, such as children's work samples (Livy, Downton, & Muir, 2017) or providing opportunities for PSTs to observe their MTE teaching in a primary classroom (Livy & Downton, 2017). We believe that a focus on the transition to teaching in a real classroom can be improved by collaborating with schools and their teachers, such as inviting them to assist with teacher preparation during tutorial experiences. For most PSTs, working with practicing teachers only occurs through their professional placement experiences, and there are no guarantees that these experiences provide PSTs with examples of 'best practice', current research or exposure to quality mathematics teaching. Research, however, has shown that connections between theory and practice of mathematics classroom teaching can be challenging for MTEs, but achieved by making university experiences related to primary school classroom experiences (e.g., Beswick & Muir, 2013).

Frameworks that inform knowledge for teaching

There is ongoing concern of a possible disconnect between how PSTs make sense of what they learn in their tertiary classrooms with school-based practicum placements (TEMAG, 2015; Zeichner, 2010). As MTEs, our teaching and research is informed by frameworks and explanations of terms to guide our thinking about the knowledge an effective mathematics teacher might use. For example, Shulman's (1987) seminal study has guided many researchers as they consider important categories of a teacher's knowledge base such as *knowledge of content*, *pedagogical knowledge* and *knowledge of learners*. Others have elaborated by describing *specialised content knowledge* when referring to a unique kind of knowledge mathematics teachers demonstrate (Ball, Thames, & Phelps, 2008). Further, Rowland, Turner, Thwaites and Huckstep, (2009) provided four categories of the Knowledge Quartet: *foundation knowledge* (including knowledge of content and pedagogical knowledge); *transformation* (representing the mathematics); *connection* (e.g., coherence of planning, sequencing of instruction); and *contingency* (when the teacher responds to classroom events) that can be used to develop and deepen teachers' (and PSTs') mathematics knowledge. These frameworks can be useful for guiding MTEs' instruction and helping PSTs to become confident teachers of mathematics. In particular assisting PSTs to develop their foundation knowledge, including beliefs about how and why mathematics is learnt (Turner & Rowland, 2011), is likely to impact on their ability to adopt appropriate pedagogical practices into their future classrooms.

When conducting lessons, evidence of teachers' Pedagogical Content Knowledge (PCK) can be enacted through the way in which they *transform* and *connect* the knowledge they are teaching and how they respond to *contingencies* (Rowland et al., 2009). When a teacher is using their *specialised mathematical content knowledge* (Ball et al., 2008) they are relying on their *mathematical knowledge* to consider ways to represent a mathematical idea. These mathematical ideas can be 'transformed' to students by using representations or materials that demonstrate and *transform* what the teacher knows when helping their students learn (Rowland et al., 2009). The third category of the Knowledge Quartet, *connection,* is apparent when a teacher makes choices and decisions: making connections between procedures or concepts; as they consider the complexity of the learning; when making decisions about the sequence of the lesson; and recognising the conceptual appropriateness (Turner & Rowland, 2011). Within any given lesson, it is reasonable to expect that situations will occur when students might provide an incorrect response, or the lesson takes a different direction than that originally intended. Such instances are referred to in the Knowledge Quartet as *contingencies* because the teacher must consider how to respond to an occurrence that was unexpected (Rowland et al., 2009).

Framework for facilitating mathematical discourse

Facilitating mathematical discourse is an effective teaching practice (National Council of Teachers of Mathematics [NCTM], 2014) that also applies to MTEs' practice. Staples and King (2017), developed a framework (see Figure 1) that includes three key functions teachers rely on when guiding students' mathematical discourse: eliciting student thinking; supporting student-to-student exchanges; and guiding and extending the mathematics. The key functions and inter relations between the functions support students' participation in mathematical discourse.

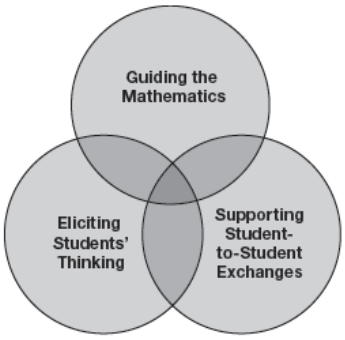

Figure 1. Three key functions of the teacher's role in facilitating meaningful mathematical discourse (Staples & King, 2017 p. 39).

When supporting student learning, Staples and King (2017) suggest providing tasks that make learning accessible to all, as well as a student-centred approach to discussions where the teacher relies on a variety of strategies for guiding learning. When this occurs, the classroom can be transformed into a community of practice, where groups of people interact and contribute to a common interest, actively participating and sharing information, stories, and experiences to gain knowledge and skills (Wenger, 1998). Such interactions are important because a recent review of literature within Australia recommended that collaboration and sharing of practice with other education communities is required if teacher educators are to improve their practice (Anthony, Cooke, & Muir, 2016). Others have also identified a community of practice as an effective perspective for examining co-teaching situations when teaching mathematics. For example, Enfield and Stasz, (2011) found that in their study, a community of practice helped PSTs to develop as reflective professionals by encouraging reflection in action and discourse that forced explicit thinking or metacognition about an activity.

The mathematical discourse that occurs in a tertiary classroom setting, therefore, must include discussion that advances PSTs' *breadth* and *depth* (Ma, 1999) of mathematical understanding. Discourse, in this context, is an approach to teaching which encourages students to discuss the mathematics as they reveal their understanding of concepts and engage in reasoning and debate (Cobb, 2006). Mathematical discourse is more than spoken words, rather learning that supports meaningful discussion (Staples & King, 2017). Having clarified the knowledge that is needed for primary teaching it is also important to consider the role of the MTE in facilitating purposeful learning. An important role of teachers is to use 'talk move' strategies that encourage thinking such as turn and talk, think pair share (Kazemi & Franke, 2014; Staples & King, 2017). These strategies can also support PSTs to develop their mathematical discourse as well as model approaches to teaching. Similarly, students might be asked to share their ideas with the class and explain their thinking and strategies.

Adapted framework for pre-service teaching

Staples and King's, (2017) framework for facilitating meaningful mathematical discourse was useful for our study, along with the Knowledge Quartet (Rowland et al., 2009). In addition, a community of practice was also a key function for promoting reflective discourse of PSTs (Enfield & Stasz, 2011). As already indicated within the review of literature, many categories of teacher knowledge are useful when helping PSTs to develop links between theory and practice, which became the centre of the revised framework (Figure 2).

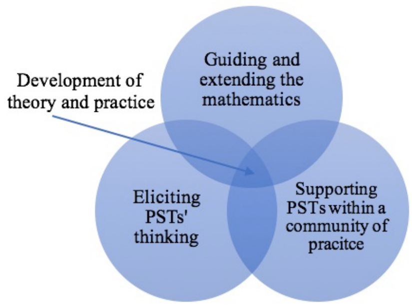

Figure 2. Three key functions of the co-teachers' role in facilitating meaningful mathematical discourse (adapted from Staples & King, 2017 p. 39).

Figure 2 shows three key functions:

1. Guiding and extending the mathematics, including pursuing common misconceptions to advance the learning of the class (Staples & King, 2017);
2. Supporting PSTs within a community of practice, establishing a supportive setting to help PSTs to learn as reflective professionals;
3. Eliciting student thinking, including providing opportunities for students (PSTs) to generate ideas with the class (Staples & King, 2017).

Within the classroom these three functions may overlap. For example, one and three overlap when the discourse includes making connections such as developing conceptual understanding of why a rule might work when calculating the formula for the area of a triangle. One and two overlap when the exchanging of ideas relates to guiding the mathematics understanding using one of the talk moves. Similarly, two and three overlap when the exchange of ideas relates to making connections and includes the talk moves (Kazemi & Hintz, 2014). When all three functions overlap they assist PSTs to develop knowledge of theory and practice for primary mathematics teaching.

Methods

Both case study research design and qualitative methods were used in this study. A case study may answer how or why questions (Yin, 2009) and describes specific instances or phenomenon (Merriam, 1988). The following case study research was designed to assist with reporting on the phenomenon of co-teaching in a tertiary classroom whereby the co-teaching partnership assisted the MTE by helping the PSTs to make connections between theory and practice through meaningful mathematical discourse.

Participants and co-teaching program

The participants in the study included one cohort of PSTs (N=35), their mathematics MTE, Sally, and a primary school teacher Sam (pseudonyms used throughout.) The PSTs were enrolled in a four-year teacher education Honours degree specialising in primary education. All PSTs complete 80 days of professional experience in primary school settings during their degree, and as part of the course structure studied two units that focused on preparing them for primary mathematics teaching. The first unit in second-year assisted PSTs to develop knowledge, skills and dispositions related to mathematics and numeracy education in the early years.

The second unit, which is reported on in this study, was undertaken in third year and designed to extend PSTs' knowledge for teaching mathematics and numeracy by focusing on upper primary levels. Each week the PSTs attended two-hour tutorials in their tertiary classroom during semester (10 weeks). The PSTs were expected to engage with independent study, to complete two assessment tasks: a critical analysis related to an issue in mathematics education as well as planning and facilitation a lesson to their peers, then to reflect on their and other PSTs' experiences.

The MTE had applied for a co-teacher as part of an initiative to improve the quality of teacher education at the university. This was also in response to a call for teacher education providers to work together and assist pre-service teachers to develop a connected knowledge of theory into practice (TEMAG, 2015). Sally invited Sam to join her for the semester, once a week, as a co-teacher in 2017, mainly because of his experience as a leading mathematics teacher in his school. Sam agreed to participate in the program and was keen to share his expertise with future teachers.

Each week, prior to teaching, the co-teacher and MTE planned together. Planning included suggestions of artefacts that the MTE might bring to contribute to the lesson such as children's work samples, teaching resources or photographs. Each weekly tutorial followed a similar format, and usually included solving and discussing different methods of solutions to challenging tasks (e.g., Sullivan, 2017). The co-teacher regularly taught these tasks before the tutorial, bringing student work samples to share with the PSTs so as to extend discussion of strategies children might use when solving these problems. Many of these experiences also helped the PSTs to extend their mathematical content knowledge and to revisit the mathematics they were taught in schools. When eliciting, supporting and guiding mathematical learning as suggested by Staples and King (2017), discussion "focused on concepts, procedures, problem-solving strategies, representations, or reasoning" (p. 37).

Data collection and instruments

Data were collected from PSTs during Weeks 6 and 7 of a 12-week semester (Table 1). Data consisted of tutorial observations, post-tutorial reflections and interviews, and were collected by two co-researchers, Amanda and Annabel (Note, there was no PST focus group interview in Week 7 as no-one offered to participate, most likely as many assignments were due this week). Both researchers took field notes of the observed tutorials which formed the basis of the vignettes presented later in this paper.

Table 1
Participant cohort information in weeks 6 and 7 (N=35)

Cohort	Focus group Participants	Survey responses
Week 6	6 (Groups of 4 and 2)	32
Week 7	0	29

Table 1 shows the number of PSTs who volunteered to participate in the focus groups. Interview questions included: To what extent was today's tutorial typical of the weekly classes? How does the co-teacher support your learning? How might this co-teaching approach differ to other tutorial experiences? Each interview took 15 minutes, was audio-recorded and fully transcribed.

After both tutorials (110 minutes each), most PSTs completed a written reflection about their learning experiences. They responded to the following questions: What is something Amanda/Annabel will notice about today's tutorial? How do you benefit from having a co-teacher in your mathematics tutorials? Was there a mathematical activity you needed help with today and who assisted you?

Data analysis and coding

Following an interpretative paradigm in qualitative data analysis, the authors coded the vignettes to identify evidence of meaningful mathematical discourse for supporting the PSTs to make connections between the theory and practice of mathematics teaching. Independently, the first two authors used open coding to find evidence of the three functions for facilitating discourse (Figure 2), evidence of the dimensions of the Knowledge Quartet and evidence of supporting a community of practice. In collaboration, the authors conducted a further cycle of coding to derive agreed categories. These are presented in Table 2 together with illustrative examples documented in the tutorial observations.

Table 2
The categories derived from the analysis of the observed tutorials and transcripts, with examples

Categories	Examples from the Tutorial Observations
1. Foundation knowledge	How would you define perimeter, area and volume?
	Articulating the meaning of volume verses capacity
2. Making connections between concepts	Considering the relationship between area and surface area of a cube.
3. Eliciting PSTs' thinking	Sam reframed the question to elicit more responses from the PSTs
	Talk to your friend about your thinking
4. Connecting to how children learn mathematics	Sharing insights of classroom experiences and student work samples
5. Establishing a community of practice	Being co-constructors of the ideas with students
	The interplay between co-teachers and PSTs

These categories link to some of the dimensions of the Knowledge Quartet, the three functions for facilitating meaningful mathematical discourse (Figure 2), talk moves (Kazemi & Hintz, 2014) and evidence of establishing a community of practice (Enfield & Stasz, 2011).

Results and Discussion

The following vignettes report two episodes of Sally and Sam's co-teaching to provide details of how they elicited PSTs' thinking, supported and guided mathematical discourse that helped PSTs' to make connections between theory and practice. Throughout the discussion links are made back to the literature including the frameworks that guided our study (Figure 2).

Vignette 1: Algebraic Thinking tutorial

The tutorial was observed by Amanda and began with Sally asking the PSTs to share what they thought algebraic thinking involved. When there were few responses, Sam reframed the question and elicited some more responses from the PSTs. Sam recorded the responses on the whiteboard, which included 'solving problems', to show how components are related, and to make predictions. Sally then directed the PSTs' attention to the PowerPoint slide which contained the three big ideas associated with algebraic thinking: Patterns, Equality, and Function. Evidence of Sally's foundation knowledge was demonstrated through her focus on the mathematics of algebraic thinking, and both Sally and Sam's questioning enable them to elicit PSTs' thinking.

Sally then provided a general overview of where to source teaching activities, including websites and professional journals. At this point, Sam cautioned students to be critical when accessing websites and to "resist the temptation to use a resource without thinking about how it fits in with developing students' conceptual understanding". Again, aspects of Sally's foundation knowledge were demonstrated through her knowledge of resources, and the way that Sam and Sally interacted and complemented each other when discussing websites, established them as members of a community of practice.

Sally then directed Sam to "go over the curriculum for us". Sam spoke to a slide that contained an overview of algebraic thinking at different year levels. He then placed a line of tiny teddies on the desk and asked PSTs what they would do if students could not identify and then follow the pattern to place the next tiny teddy. When there was no response, Sam kept rephrasing until there were a few suggestions given. Photographs showing students from Sam's class with examples of extending patterns were then shared with the PSTs. This was a clear example of connecting to how children learn mathematics and another example of the co-teachers' attempts to elicit PSTs' thinking.

Modelling and representations

Sam then modelled how a balance scale could be used to demonstrate equality to show that the concept of 'balancing numbers actually works'. He emphasised the importance of students' opportunity to 'make, name and record' and shared photographs and examples of students' work (connecting to how children learn mathematics). The PSTs were directed to use the balance scales at their tables to demonstrate the equality of number, which provided an example of making connections between concepts.

Further on in the tutorial the PSTs were asked to construct patterns using yellow and red counters that Sally distributed. After PSTs spent about 5 minutes making their patterns, Sally directed the class to move to one group's table and asked Mark to continue the pattern. She then asked the group to name the type of pattern [repeating] and then used the counters to show a growing pattern and asked PSTs to identify how the pattern was growing. The PSTs were then asked to use the counters to model triangular numbers as an example of a growing pattern. She then directed the PSTs back to the PowerPoint and provided an example of a 'Year 5 lesson where 'Nick' wrote $8 + 4 = \boxed{12} + 5$.

PSTs were then asked to discuss why they might agree or disagree with that, and then looked at some similar work samples in their table groups. After sharing, Sam stated, "Here's a different perspective on this. What do you think they would have come up with if there was no 12 in it?". After some prompting, Sally then suggested that "Maybe equal means that's where the answer goes". She then explicitly explained to the PSTs that "We were trying to show you an example of students' thinking which shows a misconception" and Sam responded with "What Sally's struck on is fundamental – what do you think students are doing? I often get them (students in class) to teach me—I will deliberately make a mistake and get them to teach me how to do the maths correctly". Again, the use of students' work samples and the specific reference to the misconceptions demonstrate Sally and Sam's attempts to connect to how children learn mathematics.

The remainder of the tutorial was spent on discussing algebraic 'rules', patterning and generalisations, and PSTs were required to solve a problem involving seating people at six tables. Sharing of solutions provided another example of eliciting PSTs' thinking and establishing a community of practice.

The final activity focused on functional thinking. Sally shared an example of a function machine and then shared the story *Two of Everything* (Hong, 1993) to talk about the 'rule' and "What would my machine show if I put in 10 and 30 came out?". Sam then connected this with how children learn mathematics through providing examples of multiplicative thinking from his students.

Post-tutorial feedback from the PSTs

Following the tutorial, 32 PSTs completed post-tutorial reflections and 6 PSTs participated in two focus group interviews. The PSTs were asked what they learnt as a result of the tutorial, both in terms of mathematics and how to teach it, and how each of the co-teachers contributed to their learning. PSTs consistently made reference to the practical nature of the class (25 responses) and the connections made with students' learning (22 responses), as the following comments illustrate:

> It's always a practical approach … we often get presented with a problem we have to solve and then discuss [Missy, focus group interview]

> [Sally] gets us to do the activities ourselves so I guess that we can see how they work, how the children might think and then we compare how everybody else has worked out a solution [Sui, focus group interview]

> [Sally] gave me lots of beneficial and useful ideas to use in the classroom including some great hands on activities … [Sam] was always challenging my thinking and gives recent classroom advice and ideas [Casey]

Misconceptions, such as the example of the equals sign, also enabled the PSTs to make connections with classroom practices and how students learn:

> [Sally] breaks down misconceptions in most classes and we have to think about getting rid of those now so that when we go into a classroom, we don't have those misconceptions ourselves [Jamie, focus group interview]

PSTs' comments also showed that they identified as members of a community practice (13 responses), which was facilitated by the co-teaching of Sally and Sam. The following comments are illustrative of the feedback given:

> [Sally] encourages us to share different answers because in a classroom when we ask a question, we have to be able to understand all the students' responses, so she gets us to all respond in different ways to the same question. [Nina, focus group interview]

> Sam always comes around and helps us throughout, asking questions like, "What are you doing?" [Kelly]

Even in a small group he'll come over and he'll look at what we've done and he'll sort of say, "And what would you do if you were to change the level?" … so, he tries to get us to expand on what we've done and questions us differently. [Sui]

The PSTs also commented on the relevance of Sam's classroom experience (18 responses), which enabled them to make connections with the practice of teaching:

It's quite helpful that Sam is actually teaching in a primary classroom and his teaching is very current. [Kelly, focus group interview]

Having a current teacher in the classroom has really helped me develop and understand what children understand and how they learn [Gemma]

Vignette 2: Measurement tutorial

The second tutorial, a week later, was observed by Annabel. It focused on perimeter, area and volume and the PSTs' MCK and PCK of these topics. There were three core tasks in the tutorial: Finding the area of a triangle (computer program): Blocks and boxes (Finding the dimensions of a rectangle prism constructed using 48 cube blocks) and writing a report about the different prisms made; and constructing a rug for Granny's hallway that is one square metre, using newspaper, but it cannot be a square shaped rug (Downton, Knight, Clarke, & Lewis, 2006). The subheadings reflect the categories that emerged from the analysis of the tutorial.

The session began with Sam asking the PSTs to individually brainstorm everything they knew about measurement and he recorded their ideas on the board. Sally asked questions to elicit PSTs' thinking and to go deeper, "What are some attributes that we use but cannot see?" (e.g., time, temperature). "Can we measure all the things listed on the board? Is shape measureable?" Doing so also generated discussion at their tables.

Sally drew on her foundation knowledge when she suggested that angles are aspects of shapes that can be measured, and that shapes are part of geometry. Her intention was to help the PSTs to make connections between concepts and highlight the importance of using correct mathematical language. To explore their foundation knowledge, she asked them to record their definitions of perimeter, area, and volume. Sam then recorded PSTs' examples on the board. He suggested that asking Year 5/6 students to generate their own definitions during mathematics lessons was helpful as it gave the teacher an insight into their thinking. This was an example of connecting to how children learn mathematics and use of formative assessment. The constant interplay between Sally and Sam as they discussed key ideas with the PSTs and how they might explore and model mathematical concepts with students of different year levels, illustrated a community of practice. There were several links to the classroom practice during this discussion.

When helping children learn correct mathematical language, Sally mentioned the importance of having word walls in the classroom. Sam agreed, but made the point that these should be co-constructed with students and that displaying students' work is much more meaningful than using commercial posters. Sally asked them to articulate the meaning of volume versus capacity and used a drink bottle to illustrate the difference and highlight the misconception that some PSTs and students hold—that both volume and capacity are the same—also held by some teachers. Again, Sally made a conscious decision in the moment to challenge the PSTs misconception and did so using a practical example. Linking to the classroom experience Sam asked the PSTs what they might do to introduce a unit on the topic of capacity. He modelled the strategy he uses in his classroom of, "Talk to a friend about your thinking", to get all students discussing this. He then invited different PSTs to share their thinking with a partner.

Making connections and challenging PCK

The discussion shifted to when and how to introduce formulas and the steps taken before this. Sally drew a rectangle on the board and asked PSTs to record the area and perimeter, then to share their thinking. Some suggested the step before would be to fill in a grid and to model it with understanding. Sally suggested linking to multiplicative thinking and the use of arrays, thus illustrating the importance of knowing the mathematics and making connections between concepts. She used this discussion as a link to the 'Blocks and Boxes' task, asking them how they would find the volume of a cube without using the formula. A PST (Kelly) made a model of a cube and Sally asked how many blocks on each layer and how many altogether and how we record cubic measures. She extended this by asking if they could work out the surface area of the cube as well. Sam made the point that this is a typical National Assessment Program Literacy and Numeracy (NAPLAN) question. Sally then asked Sam, "If we were to make a cubic metre how many cubes would we need?" Sam drew the PSTs back to the surface area problem of a rectangular prism. One PST proceeded to find the surface area for each of the external faces would be 2 × (4×6), 2 × (2×8), 2 × (6×2). Seeing that she was struggling Sally provided an enabling prompt, "Draw the net of rectangular prism". Sam said it would be better to build the net. Sally invited another PST (James) to use the model to visualise the net and then to record it on the board. To draw other students into the discussion, Sally asked different students to explain their understanding of surface area. She said that this experience was an example of a 'teachable moment'. Sam then linked back to the 'Blocks and Boxes' task and the need to assess students' prior knowledge. In the post-tutorial interview he said:

> Some students come with knowledge of formulas and need to unpack it so getting students to break it down helps others to understand how the formula was developed and what Sally did was gold as she broke the concept down using the covering of the grid. In school, we need to get students to understand the concepts not focus on learning the rule.

Sally reinforced the importance of having a lesson plan, "… to ensure you are aware of the mathematics, the questions and the key mathematical language". She then introduced the area of a triangle task and both she and Sam roved and challenged the students as they worked. Sam invited different students to share their learning on the board and Sally then challenged them, "How could you find the area of a trapezium?" Sam asked them if they noticed the pedagogical action Sally used. He said, "Going from the known and applying it to a new structure. It comes back to knowing the mathematics and deepening the students' experiences".

Making connections between perimeter and area

The session concluded with the PSTs working in groups of four to do the Granny's Rug task. Both Sally and Sam moved around the groups asking them to explain and justify the process they were using. Due to limited time only one group was selected to share. Sally asked, "What is the same and different about the rugs? What do we want the children to understand? It is important that they have experiences such as this so that they develop conceptual understanding? You need to think about the big idea of the lesson and the enabling and extending prompts, and to get children to explore all different possibilities." The PSTs then completed their reflections.

Post tutorial feedback from the PSTs reflection:

Following the tutorial 29 PSTs completed a post tutorial reflection in which they responded to questions related to their mathematics learning, the co-teaching experience, and how they were assisted. From the analysis of their responses, the most frequent comments related to the hands on nature of the tutorial and use of open tasks and problem solving (21), how they felt Sally and Sam supported their learning (19), the connections made to primary mathematics classrooms (18), how they felt Sally and Sam supported their learning (16) and opportunities to share their thinking and strengthen their own mathematics knowledge (16). Comments illustrative of these include:

> Sally helped me make connections between the area and perimeter before we cut Granny's rug, then after finding out that the area stays the same, perimeter changes. [Jess]

> Sally and Sam helped me to understand the difference between volume and capacity using practical examples and cleared up our misconceptions. [Jules]

> It was difficult trying to explain why the perimeter can change but the area can remain the same. Sally helped me to see the difference and Sam reiterated instructions of the task to make it clear. [Sui]

PSTs indicated that both Sally and Sam supported them within a community of practice and encouraged them to share their thinking, as illustrated by these comments:

> The active participation in the learning and the knowledge shared by Sally and Sam helped our learning of measurement. [Millie]

> The tutorials were a collaborative process of all of us working together and learning together, supported by Sally and Sam. [Zac]

> They encouraged us to discuss our ideas and share them with the class on the board. [Kelsie]

> I needed to clarify my prior knowledge of a square metre. The class discussion led by Sally and Sam and recordings of the models on the whiteboard clarified it for me. [Nina]

> We can receive support/guidance, even if the other teacher is occupied. [Miff]

The PSTs also commented on Sam's classroom experience.

> Having two different approaches to learning and teaching, and Sam sharing his stories from different classroom experiences. [Anna]

> Having the examples from the classroom and seeing how students respond made the learning meaningful. [Jamie]

> I like the way Sam brings the learning back to practical ideas for the classroom. [Casey]

> Sam's classroom background, and each week he brings new ideas and insights to share with us about planning, ways to engage students in class discussions and tips to support students' mathematics vocabulary. [Kim]

Other comments were more general and related to the dynamics between Sally and Sam and the PSTs own engagement, such as:

> With co-teaching tutorials, I find I can remain more focused and engaged. [Gemma]

> Sally and Sam have different ways of explaining things that cater for different people and they work well together. [Issie]

Insights from the tutorial observations

Throughout each tutorial Sally drew on her foundation knowledge as she guided and extended the PSTs' mathematical content knowledge. She used questioning effectively to probe and challenge the PSTs. Sam also capitalised on opportunities to challenge the PSTs' knowledge and highlighted the importance of knowing the mathematics when responding to a teachable moment such as a student misconception. It was evident that some students were challenged by the mathematical content in both tutorials, while others relied on their procedural knowledge rather than demonstrating conceptual knowledge and understanding. It could be argued that these issues would be identified in a regular class of PSTs taught by Sally due to her strong MCK and PCK. However, Sam provided additional support to the PSTs and an opportunity to challenge others to consider the implications for classroom practice. Having a co-teacher from a primary school provided a practitioner's perspective and enriched the PSTs' experience and discussion. There were several examples throughout both tutorials where Sam made links to his classroom experiences and provided examples of pedagogical actions that he used in his classroom, which complemented those modelled by Sally.

Evidence presented in the vignettes, PSTs reflections and post tutorial interviews highlight the dynamics between the co-teachers and the rapport they developed with the PSTs to create a community of practice. Within such a community the PSTs were required to explain and justify their thinking, and communicate with and respond to the views put forward by others. As Enfield and Stasz (2011) indicated, a community of practice helps PSTs to reflect on their learning during the process of having to share their thinking. Others (e.g., Anthony, Hunter, & Hunter, 2015b) argue that "attending, interpreting, and responding appropriately to students' mathematical thinking is a specialised pedagogical skill that needs to be explicitly taught within teacher education courses" (p. 8). For these reasons, the co-teachers made a conscious decision to focus on developing meaningful discourse within their tertiary classroom. In their planning and enactment each week they drew on the talk moves, dimensions of the Knowledge Quartet (Rowland et al., 2009) and the three-function model to facilitate meaningful mathematics discourse (Staples & King, 2017).

Refined model

From the analysis of the data using the dimensions of the Knowledge Quartet (Rowland et al., 2009) and the adapted three-function model (Staples & King, 2017) (see Figure 2 and Table 2), we realised that categories of Knowledge Quartet could extend the application of the Staple and King (2017) discourse model. We derived the relationship between the intersections of the three quadrants, which showed a relationship between the frameworks and practices used. These are presented in Figure 3.

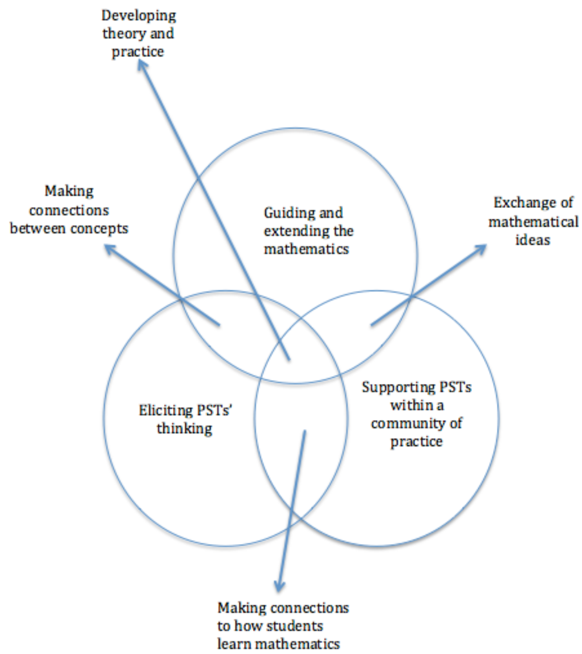

Figure 3. An interconnected model to assist PSTs to link theory and practice (adapted from Staples & King, 2017 p. 39).

Note that in Figure 3:

1. *Guiding and extending the mathematics:* Links to Foundation knowledge (KQ), beliefs, PCK and MCK, and transformation (KQ)
2. *Exchange of mathematical ideas:* Links to guiding the mathematics and supporting a community of practice (Staples & King, 2017).
3. *Supporting PSTs within a community of practice* links to talk moves, and supporting PSTs' exchange in a community of practice.
4. *Making connections to how students learn mathematics* links to making connections to how children learn in the classroom, the affective domain and how we believe our PSTs should learn.
5. *Eliciting PSTs thinking* links to making connections (KQ), and dealing with contingencies (KQ)
6. *Making connections between concepts* links to talk moves, guiding and extending the mathematics, making connections between mathematical concepts.

While this interconnected model can be used by MTEs to assist PSTs to make connections between theory and practice, a co-teaching situation can strengthen these links through the shared experiences and expertise of the classroom teacher.

Final remarks

We have presented a small study relating to the experience of a MTE and an experienced primary school teacher working collaboratively in a co-teaching situation in a tertiary setting. Our intention was to identify whether such an approach would bridge the gap between theory and practice, and to gauge whether a co-teaching situation provides opportunities to elicit PSTs' mathematical thinking. From our findings, we identified four key benefits of engaging in a co-teaching situation. First, having a practicing primary school teacher as one of the co-teachers provided a direct link to the classroom and assisted the PSTs to make connections between the theory and practice of mathematics teaching in a different context from that experienced through practicum. Second, having a classroom teacher working with the MTE in the PSTs' course on a weekly basis enabled greater PST engagement, than might otherwise be the case, and at times challenged their MCK and PCK. It also enabled them to see that an essential part of being an effective teacher of mathematics is having breadth and depth of mathematical understanding (Ma, 1999) and specialised pedagogical skills. Third, there was greater opportunity to elicit PSTs' mathematical thinking, facilitate discourse, and to develop a rich community of practice, from would otherwise have been the case, with just a MTE taking the tutorials because of the interplay that could occur between two teachers. Fourth, having a co-teacher in the room with the MTE provided additional support to PSTs who needed it, and providing for more individualised instruction and assistance. It was also evident from the observations that the PSTs respected and valued the contribution the classroom teacher made to their learning.

Key features of the co-teaching partnership when planning and teaching included evidence of the categories of the two frameworks (Rowland et al., 2009; Staples & King, 2017) by eliciting PSTs' mathematics thinking, facilitating meaningful discourse during tutorials, and establishing a community of practice. As indicated earlier, this interconnected model (Figure 3) could be used by other MTEs to assist PSTs to make connections between theory and practice, with a co-teaching situation, further strengthening this link through the shared experiences and expertise of the classroom teacher.

A limitation of this study was the small sample size, with only two of the ten weeks of the course observed by the researchers. This was due to time constraints. Other limitations included only interviewing a small sample of PSTs after one tutorial, rather than both; the post tutorial reflection questions could have been more focused; and possibly collecting data from a different academic/practitioner set. These limitations will be considered in subsequent studies.

A key implication for future studies and practice is for the MTE to consider co-opting classroom teachers to co-teach in the university context because of the benefits they can provide. Within the co-teaching situation presented in this study, it was evident that there was a good working relationship between Sally and Sam, a key factor that contributed to how PSTs engaged during tutorials.

In summary, the findings suggest that the PSTs found the experience beneficial, particularly the sharing of student work samples and experiences from the primary mathematics classroom. It is hoped that the findings will deepen MTEs' understanding of the need to make explicit connections between theory and practice in their courses, and demonstrate how a co-teaching model can assist to bridge this divide. Using a co-teaching model can also build strong relationships between tertiary educators and those in the profession.

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