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Practitioner Research: Reflecting on Minoritised Student Agency in a Reform-Based Secondary Mathematics Classroom

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There is a global demand for mathematics teachers to learn how to facilitate discussions where students make sense of mathematics via problem-solving and reasoning activities. Despite this call for meaningful discussions, they are rarely found in secondary instruction, especially in low-socioeconomic communities. A practitioner research approach was used to document the types of agency students and teacher employed during whole class discussions in a racially and socioeconomically diverse classroom in the United States. Cobb, Gresalfi and Hodge's (2009) interpretive scheme was used to analyse the ways students chose to identify with the normative expectations of the classroom, documented in the daily field notes and audio recordings. In this paper, two transcripts were used to illustrate examples of student and teacher participation in a 9th–11th grade mathematics classroom. Findings from this research demonstrate the small shifts that occurred allude to the structural constraints of traditional school systems.

Keywords: practitioner research · equitable whole-class mathematics discussions · agency · status

Introduction

Worldwide, reform-based classroom instruction has been perpetuated in mathematics education research for decades. Many STEM education practitioners and researchers have argued that reform-based practices, such as student exploration of procedural and conceptual ideas, can strengthen learners' mathematical understandings (e.g., Johnson, 2012) and simultaneously empower students by valuing their ideas. The authors of the recent Common Core Standards in the U.S. state that all learners should speak about mathematics in ways that students believe are meaningful and rigorous (Yonezawa, 2015). In New Zealand, one of the goals written by the Ministry of Education (2007) urges learners to be able to interpret and communicate contextual problems. Asking students to explain what they know by engaging in mathematical practices can strengthen mathematical conceptions for all participants in the classroom (Kazemi & Stipek, 2001). Therefore, implementation of whole class discussions of students' mathematical ideas is a meaningful endeavour.

Despite the importance of teaching for understanding, teachers struggle to implement reform-based pedagogy (Sherin, 2002), especially in classrooms in low socioeconomic areas. Traditional mathematics classrooms tend to prioritise rote practice of procedural skills over critical thinking or conceptual understanding (Stephens, Landeros, Perkins, & Tang, 2016). In the U.S., students in low-socioeconomic communities or in urban areas receive less exposure to problem-based lessons or whole class discussions than their more affluent counterparts (Spencer, 2009). Without proper support from experienced teachers, reform-based instruction can also advantage students that are already privileged (Lubienski, 2004) and exacerbate inequities for multilingual students (Secada, 1996). This global phenomenon is also evident in New Zealand where there are disproportionate achievement results between indigenous Māori and Pāsifika students compared to their European counterparts.

Learners who identify with traditional mathematics instruction typically identify with being white, affluent, and male (Boaler & Greeno, 2000), requiring minoritised learners to leave their identities at the door (Varelas, Martin, & Kane, 2013). Students who do not identify with normative behaviours of the classroom are perceived by their teachers from a deficit lens (Martin, 2000). Similar to black and brown students in the U.S., Māori and Pāsifika students' cultural ways of being and home languages are often neglected or devalued in New Zealand, which constrains students' opportunities to demonstrate multiple mathematical competencies (Hunter & Anthony, 2011). This disparity also resembles the dichotomy between Aboriginals and Europeans in Australia (Ministerial Council for Education, Early Childhood Development and Youth Affairs, 2009). Worldwide, English-speaking students of European descent are perceived by teachers as more capable than multilingual students from diverse ethnic backgrounds (Vanneman, Hamilton, Baldwin Anderson, & Rahman, 2009), limiting opportunities for minoritised students to express their mathematical ideas. In this context, the term minoritised is used to include any person who identifies with characteristics other than the white, English-speaking, heterosexual, educated, affluent, patriarchal ways of being. It is imperative that researchers are aware of the ways marginalised learners develop their mathematical understandings by examining the agency and status minoritised students display when learning mathematics (Martin, 2012).

In this paper, the agency minoritised students in the United States employed in a ninth-grade mathematics class was explored. I was the researcher-teacher (cf. Ball, 1993). Reform-based methods were used, predominantly Chapin, O'Connor, and Anderson's (2013) Talk Moves, to encourage all students, not just those who score high on achievement tests, to discuss their mathematical ideas. The goal was to shift the traditional classroom to a place where students who have historically been marginalised from mathematics instruction felt empowered to collaboratively make sense of new mathematical concepts. Cobb, Gresalfi and Hodge's (2009) identity scheme was used to analyse how minoritised high school students chose to employ mathematical agency during these discussions.

Theoretical Framework

Sociocultural learning theory was used to guide instructional decisions and Cobb et al.'s (2009) interpretative scheme was used to analyse the ways students responded to prompts to promote student voice. Stemming from the theory that learning occurs collectively through interaction with others (e.g., Rogoff, 1990), participation in whole class discussion is important for student learning and agency. Hearing multiple voices allows the teacher and students to be aware of different ways of viewing the concepts (Forman & Ansell, 2001). When individuals articulate their own thinking they come to a better understanding of the ideas they are expressing (Stein, Engle, Smith, & Hughes, 2008). Mathematics classroom discussions are also helpful for the teacher as he/she uses students' explanations to adjust instruction and to address the needs of their emerging conceptions (Moschkovich, 1999). Students can discuss and argue their current understanding of the material while they build deeper understandings (Forman, McCormick & Donato, 1998). Stein, Grover and Henningsen (1996) promoted the use of tasks that allow for multiple solution strategies so teachers can elicit students' explanations about what they understand. Using students' prior understandings to guide instruction can create a collaborative learning experience (Staples, 2008) which is similar to that of practising mathematicians by requiring students to access each other's ideas, critique other's thinking and collectively generate new approaches (Ball, 1993; Lampert, 1990).

Cobb, Gresalfi and Hodge's (2009) mathematics identities framework was drawn upon to analyse the ways learners chose to contribute to the whole class discussions. Each classroom is a unique microculture with specific social and mathematical norms and obligations that are established and maintained by participants (Yackel & Cobb, 1996). For example, the expectation that the learning community will collaboratively make sense of mathematics is a different obligation than offering brief right-or-wrong responses to the teacher's questions (Lampert, 1990). Cobb and colleagues provided a frame for learners' (students and teacher) expectations of how to do mathematics and how they identify with mathematics within class microcultures. They built the scheme using two of the four levels of Martin's (2000) mathematics identity framework: the *school* level and the *intrapersonal* level. Cobb and colleagues theorised that there are *normative* and *personal* identities of learners' roles in the classroom.

The *normative identity* refers to what it means to do and know mathematics in a particular classroom. This constitutes the general obligations (the ways students demonstrate agency and how they are held accountable) and the mathematical obligations required to be successful in a particular classroom. Participants have the power to shape the culture of the classroom by choosing how they want to engage, and are held accountable by the students and teacher. These choices are referred to as agency. An examination of the normative identity requires one to explore who is in charge and how participants choose to interact. The *personal identity* examines students' agency to identify, resist, or comply with the general and mathematical obligations of the classroom. In this paper, I use the normative and personal identities to analyse how students demonstrated agency in our classroom. As the teacher, I reflect on who had responsibility for the learning processes by taking the initiative to share their mathematical ideas, complied with my prompts to participate, or resisted.

Review of the Literature

Student Voice

Communicative practices have the potential to empower students who have traditionally been silenced in a mathematics classroom. This can be facilitated through individual journal writing (Brenner, 1994) partner talk (e.g., Cobo & Fortuny, 2000), small group collaboration (e.g., Staples, 2008), or whole class discussions (e.g., Kazemi & Stipek, 2001). In a case study of a middle school mathematics class, students valued the knowledge produced by themselves and their peers when they learned through student presentations and small group work activities (Forman, 1996). Hufferd-Ackles, Fuson and Sherin (2004) stressed the importance of utilising students' mathematical ideas to guide whole-class and small group discussions. They found that students in a third-grade U.S. classroom, with predominantly Latinx learners, took up opportunities to explain their thinking and ask each other questions, regardless of accuracy. This math-talk learning community developed because of a slow release of responsibilities from the teacher to the students. Similarly, Hunter (2010) described an Intermediate teacher in New Zealand that positioned Pāsifika students' cultural norms as assets in their classroom community. The students were able to communicate their ideas effectively and have agency over their own learning processes when they felt they were contributing to a learning community. The agency students employ over their own learning processes is dependent on the roles they take up within the structure of a learning community.

Whole class discussions are effective when all learners are pressed to share and justify their mathematical ideas (Kazemi & Stipek, 2001), not just those who are fluent in the language of instruction (Planas & Gregorió, 2004), or those who score high on achievement tests (Ball, 1993). Herbel-Eisenmann, Steele, and Cirillo (2013) illustrate of the *power of positioning* when analysing whose knowledge is valued and whose voice is heard. People often value the ideas of those positioned as having high status while those with low status are excluded from discussions. Instead, learning communities need to be designed as places where all voices are valued and taken up (Hunter, 2010). For example, complex instruction (CI) teachers implement rigorous, group-worthy, open problems and employ strategies that enable all students to learn via communication with each other, regardless of perceived status or rank (Cohen, Lotan, Scarloss, & Arellano, 1999). These structures differ from traditional classrooms where the teacher is the sole authority of knowledge, positioning students as powerful agents.

Agency and Identity

The types of agency students are able to employ vary from classroom to classroom. Turner (2003) defined mathematical agency as, "students' capacity to identify themselves as powerful mathematical thinkers, who construct and use mathematics in ways that are personally and socially meaningful" (p. 11). Thus, mathematical agency can be an empowering experience for students who learn to read and write about the world using mathematics (Gutstein, 2003). Holland and colleagues (1998) refer to the different ways people take up roles in various situations as the agency they have in each of their Figured Worlds. They explained that from one context to the next, people choose to act in certain ways depending on how they identify with those situations. They referred to these choices as *agency*. The behaviours students choose to engage in fluctuate depending on how personally and socially meaningful the structures (tasks, norms, activities) are in the classroom (Varelas, Tucker-Raymond, & Richards, 2015). Dialectically, the classroom structures also change depending on how participants choose to interact. Learners who lack confidence in their mathematical abilities may be less inclined to contribute to whole class discussions in mathematically meaningful ways. This is especially true for minoritised groups of people who have traditionally been marginalised from mathematics (Cohen et al., 1999). Therefore, proper structures need to be established to create a supportive learning community where all participants value each other's ideas.

I draw on these examples because they provide evidence of how learners who have traditionally been perceived from a deficit perspective explored their mathematical ideas with the support of the classroom learning community. All ideas were valued and discussed, regardless of accuracy, to deepen the understanding of the whole class. Wagner and Herbel-Eisenmann (2009) described the fluidity of passive and active positioning done by oneself or onto another, intentionally or unintentionally, as people perceive storylines about their interactions. I use the term, *student voice*, to capture moments when students were positioned as mathematical agents of their own learning process, without teacher prompting. Building on prior research, I list five student voice principles below that guided the instructional decisions and daily reflections in my practitioner research plan:

Students are talking. Participation must be inclusive so that all students, including non-dominant students, feel welcome to contribute to the classroom community using verbal and nonverbal forms of communication (Planas & Setati-Phakeng, 2014; Planas & Gregorio, 2004).

Talk is about mathematics. Learners understand mathematical concepts better when they argue and discuss their reasoning processes (Moschkovich, 1999). Students learn to guide, anticipate, probe, question, argue, and critique each other like mathematicians as they make meaning out of the topics they discuss (Yonezawa, 2015).

Multiple repertoires of academic language are used. Students practice when and how to appropriately use multiple repertoires (formal and informal talk) at different times (Herbel-Eisenmann et al., 2013). Open-ended questions can be answered in any domain to position learners as agents of their own knowledge production (Cope and Kalantzis, 2009). Multiple languages, various levels of academic literacy, and a range of presentation modes should be used to describe mathematical concepts in order to privilege the ideas being discussed, not the language used to discuss them (Enyedy, Rubel, Castellón, Mukhopadhyay, Esmonde, & Secada, 2008).

Ideas must be from the students. Students learn when they express and question genuine thoughts rather than mimicking what the teacher or the text told them to think (Forman et al., 1998).

Everyone's ideas are taken up. The class must acknowledge each idea presented, no matter who shares it or how accurate it is (Forman et al., 1998). Students in the collaborative learning community benefit from hearing all forms of mathematical understandings and logical explanations (Cohen et al., 1999). Many of the studies were conducted in ideal situations, such as advanced courses (e.g., Boaler & Greeno, 2000), with students who were familiar with communicative practices, or with teachers who had professional development and researcher support. The afore-mentioned examples of studies of student voice were conducted in primary or intermediate classrooms. More research is required to explore the outcomes of whole class discussions in classrooms that resemble similar demographics to high school entry-level mathematics. In this paper, these examples are expanded upon by reflecting on efforts to promote student voice with minoritised learners in a high school mathematics classroom.

Methods

Research Statement

This paper addresses findings regarding the research question, in what ways do minoritised high school students demonstrate mathematical agency during whole class discussions in a reform-based mathematics class? Audio recordings were used to demonstrate the ways learners (students and teacher) personally identified with the normative expectations of our reform-based mathematics classroom community. All of the participants in the transcripts were non-white (Latinx, black, Filipina, Chinese American), from a low socioeconomic area, and several are multilingual. Therefore, because they all have some minoritised characteristics in the general sense I differentiated between those who felt confident participating often and were perceived as "smart" by their peers compared to those who rarely initiated mathematical conversation or expressed low confidence in themselves as mathematicians.

Context

I was the researcher-teacher (cf. Ball, 1993) for two Integrated Mathematics 1 classes at an ethnically and economically diverse public high school in the United States for one academic year. This paper will focus on the findings from one of these two classes. Integrated Mathematics

1 is the first required high school mathematics course for ninth graders (similar to Year 10 in New Zealand) and consists of concepts from algebra, geometry, and statistics. Students who do not pass this entry-level mathematics course in ninth grade are required to retake the course until they pass. The high school had traditional school structures (e.g., 50-minute periods, students moved from class-to-class each period, common assessments, pacing calendar). Most of the teachers, including myself, were white and middle class. The class had a mixture of 9th–11th grade students (aged 13–17) with a majority of ninth graders. The racial demographics were 52% Black, 29% Latinx, 16% Asian/Pāsifika (Samoan, Filipino and Chinese), and 3% white (one person). Seventy-three percent of students were on a government subsidised lunch plan. The total number of students fluctuated around thirty-five as students transferred in and out throughout the year. This fluctuation led to a wide distribution of skills and confidence among students.

The range of skills, ages, ethnicities, and socioeconomic backgrounds contributed to the ways students chose to engage with the social expectations of the classroom. At the beginning of the year the students were asked to write a mathography, an autobiography of their mathematics experiences. Fifty percent of the class wrote on their mathographies that they have never felt successful in mathematics. Conducting this exercise in the beginning of the year gives teachers an opportunity to get to know the students, gives students an opportunity to express their mathematics identities, and establish a foundation for the mathematics learning community (Allen & Schnell, 2017). Some students expressed that they did not feel fluent in mathematics by saying comments such as, "I need to go back to third grade. I don't know how to multiply. That's why it doesn't make sense when you're talking about writing equations," or "I just don't apply myself in this class." Initially, students did not position themselves or each other as *doers of mathematics* (Cobb et al., 2009), which made it that much more important to create a supportive mathematics learning community.

The district was in its second year of implementing Integrated Mathematics classes using the Carnegie Learning curriculum (Bartle, 2012). Teachers were held accountable for preparing students for the department unit tests. I closely followed the district's pacing guide using the suggested text while supplementing some tasks from the Integrated Mathematics Project (IMP) (Fendel, Resek, Alper & Fraser, 2003), Discovering Algebra (Kamischke & Murdock, 2007), or Rethinking Mathematics (Gutstein, 2006) texts. Some problems were better suited to provide opportunities for students to discuss their mathematical understandings in small group and whole class interactions than others. Regardless of the task, I frequently used Chapin, O'Connor and Anderson's (2003) Talk Moves to encourage student voice: revoice, restate, apply own reasoning, prompt for participation, and wait time.

Practitioner Research

Practitioner research is an informative but rare contribution to the mathematics education field (Herr & Anderson, 2014) because it provides an opportunity for practice to inform theory, as well as aiding the researcher-teacher to systematically reflect on one's practice through participant observation (Hammersley & Atkinson, 1983). Three prominent practitioner-researchers who focused on problem-solving discussions during an era of mathematics education reform were Lampert (1985), Ball (1993), and Chazan (2000). Each of the researchers facilitated classroom discussions around cognitively demanding problems to deepen students' conceptual understandings. Gutstein (2003), Frankenstein (1990), and Brantlinger (2013) were three other practitioner-researchers who taught critical mathematics using social and political issues. Their goal was to provide a space for students to discuss and reflect on social inequalities within our global society by creating lessons that developed students' critical consciousness using mathematics to examine social injustices, such as comparing proportions of liquor stores to movie theatres in different neighbourhoods. These practitioner-researchers put theory into practice by analysing and reflecting upon their own teaching practice.

In this paper, the two goals of deepening students' mathematical understandings and teaching for social justice were combined by using a practitioner research approach to implement reform-based lessons and reflect on how students chose to participate during those lessons. Data collection and analysis focused on the ways students offered up their own conjectures to a problem, as stated in the Common Core standards (Yonezawa, 2015), and on instructional methods that gave all students opportunities to use their voice. The goal was to create an environment where all students felt comfortable making mistakes, asking questions, and critiquing each other's reasoning to strengthen their mathematical understanding and simultaneously empower those who have historically been left out of traditional mathematics curriculum. I constantly reflected and revised my plan as I evaluated students' effectiveness in communicating their ideas with each other. Data were analysed to explore how students employed their agency to initiate discussion on their own, comply, or resist the classroom expectations, as described below.

Data Collection

Daily audio recordings, daily lesson plans, and daily field notes were collected to document the ways students talked about mathematics. Lesson plans were electronically stored to use as a resource when analysing field notes and audio recordings. Field notes were written at the end of each day or the next morning. These field notes were written to illustrate the five principles of student voice: *multiple students are talking, including minoritised students; talk is about mathematics; multiple repertoires of academic language are used; ideas must be from the students; everyone's ideas are taken up*. These voice principles guided the observations as well as the overall plan to facilitate whole class discussions focused on sense-making.

Although there are many nonverbal forms of participation (Moschkovich & Zahner, 2018), analysis was limited to what was captured in the audio recordings. After summarising themes at the end of each field note entry, and reflecting on who spoke and how they contributed using the five voice principles, themes were used to determine appropriate practices for the succeeding lesson plans. For example, if many students made contributions but the discussion required a low level of challenge, use more challenging problems. If many students shared solution strategies that were previously learned, use open-ended problems for the subsequent lessons. Upon reflection, other themes emerged that were related to the guiding principles, such as students' resistance or compliance to these goals, and how they positioned themselves as learners. These themes were documented daily as my practice changed in response to students' mathematical success in the reform-based classroom.

Data Analysis

Field notes and reflections were analysed the following day by inserting theoretical and methodological notes (Watson-Gegeo, 1997) as comments when an observation or reflection seemed to connect to an overall theme. Daily themes and planning for the next steps were summarised at the bottom of the field notes. In addition to the five voice principles that guided instruction, twelve more themes emerged from the data (Strauss & Corbin, 1990) (e.g., *social positioning; mathematical positioning; racial positioning; resistance; comments about norms; prompted students to talk to each other; student agency*).

One hundred and seventy-two days of field notes were systematically coded using Microsoft Excel, documenting the fidelity of the five voice principles as well as when/how the daily summarising themes occurred. Analysis focused on how many students spoke up (most of the class, some of the class, or only a few), if the ideas were discovered by the students or if they restated previously learned ideas, how rigorous these contributions were, how the community responded to students' ideas, and how minoritised students contributed (those who did not usually participate, did not score high on tests, or shared that they did not feel confident in mathematics) compared to those who readily identified with the reform-based classroom norms. The Excel sheet also contained how often teacher reflections included one of the twelve emergent themes and plans to support more student voice.

The field notes were used to determine ten audio recordings of whole class discussions for transcription. Strategic sampling (Merriam, 1998) was utilised to identify three lessons from the beginning, middle and end of both semesters that seemed to meet at least one of the above-mentioned student voice criteria. Four more discussions were transcribed from the second semester when students contributed who did not usually participate. The whole class discussions were analysed by coding these ten transcripts using the literature to categorise the quality of the discussions. The number of students who shared mathematical ideas and if these contributions were initiated by the students or the teacher was recorded to find examples of how students identified with the normative expectations of the reform-based classroom – moments where multiple students took initiative to contribute to the discussion without prompting from me, the teacher. Although some aspects of effective whole class discussions were met, there was never a time where we enacted a high quality discussion as defined in theoretical research papers (see Table 1). We never accomplished multiple participants, high rigor, and students' initiation of ideas simultaneously.

Table 1.
Summary of ten transcribed whole class discussions.

Lesson	Number of students who spoke/ Time	Quality of discussion	Teacher prompted or student initiated? Social or mathematical norms?
FIRST SEMESTER			
10 Sept Describe the graphs	13 10 min	Low-level: Students responded to the teacher using brief descriptions of the shape of the graphs. Teacher pressed for lengthier answers but no exploration. Teacher up front writing students' ideas.	One person from each group required to share out
15 Oct Solve for a variable	6 6 min	Medium level: Started with student up front explaining steps to solve for height of a triangle. Teacher stood in back and pushed audience to ask questions and make sense of solution. Discussion focused on procedure without comparison of alternative strategies.	One presenter required to present

7 Dec Exponential growth or decay?	9 15 min	Medium level: T stood in back while groups took turns presenting procedural problems. Students were pressed by teacher and each other to explain how they knew an exponential function was growing or decaying.	Each group required to present prepared procedural problem on a poster
SECOND SEMESTER			
8 Jan Who has the better deal?	11 23 min	Medium level: Contextual problem involving a system of two linear equations. Students justified from their seats why they agreed/disagreed w/ speakers' interpretation of the question, 'who has a better deal?' Students used graphs, formulas, and context to prove their claims. Teacher wrote all ideas on board. Discussion focused on context.	Teacher up front writing student-elicited ideas
14 March Relationship between crime and proximity?	15 6 min	Low-level: Contextual problem requiring students to interpret the meaning of a table of values. Students briefly responded to the question, 'what is the relationship between crime and nearness to a police station?' Student up front (with teacher) writing students' conjectures. No exploration of students' ideas.	Student Scribe up front recording student-elicited ideas
18 April Congruent transformations	13 10 min	Medium level: 'Describe the transformation and determine congruence.' Claims were from the students. Students (up front and in audience) justified their reasoning and compared ideas. Teacher asked follow-up questions from back of room.	Student volunteered to present claim up front
26 April Critique Distance Formula procedure	9 4 min	Medium level: 'Find the distance between two points.' Class asked the two presenters questions about their error after stating procedural steps. Students justified their reasoning without prompting. Discussion focused on procedures. Teacher stood in back prompting audience to write what was changed.	Two students volunteered to present procedure up front
11 May Parallel or perpendicular?	7 10 min	Medium-level: 'Are the lines parallel, perpendicular, or neither? Explain your reasoning.' Discussion started with two students up front comparing different claims. Teacher stood in back and pushed audience to agree/disagree with presenters. Some audience members contributed.	2 Students presented different claims up front
19 May Transformations	6 7 min	Medium-level: Describe the transformation and determine congruence of $(x, y) \rightarrow (x, -y)$. Students provided short answers with no justification. Teacher and student Scribe up front asked audience to clarify. Teacher asked audience to evaluate students' ideas. Students revoiced and extended congruence idea.	Student scribe up front
23 May Describe graph of systems of inequalities	5 3 min	Low-level: Graph of system of inequalities displayed up front. Multiple students provided brief descriptions of the graph. Teacher is only questioner. Students' ideas were compared. Students revoiced the solution in their own words. Student Scribe up front wrote students' descriptions. Students projected graph up front.	Student Scribe up front recording student-elicited ideas

Cobb et al.'s (2009) interpretative scheme for mathematical identities was applied to analyse how students identified with the general and mathematical *normative* expectations of our classroom community. The ten transcripts were utilised to document examples of when students expressed legitimate agency by taking the initiative to publicly volunteer their mathematical ideas and how they were held accountable to the expectations. Students' *personal* identities were analysed by noting how they chose to identify, comply, or resist prompts to discuss mathematical concepts. Findings indicate that the types of contributions varied. For example, some transcriptions had a lot of student participation but only consisted of brief responses without any justification. Sometimes students who rarely participated presented a prepared response, but the class did not explore their mathematical conceptions. Sometimes students complied with the normative expectation that multiple students, including minoritised learners, share mathematical ideas, but we never reached a high level of rigor. Depending on the rigor, whether multiple students participated, and if ideas were from the students, each of the ten transcripts were labelled as either low or medium level of quality.

In this paper, two out of the ten transcripts were used to illustrate moments where students employed *some* mathematical agency over their learning processes. Evidence is provided to demonstrate how my in-the-moment pedagogical decisions supported (or consequently constrained) the mathematical ideas that students shared. These excerpts were used to illustrate the social and mathematical norms (Yackel & Cobb, 1996) that were evident in the high school mathematics classroom during the last term of the school year. The various positions (Wagner & Herbal-Eisenmann, 2009) students and I took up as we interacted with each other are also highlighted.

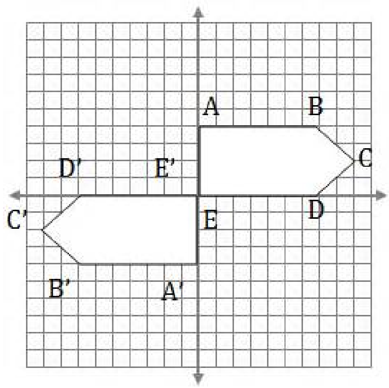
Results

This first example is used to illustrate some of the classroom norms that we established and some examples of legitimate expressions of agency (Cobb et al., 2009) that students and I employed. The discussion section is a reflection on how students' ideas were taken up and what the literature states about opportunities to explore students' thinking. Both transcripts are from the fourth term of the school year, therefore the social expectations of the classroom (norms) had already been established.

"Congruent Transformations" Whole Class Discussion

Martin (pseudonyms were used for all participants) stood up front and wrote his conjecture for the transformation used in *figure 1* below. He wrote, "the shape rotated 180 degrees". Jasmine stood up front and recorded the names of people who participated. Tiana read the prompt from her seat. I stood in the back of the room:

The shape rotated 180 degrees



reflection

Figure 1. Picture of task and Martin's writing on the board during the congruent transformations whole class discussion.

Tiana: Describe in detail the steps you took to find A, B, C, D, E in figure A.

T: It's prime. Say prime.

Tiana: Oh, A prime, B prime, C prime, D prime, and E prime.

T: Thank you Tiana. Martin, what type of transformation do you think happened from this shape to this shape? What do you think happened?

Martin: I'm going to stick with what I said first. One-hundred-and-eighty-degree rotation, that's what happened.

T [to class]: Do you agree or disagree, 180-degree rotation?

[Audience members raised their hands]

T: Yeah. Martin, why do you think that?

Martin: It's because if you rotate it, if you have like, something to rotate, it would go exactly the opposite of this [pointing to the board], but facing this way, I mean, not exactly, but facing this way on the other side of the graph.

T: Okay, one of our tools is transparency paper. Can you show us a 180-degree rotation? (Audio, 4/18/16).

In the above transcription, Martin started the discussion by standing up in front, pointing to the graph and his written answer. I prompted him to use transparency paper as a tool to show the rotation he tried to describe.

After Martin explained why he believed the transformation was a 180-degree rotation, Jerry and KC shared different conjectures from their seats. Martin and Jasmine stood up front and paraphrased what was said by writing on the white board:

Jerry: You can reflect over the x-axis and the y-axis.

[Martin wrote, 'reflection' on the white board]

T: Reflect over the… reflect over the x-axis and the y-axis. So, reflect and reflect.

[T walked up front to label the graph with arrows to illustrate what Jerry said]

Jasmine: *And* it's a rotation?

T: You could say either one, reflection or rotation.

Martin: So … to say it exactly, wouldn't [it] be over the y-axis?

T [used transparent paper on the white board to explore Martin's understanding]: If you reflect it over the y, it would still be...

Martin: Oh! Then reflect it to the x (audio, 18/4/16).

Jasmine, Martin and I stood up front summarising Jerry's idea using words, pictures and transparency paper. By revoicing Jerry's input, I hoped that his ideas were valued and that the audience internalised the variety of solutions.

Last, KC offered a third description from the audience, "I was going to say it can also be a reflection over y then x" (audio, 18/4/16). As the teacher, I was happy to see multiple students willingly share their own ideas. KC, Jerry, and Martin took the initiative to offer their thoughts, without waiting to be prompted by the teacher. They shared three different transformations to describe the change from figure ABCDE to A'B'C'D'E'. Their contributions supported the mathematical practice of making sense of multiple solution strategies.

"Parallel versus Perpendicular" Whole Class Discussion

I use a second example to demonstrate some instances where student agency did and did not take place. This discussion took place one month after the *congruent transformations* whole class discussion, giving students slightly more time to familiarise themselves with the sociomathematical norms of the classroom before engaging in this mathematical argument. Similar to the excerpts above, my goal was for students to develop their mathematical identities by contributing to the whole class discussions, especially minoritised students and those who have not felt successful in the past. I chose the transcript below to illustrate the opportunities students had (and did not have) to share their thinking as a class regarding equations for parallel and perpendicular lines.

Justin, KC and Mona stood up front with white board markers in their hands. I stood on the side of the room by the door (field notes, 11/5/16). I asked Justin and KC to present to the whole class the argument they had during small-group time. Mona stood up front as a recorder (similar to Jasmine's role in the previous example). Justin and KC were positioned by their peers as having high mathematics status. They were not positioned as having high social status. Mona was the opposite. She had high social status but low mathematics status. Martin, Jorge and Kayla contributed from their seats.

These words were projected up front:

Determine if the lines are parallel, perpendicular, or neither. Explain your reasoning.
$f(x) = -5x + 5$ and $f(x) = {}^{1}/_{5}\,x - 5$

KC started the conversation by sharing what she knew about the slopes of parallel lines:

KC: We both know that it's not parallel because they don't have the same slope. But I think it's neither. I don't think it's parallel or perpendicular.

T: You don't think it's parallel or perpendicular. Why?

Mona: Why? Why?

KC: Because the equation parallel to this one is... [Draws on board]

Martin: Ms. Teacher, I know!

T: KC, why do you think that it's neither... Why do you think that those two lines are neither parallel nor perpendicular? Why?

KC: I just think that, because this equation is parallel to this one [points to board], but this doesn't match this one because it has minus five, this is plus five.

T: So, you're looking at the y-intercepts?

[KC nodded her head yes]

Martin: Wait!

T: Hear Justin's argument before you hear Martin's. Justin, what do you think?

I realised that although KC understood that parallel lines have the same slope she was looking at the y-intercepts of the two linear equations, not the number that represented the slopes of the lines. I waited to see if anyone else in the class was inclined to build upon this understanding. Instead, Justin showed the class a graph of the two perpendicular lines and announced, "I'm not really sure, but I used Desmos [online graphing calculator] and it looked like it intercepts at a 90-degree angle." Justin had just shown the class a correctly plotted graph to prove his idea. I wondered if Martin's enthusiasm would build upon KC's claim so I turned the discussion towards him:

T: Okay, so you used a graph? Martin, what's your opinion?

Martin: I go with KC because of the slope.

T: So, what'd you say?

Martin: The slope is the most important thing to determine if it's parallel, perpendicular, or neither.

T: So which number is the slope?

Martin: Five.

T: Point to the slope in both equations… so Martin, what should they be comparing if they're supposed to compare the slopes? What should they be comparing if they're supposed to compare the slopes? Which number are they supposed to be looking at?

Martin: The y.

At this point, I realised that both Martin and KC had a similar understanding of linear equations. I heard both of them say that the slope of a line determines if it is parallel, perpendicular, or neither, which was accurate. In the equation, both Martin and KC used the y-intercept as proof, which was inaccurate. At this point, I tried to use Justin's graph and Kayla's correct answer (below) to connect the numbers in the two equations to the graphical representation of the functions, $f(x) = -5x + 5$ and $f(x) = {}^{1}/_{5}x - 5$.

Kayla: The one over five.

T: Can you point to that please?

[Justin points to projected equation]

Kayla: One fifth.

T: KC, point to the slope in the other equation.

[Both presenters (KC and Justin) point to board. T is back on the side front of room by the door.]

T: What do you know about the slopes? Jorge, what do you know about the slopes? Angelica, what do you know about the slopes?

Angelica: I don't know. Can you sign this though?

Jorge: It’s increasing more. The first one is increasing more.

T: This one goes down 'cause it's negative. Thank you Jorge, good catch. If it's going down, it's decreasing more? Mona, tell the class what you just said, but make sure they're listening…

Kayla and Jorge correctly compared the slopes of the two lines, -5 and $^{1}/_{5}$, to determine the shapes of the linear graphs. Jorge said, “It’s increasing more. The first one is increasing more.” I focused on the correct part of his statement, which was the word, “more.” At this moment, I took authority over the learning process by quickly correcting his understanding from “increasing” to “decreasing” when I said, “This one goes down 'cause it's negative. Thank you, Jorge. Good catch. If it's going down, it's decreasing more?…”

Next, Mona walked over and circled the two slopes in the projected equations, -5 and $^{1}/_{5}$. I noticed she and Kayla were on the right track so I asked her to share what she noticed with the class:

T: Who do you think is right? Mona, what'd you just circle? Mona, what did you just circle on the board?

Mona: Oh, the slope.

[Mona walked up to front and looked closely at the projected equations].

T: And what do you notice about those two numbers?

Mona: They're different.

Martin [from his desk]: They had something similar.

T: What is similar?

Martin: The five one and one five.

T: So, they're reciprocals.

Mona: They're the opposite.

Martin: They're the opposite.

T: Please write opposite reciprocals. [Mona writes the words on the board].

T: Slopes are opposite reciprocals. What does that tell you about the two lines?

Martin: They're perpendicular.

Justin [quietly from up front]: They’re perpendicular. Was I right? Yes?!

[T nods her head yes. Justin smiled and walked over to KC and the projected equations].

Jorge: They’re going to cross (audio 11/5/16).

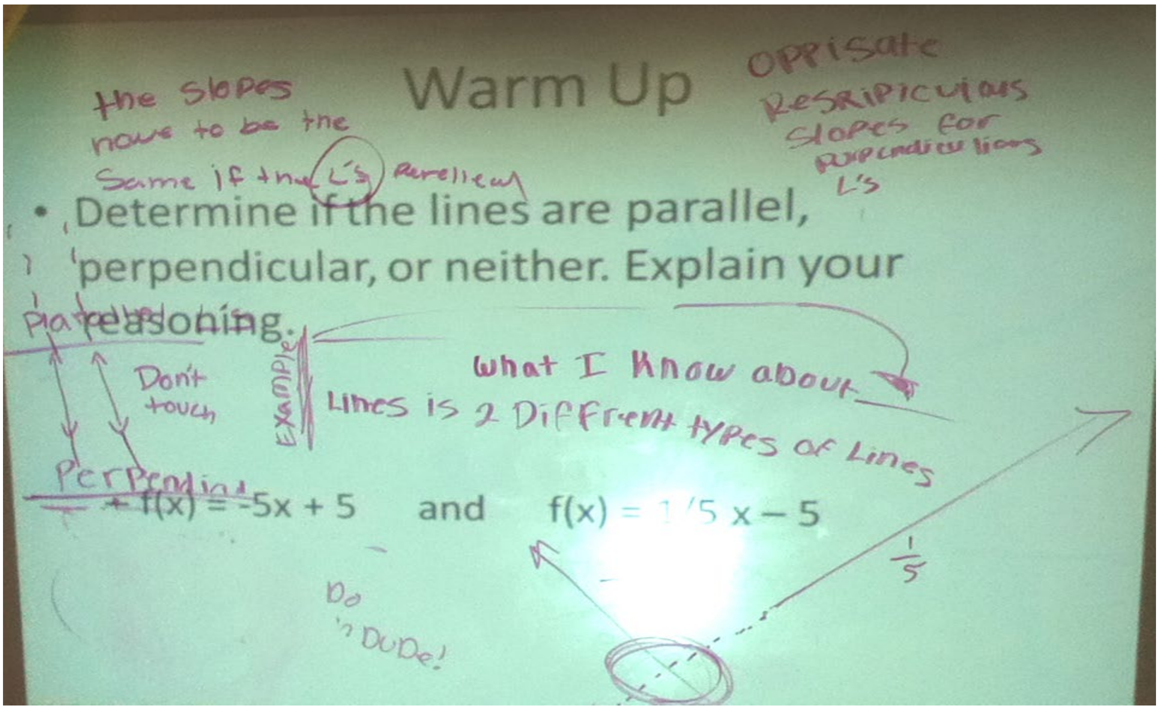


Figure 2. Picture of task and KC, Justin and Mona's writing on the board during *parallel vs. perpendicular* discussion.

Discussion

Identifying with the Specifically Mathematical Classroom Obligations

In the first transcript, Martin, Jerry and KC employed legitimate agency (Cobb et al., 2009) when they volunteered their ideas about transformations without any prompting. Martin started the discussion by stating that he thought the transformation was a rotation. Jerry and KC offered two alternative transformations using reflection. These three students shared their mathematical ideas by providing multiple solutions (Stein et al., 1996) for the class and helped to establish the norm that we discuss all potential responses, which is a mathematical practice (Yonezawa, 2015). Martin and KC were two students who readily identified with the expectation that students publicly share their mathematical ideas. KC and Jerry scored high on assessments. Therefore, their participation set an example for the type of participation I hoped the rest of the class would eventually contribute.

It is important to note that the task in the first transcript was not very cognitively demanding (Stein et al., 1996). Students were simply asked to describe the steps they took to find the new shape. With the exception of Martin's hand movements and transparency paper, the volunteers did little to justify their ideas (Kazemi & Stipek, 2001), which would have demonstrated more conceptual agency than simply stating transformations. A justification requires one to prove why they believe their idea makes sense. This is more sophisticated than describing a procedure or stating a brief answer without any reasoning (Kazemi & Stipek, 2001). The rest of the class would have had an opportunity to employ conceptual agency if I asked them to evaluate Martin, Jerry or KC's ideas. Lastly, KC's statement, "I was going to say it can also be a reflection over y then x," would be more precise if KC used the terms, "x-axis" and "y-axis."

The second transcript also demonstrated some moments when students had authority over their own learning processes. For example, KC, Justin, Martin, Kayla, Jorge and Mona took the initiative to volunteer their opinions without much prompting. They engaged in the mathematical practices by justifying their reasoning (Kazemi & Stipek, 2001) using multiple representations (Stein et al., 1996) and multiple repertoires (Cope & Kalantzis, 2009). KC used the numbers in the equation to prove her argument. Justin chose to use Desmos.com, the online graphing calculator. Mona drew on the white board. Martin, Jorge and Kayla verbally contributed from their seats. These students had the confidence to argue opposing ideas in front of their peers, using multiple resources. Justin and Kayla were also frequent participants like the three mentioned above. Jorge and Mona contributed less often. By actively contributing to the discussion (Wagner & Herbal-Eisenmann, 2009), these students shifted our classroom norms towards a collaborative learning community, even though it remained within traditional school structures.

Identifying with the General Classroom Obligations

Jasmine's and Tiana's roles as the scribe and the reader in the first transcript and Mona's role as the scribe in the second transcript matched the social expectations of our classroom learning community, although they were not as specifically mathematical as Martin, Jerry, KC, Justin, Kayla and Jorge's contributions. Earlier in the academic year, the three girls resisted the classroom norms by occasionally being disruptive or absent. Jasmine was the student who shared, "I need to go back to third grade. I don't know how to multiply. That's why it doesn't make sense when you're talking about writing equations." They did not initially perceive themselves to be *doers of mathematics*.

In the first transcript, I was pleased to see Tiana participate in an accessible way by reading the question. She publicly practiced using mathematical terminology by revising her language to use the word, "prime" to represent the new shape. This is a specifically designated academic instruction in English (SDAIE) strategy for teaching academic language to second-language learners (Bunch, 2013). Although this was a low-risk contribution, her revised language was helpful for the entire class as they learned how to speak about transformations using precise vocabulary, which is good mathematical practice (Yonezawa, 2015), and for Tiana to play a role in our learning community. Similarly, Jasmine's role as the scribe (Kagan, 1989) was a low risk way to participate in the whole class discussion. She publicly displayed her peers' ideas as a way for her to play a central role in the discussion, even though it was also passive (Wagner & Herbel-Eisenmann, 2009) like Tiana's contribution. Although Jasmine and Tiana did not exhibit the agency to share their own mathematical ideas, they chose to engage in the social expectations (Yackel & Cobb, 1996) of our classroom.

In the second transcript, I asked Mona to stand up in front with the marker because she was talking to her friends when it was time to start our whole class discussion. I leveraged her social skills by assigning her the role of the public scribe, similar to Jasmine in the first transcription. I prompted this initial positioning because Mona was resisting the classroom expectations for learning. Mona took up the role of the scribe by summarising the discussion when she took the initiative to circle the two slopes and announce that they are different numbers. She showed she was listening to her peers by connecting all the ideas that were shared by KC, Justin, Martin, Kayla and Jorge when she circled the correct numbers that represented the slopes of the two lines. Mona also pointed out to the whole class that -5 and $^1/_5$ were "opposite." Mona's brief comments played a valuable role in the discussion as both the scribe and connector. Connections are an important aspect of conceptual understanding (Stein et al., 1996). Hearing students' ideas was helpful for me as the teacher so I could adjust instruction to address the needs of students' emerging conceptions (Moschkovich, 1999). Mona shifted from resisting the classroom expectations to complying with the general obligations of the class. At the end, she briefly contributed mathematical connections which changed her position three times.

Positioning of Student Voice

Some of my goals to create an environment where students had opportunities to employ agency over their own learning processes came into fruition. As the teacher, I chose to stand on the side to prompt students to take the lead in the discussion. I positioned the audience as authorities over the knowledge in the first transcript by asking them to evaluate the correctness of Martin's conjecture, "do you agree or disagree, 180-degree rotation?" However, students did not respond to this attempt at positioning. I encouraged Martin to explain why he thought that transformation made sense. I also revoiced (Chapin et al., 2003) Jerry's idea when I restated, "Reflect over the… reflect over the x-axis and the y-axis. So, reflect and reflect." This allowed me to highlight Jerry's transformation for the development of the class. In the second transcript I asked students to determine if the lines were "parallel or perpendicular?" I also asked, "What kind of lines have opposite reciprocal slopes?" These questions promoted a conceptual fluency because students were required to justify their ideas in the way mathematicians are expected to prove their claims (Lampert, 1990). I waited for students to come to an agreed-upon answer via a student-lead discussion. I physically positioned myself at the back of the room and prompted students using Chapin and colleagues' (2003) talk moves to cultivate an organic discussion of mathematical ideas, similar to how mathematicians interact when determining correctness of proofs (Lampert, 1990). I purposefully chose to create an environment that prioritised student sense making with the hope that students would be agents of their own learning processes.

There are challenges involved when interacting in a spontaneous dialogue (Sherin, 2002). After reflecting on what occurred, I realised there was more I could have done to facilitate an effective discussion of ideas. For instance, I took authority over the knowledge in the first transcript when I responded to Jasmine's question about multiple answers, "*and* it's a rotation?" I said, "you could say either one, reflection or rotation." Jasmine took the initiative to ask a question about the two different descriptions of the transformation. Wagner and Herbel-Eisenmann (2009) would say this question positioned her as an initiator in the discussion, which is a powerful role. Instead of exploring her understanding, I responded by announcing my own evaluation of Jerry and Martin's responses, as traditional teachers do.

In the second transcript, I steered the discussion towards the correct answer when I pointed out that KC was using y-intercepts to compare lines by saying, "So you're looking at the y-intercepts?" and after I heard Jorge's contribution, "It's increasing more. The first one is increasing more." Instead, I focused on the part that was correct by focusing on Jorge's word, "more," and I quickly corrected the misunderstanding of the word, "increasing." I did the same thing when I corrected Mona and Martin when they referred to -5/1 and 1/5 only as "opposites" but not reciprocals. Although Jasmine, KC, Jorge and Mona informed me of their mathematical understandings by sharing partially correct ideas (Moschkovich, 1999), these could have been opportunities for me to press (Kazemi & Stipek, 2001) the class to make sense of what they said, which would have promoted understanding of slopes of lines, increased participation, and illuminated more students' understandings of parallel and perpendicular lines. Overall, these interactions would be strengthened if there was more student authority over the knowledge (Herbal-Eisenmann et al., 2013), academic language (Yonezawa, 2015), increased cognitive demand (Stein et al., 1996) or cultural relevance (Hunter, 2010) of the task. Students would have had more agency over the process if they controlled more of the discussion.

Conclusion

In this study, the ways students from a diverse range of ethnic and socioeconomic backgrounds contributed to the whole-class discussions was explored by focusing on the agency employed by the students and me, the researcher-teacher. Using two of the transcribed audio recordings, findings indicate that the successes were that we normalised the mathematical practices, students with low mathematical status played active and passive roles in the discussions, and some discussions built on students' ideas. It was normal for students to take the initiative to volunteer multiple representations (Stein et al., 1996), descriptions or claims. Students also engaged in the mathematical practices when they justified their reasoning for opposing conjectures (Hufferd-Ackles et al., 2004) and by choosing to use tools to argue an idea (Yonezawa, 2015) using transparency paper or a graph. Student presentations up front were a normal part of our classroom activities, as well as a student facilitator (Kagan, 1989) who summarised the shared ideas. Students such as Jasmine and Mona, who initially expressed a lack of confidence or resistance to the expectations, utilised their social skills when summarising discussions.

These small shifts veered away from the traditional initiate-respond-evaluate model of instruction but remained constrained by the low-level of the task and the teacher authority over knowledge (Herbel-Eisenmann et al., 2013). For instance, both Jasmine and Mona stood up in front of their peers with a marker, playing a leadership role by facilitating the discussion. Although this did not empower them to share their own mathematical ideas, as Turner (2003) described, it supported the social norms (Yackel & Cobb, 1996) that we developed in our classroom: students should contribute to the discussion. Playing the role of the facilitator utilised both Jasmine and Mona's strengths as leaders without the risk of sharing their mathematical understandings.

Learning the social norms of a reform-based classroom is important because students need to learn how to interact with each other productively before they feel confident collaboratively discussing more rigorous mathematical concepts (Fendel et al., 2003). It takes time to develop a community of learners where students, especially those traditionally marginalised from mathematics, feel safe discussing their mathematical ideas (Gutstein, 2003). These small shifts are significant because students started to learn how to respond to each other's ideas and play different roles than in traditional classrooms. This shift would be strengthened if there was systemic change in schools and mathematics classrooms regarding people's expectations of what is mathematics and how people do mathematics.

Learning to respond spontaneously to students' ideas is a challenging endeavour (Sherin, 2002). Similar to Secada's (1996) warning of exacerbating inequities, my pedagogical decisions and the classroom structures (tasks, norms, curricula) may have consequently further marginalised some students, which limited the agency they were able to employ. For example, there were moments when I could have turned the discussion towards the students to evaluate correctness instead of guiding students towards the correct idea. Learners are only able to be agents of their own learning processes within the constraints of the classroom structures (Varelas et al., 2015). I encourage other educators to build on this experience so that mathematics education can improve for diverse learners.

Limitations and Implications

This study builds on previous research by examining agency and status that occurred during whole class discussions. Unlike most studies on reform-based classrooms that were conducted under ideal conditions, these examples took place in a secondary classroom constrained by traditional school structures. The transcripts are not exemplars of ideal situations. Rather, they exist as a demonstration of real classroom norms that were established to encourage minoritised students to share and discuss their ideas using Chapin, O'Connor and Anderson's (2003) Talk Moves. Further research needs to be conducted to compare student agency and roles in classrooms where norms are already established at a level where students and teachers feel safe sharing ideas. This can be possible with systemic change regarding how we do mathematics from classroom to classroom. If students are expected to grapple with each other's mathematical ideas from one year to the next, then their collaboration skills will strengthen over time (Fendel et al., 2003). More learners will perceive themselves to be doers of mathematics by changing what counts as mathematically competent (Cobb et al., 2009) in a global society.

Additionally, the evidence captured on the audio recording or written into my daily field notes constrained my assessment of student agency. Paying attention only to spoken ideas does not capture the entirety of student thinking (Moschkovich, 2018). One must remain conscious of which students feel confident and skilled enough to share their ideas aloud. I suggest that future researchers extend this idea by incorporating nonverbal forms of communication into analysis of student agency. I encourage other mathematics educators to take these "small victories" as evidence of the shifts towards student agency that students can exercise and build from there.

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