# Patterns Linking Interpreting and Deciding How to Respond During the Launch of a Lesson: Noticing from an Integrated Perspective

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Researchers have generated a powerful framework that identifies three aspects of noticing students' mathematical thinking: attending to, interpreting, and deciding how to respond to student thinking. Previous research has tended to focus on evaluating how *well* teachers engaged in noticing, and how *well* they connected the different aspects of noticing. We describe a complementary way of studying the connections between different aspects of noticing, one that stresses the *content* of teachers noticing. We report on a study in which participants were shown depictions of students reacting to the launch of a complex task. Participants then chose among a variety of possible interpretations and teacher responses. We found that participants displayed patterns in how they decided to respond to specific interpretations. We describe some of these patterns, as well as similarities and differences between secondary and elementary mathematics teachers. We argue that developing non-hierarchical categories for interpreting and deciding how to respond to student thinking, and describing patterns linking them, reflects how teachers engage in noticing and support teachers in learning how to notice more effectively.

**Keywords** Noticing · launching rich tasks ·*LessonSketch* · pre-service teachers · pedagogical knowledge

## Introduction

Teacher noticing is deeply connected to practice and is part of the professional expertise that teachers develop over time. Jacobs, Lamb and Philipp (2010) identified three "component skills" that work as an "integrated set that provides the foundation for teachers' responses" (p. 173). These three elements of *professional noticing of children's mathematical thinking* are:

- attending to children's strategies
- interpreting children's mathematical understandings
- deciding how to respond on the basis of children's understandings.

Much of the work on mathematics teacher noticing since 2010 has focused on how *well* teachers attend, interpret, or decide how to respond. Researchers have created hierarchical categories within each of these elements in order to evaluate subjects' noticing skill (Jacobs et al., 2010; Yeh & Santagata, 2015; Stockero, Rupnow & Pascoe, 2015; Schack et al., 2013). Researchers have also studied how teachers connect these different aspects of noticing, but again have emphasised how well they did so (Jacobs, Philipp & Schapelle, 2011; Goldsmith & Seago, 2011; Amador & Wieland, 2015; Haltiwanger & Simpson 2014; Tyminski, Simpson, Dede, Land & Drake, 2015; König et al., 2014). Because they focused on evaluation, researchers have rarely described how decisions within these three areas are related; for instance, researchers have not examined which responses are likely to be enacted given particular interpretations.

We seek to enrich the study of noticing by studying the content of teacher noticing and the connections between interpretations and decisions about how to respond. Similar to previous work that has identified patterns in teacher interpretations and responses (see Hoetker & Ahlbrand, 1968; Fey, 1978, and Hiebert et al., 2005), this approach will help researchers describe teacher thinking more authentically and enable teacher educators to support more effectively teacher learning.

In the study described in this paper, we created a representation of the lesson launch of a cognitively demanding task and asked participants to interpret initial student reactions and decide how to respond. We then looked for relationships between specific interpretations of student thinking and specific pedagogical responses. In this paper, the fundamental question is not, "How well do teachers interpret or decide to respond?" nor "How well do they connect their interpretations to their decisions to respond?" but rather "What is their interpretation?" "What move did they decide to make in response?" and "How are these interpretations and moves connected?" This allows us to begin to draw a picture of what "integrated sets" of attending, interpreting and responding might look like.

## Literature Review

Over the last decade, mathematics education researchers have devoted an increasing amount of attention to professional noticing. Early studies documented differences in noticing between novice and experienced teachers (Carter, Cushing, Sabers, Stein, & Berliner, 1988) and between American and Chinese teachers (Miller & Zhou, 2007). Sherin and van Es (Van Es & Sherin, 2008; Sherin & Van Es, 2009) argued that noticing entailed not only what teachers paid attention to, but how they reasoned about what they noticed. These early studies indicate that noticing is an important pedagogical skill because what teachers notice influences what they do (Jacobs et al., 2010; Sherin, Jacobs & Philipp, 2011).

### *Noticing as an improvable set of skills*

Jacobs and colleagues (2010) synthesised this work on noticing and provided a framework for researchers and teacher educators who wanted to study and support the development of professional noticing. They proposed three "interrelated skills" as the basis of professional noticing of students' mathematical thinking: attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understandings. Because experienced teachers who had received professional development connected to students' mathematical thinking were more likely to demonstrate expertise in each of these three skills, Jacobs and colleagues claimed that expertise in professional noticing of children's thinking is a valid construct that can be empirically studied and supported by teacher educators.

Researchers have built on this noticing framework, measuring how well teachers engage in each of these skills, and how well they connect them. Some researchers have studied noticing skills in isolation, evaluating how well subjects engaged in one aspect of noticing based on a set of criteria. For example, Yeh and Santagata (2015) studied attending, rating participants who attended to teacher actions or student behaviour as lower than those who attended to the mathematics of student explanations. Other researchers addressed interpreting. Schack and colleagues (2013) described several different interpretations of student work, and judged these interpretations correct or incorrect based how aligned they were with the interpretations of experts. Goldsmith and Seago (2011), Warshauer and colleagues (2015), and Roth McDuffie and colleagues (2014) each documented how a specific intervention improved participants' ability to interpret students' thinking.

Other studies have examined how well participants connect different noticing skills. One way researchers operationalise skilful connection is by determining if subjects engage in all three subskills of noticing together, generally finding that they did not (Amador & Weiland, 2015; Haltiwanger & Simpson, 2014; Tyminski et al., 2015) or only did so loosely (König et al., 2014). A second way that researchers describe the connections between attending, interpreting and deciding how to respond is by assessing to what extent subjects use evidence from one element of noticing to inform another. For instance, if attending well means being able to describe what a student does or says when solving a problem, interpreting well means basing the interpretation on that description (Bartell, Webel, Bowen & Dyson, 2013; Goldsmith & Seago, 2011).

## *Non-evaluative categories for noticing*

Thus far, researchers have mostly conceptualised noticing on a continuum from less developed to more developed. It is also useful to report what teachers are attending to, how they are interpreting, what they choose to do in response, and to develop categories based on these descriptions without introducing an evaluative code. Without introducing evaluations about the quality of the noticing, researchers can look at the relationships between attending, interpreting and deciding how to respond, determining if there are any prevalent patterns linking these three facets of noticing. This may allow researchers to analyse the nature of the relationships among this "interrelated set" of skills.

One advantage of this way of categorising different aspects of noticing is that it aligns with how teachers think when they make decisions. Research indicates that there are patterns in how teachers attend to, interpret and respond to student thinking. For example, researchers have described IRE (Initiate, Respond, Evaluate) discourse patterns in U.S. classrooms (Hoetker & Ahlbrand, 1969; Fey, 1978; Hiebert et al., 2005). In IRE, teachers ask questions, students respond, and teachers evaluate those responses as correct or incorrect. Wood (1998) documented "funnelling," a pattern where teachers react to student struggles by asking questions specifically designed to elicit correct answers. In both of these discourse patterns, interpretations of student thinking may be limited to "correct" or "incorrect" and teacher responses are limited to "confirm correct answer" or "call on a different student" (in the case of IRE) and "confirm correct answer" or "ask leading question" (in the case of funnelling).

Patterns might also enable teacher educators to differentiate between groups of teachers according to the patterns linking their interpretations and responses. In describing a hypothetical trajectory for pre-service teachers (PSTs), Webel and Connor (2017) identified patterns in the types of questions that PSTs use in response to specific examples of student thinking. In that project, one group of PSTs was more likely to select funnelling questions in response to "simulated" students. A second, more advanced group of PSTs, were more likely to select eliciting questions, which were designed to draw out additional details about the students' solution strategies.

Identifying qualitative categories within each aspect of noticing and patterns that connect specific categories for attending, interpreting and responding may help teachers improve their practice. Categories could act as tools that teachers could use to help them attend, interpret and respond more quickly and efficiently. Furthermore, by learning to “chunk” some of these categories together into effective patterns, teachers may develop the kind of expertise that allows them to process and use information about student thinking quickly in the context of complex classrooms. This “chunking” is what experts do in a variety of complex fields (Bransford, Brown, & Cocking, 2000). We seek to make this chunking more explicit by describing how teachers may engage in it, with an eye towards determining which patterns are effective for specific learning goals.

## *Launching: An opportunity to notice using specific categories*

Teacher educators who hope to support novices in developing skill in noticing student thinking need to provide access to situations where student thinking is regularly on display. This includes the exploration of rich problem-solving tasks (Stein & Lane, 1996), because these can elicit a broad range of student ideas. The implementation of such tasks is often a subject of mathematics methods courses, and includes the skill of introducing, or “launching,” the task. In launching a task successfully, teachers need to support students in making sense of the problem situation, activating relevant prior knowledge, and clarifying the important mathematical questions and relationships without lowering the cognitive demand of the task (Jackson, Garrison, Wilson, Gibbons & Shahan, 2013; Stein & Lane, 1996).

In addition to providing opportunities for noticing, launches also provide opportunities for teachers to use specific categories to interpret student thinking (Wieman & Jansen, 2016). Some students may identify important mathematical quantities and relationships relatively quickly during the launch. An expert teacher may recognise that such a student is ready to work productively on the problem and does not need any further guidance or clarification. Such a student may also serve as a resource during the launch. By mentioning this student's thinking (without developing it towards a solution), the student may help other students begin to focus on important mathematical relationships so that they, too, can struggle productively with the important mathematics of the lesson.

Other students may harbour specific mathematical misconceptions that prevent them from focusing on the mathematical quantities and relationships connected to the learning goal. For instance, in a typical “best buy” problem, students might be asked to identify the best deal; a 10 oz. popcorn for $4 or a 22 oz. popcorn for $7. Students often initially respond by comparing only one of the quantities (Lamon, 2007; Lobato & Ellis, 2010). For instance, a student might say that the best deal is the $4 popcorn, because it costs less money than the $7. This misconception allows the student to “solve” the problem quickly while avoiding the necessary cognitive struggle that will move them toward proportional reasoning. An expert teacher may anticipate this initial reaction to a best buy problem, and plan to have a conversation with students during the launch in which they discuss how reasonable such an approach might be, eliciting a variety of viewpoints, including the importance of taking into account both price and amount of popcorn (all without sharing a specific solution method).

Another especially important aspect of student thinking that often emerges during a launch is how students are making sense of the problem's context. Students may be unfamiliar with the context (Ball, Goffney & Bass, 2005), and may need to make sense of the it before thinking about solving the problem (Jackson et al., 2013). Other students may know the context well, but may attend to aspects of the situation that are disconnected from the mathematics of the lesson (Lubienski, 2000). For instance, the goal of a lesson featuring the "best buy" popcorn task described above may be to have students wrestle with the multiplicative relationship between money and popcorn. Some students may decide they simply do not want 22 oz. of popcorn, or that they cannot afford the 22 oz. portion. These students are familiar with the context, but are attending to constraints that distract them from the multiplicative relationship between popcorn and money. For these students, it may be necessary for the teacher to clarify the context, so that student attention is focused on the important mathematical relationships.

When deciding how to respond to student thinking during a launch, teachers can choose among several categories of possible responses. These include clarifying the context, discussing a misconception, sharing some thinking about important mathematical relationships without endorsing a specific solution, or not responding at all. Teachers may choose to involve the individual student, or the entire class. They may choose to simply mention the idea, or discuss it at greater length. Finally, if they anticipate particular student thinking, they could plan to revise or enhance the task to address it (for instance, if they anticipated students having difficulty with the context, they could include pictures with the task or have students act out the problem scenario during the launch).

The choices described above are framed generally, but we assume that teachers actually engage with the specifics of the situation as they consider their next move (Jacobs et al., 2010). On one hand, they need to decide that they want to discuss an idea, how to word their questions, how to position the students, what mathematical representations to share, and so on. The options above do not capture this detail. On the other hand, there is utility in describing teaching moves more generally, so that such moves can be recognised and used across mathematical contexts. Such general descriptions have been used widely in mathematics education to refer to instructional practices that, when applied, must take into account contextual detail. Examples include the "five practices" (Stein, Engle, Smith & Hughes, 2008) and the eight "Mathematics Teaching Practices" outlined by the National Council of Teachers of Mathematics (NCTM, 2014). Knowing general moves to adapt to specific contexts is an important element of teaching expertise.

In the activities that we describe below, we similarly frame choices in general rather than specific terms. Such general categories allow us to compare patterns across different situations, which is the primary goal of this project—determining whether certain *types* of moves would be more commonly used when a participant perceives a certain *type* of situation. Without collapsing moves into larger "types," it would be difficult to detect patterns. In any case, we do not mean to suggest that the moves that we have framed are the best possible way to frame teacher choices in particular situations. Their generality simply allows us to explore any patterns that arise between interpretations and responses to student thinking.

In summary, launching a task well requires teachers to engage in professional noticing. They must attend to how students are reacting to the task, interpret what this means about students' mathematical thinking, and then decide how to act in response to this thinking. Furthermore, as described above, there are general categories for how students may initially think about a rich task that teachers may use to help them interpret, and general moves they may call upon when deciding how to respond. The practice of launching a task, therefore, provides a suitable context for exploring how novice teachers make sense of and apply categories when they engage in noticing, and determining whether there are patterns in how novice teachers connect these categories.

## *Qualitative categories in a launch: Departures from previous noticing*

The research described in this paper differs from previous work on noticing in two important ways. The first way is the context in which participants interpret and respond to student thinking. Jacobs et al. (2010) asked participants to attend to and interpret the mathematical thinking of individual students. Participants' responses were limited to asking a question of the student and were evaluated according to whether they based their question on their interpretation. We are examining noticing in the context of a simulated full group launch. In such a situation, there are a variety of possible moves, regardless of content, that teachers might make. Deciding how to respond not only involves content decisions, but also questions of audience, focus, and duration.

Another difference between our work and that of researchers cited above is our use of multiple-choice options for interpreting and deciding how to respond which was a specific pedagogical choice connected to our learning goals for participants. Based on the literature described above, we hypothesised that participants already possessed a relatively limited schema for interpreting and responding to student thinking. The multiple-choice options represent an enlarged schema that has the potential to expand participants' teaching repertoires in the context of a launch. There is a danger that in constraining participants' responses to multiple choice options we fail to capture the subtle nuances of teacher thinking. However, given the documented tendency of teachers to respond to student thinking by evaluating it as right or wrong, and then correcting or praising (see Hoetker & Ahlbrand, 1969; Fey 1978; Hiebert et al., 2005) a larger set of options might reasonably be expected to lend greater nuance to participant responses, rather than less. In addition, we included a choice of "other" for participants who felt overly constrained by any of the given choices. More detail about the affordances of the choices we designed is provided in the methods section.

## *Research questions*

Our new conceptualisation of categories *within* each of the subskills of noticing leads to a different way to think about connections *between* the subskills. Instead of simply asking whether different aspects of noticing occur together, or whether teachers apply evidence from interpretation when deciding how to respond, we can ask whether teachers consistently connect specific interpretations with specific decisions about how to respond. In this study we ask participants to choose between different categories of interpretation and different decisions about how to respond to the thinking of fictional students taking part in a simulated launch. By tracking how participants categorise examples of student thinking, and which moves they then plan to use to progress or challenge student thinking, we identify and describe connections between these categories and provide evidence that interpreting and deciding how to respond are integrated.

Our research questions are:

1. Are there patterns linking participants' interpretations with their decisions for how to respond to student thinking?
2. Do elementary and secondary candidates exhibit different patterns?

It bears repeating that we are not asking how *well* our participants interpret or decide how to respond. The categories for interpreting and responding described below represent a range that, in a given situation, may prove to be effective or not. Our concern, in this paper, is to see if participants exhibit patterns in connecting their interpretations and responses.

# Methods

## Participants

This study took place in the context of mathematics teaching methods courses for elementary and secondary mathematics teachers at three different sites.

- 17 undergraduate and 3 graduate PSTs at a mid-sized, regional public University in the Mid-Atlantic region of the United States enrolled in a year-long secondary mathematics methods course with a weekly middle-school field experience.
- 14 participants at a different mid-sized, regional, public, Mid-Atlantic university enrolled in a master's level mathematics pedagogy class. Some participants were current teachers in middle and secondary classrooms, while others had no teaching experience.
- 46 elementary preservice elementary teachers at a large public university in the Midwest region of the United States, enrolled across four sections of a hybrid content/methods courses, also coupled with a local elementary field placement.

The selection of participants was designed to help us examine general patterns across different contexts, as well as possible differences between contexts. Although the sample was relatively small, having such a diverse group of participants allows us to imply that our findings have some general application beyond a single context.

## Data collection

Participants in all sites engaged in a *LessonSketch* experience designed to support them in learning to launch a complex task. (Throughout the paper, "participants" refers to subjects of this study who engaged in the *LessonSketch* experience. "Students" refers to K-12 students depicted in *LessonSketch* experiences.) *LessonSketch* is an online platform that allows researchers to create interactive experiences involving representations of teaching. These experiences capture the complexity of classroom interactions while allowing the creator to focus participants' attention by choosing what to depict and asking participants specific questions (Herbst, Chazan, Chen, Chieu & Weiss, 2011; Wieman et al., 2016).

In the experience, participants viewed a series of slides describing:

- the context of the classroom and launch, the method of exploration, and a summary of the structure of the lesson.
- the purpose of the launch (to help students engage in the task without directing them towards a specific solution strategy or lowering the cognitive demand).
- the task the students would be working on (which participants had, themselves solved previously).

- the mathematical goal of the lesson.

The specific tasks launched in the experience were different, depending on the participants' grade level focus. Two tasks were designed to elicit a range of initial student reactions, thus providing opportunities for participants to interpret and decide how to respond to a range of student thinking.

The secondary participants were shown a proportional reasoning task:

> I built my children a sandbox, and I need to buy some sand to fill it up. There are two different places near my house that sell sand, the lumberyard and the hardware store. At the hardware store they sell thirty-pound bags of sand for six dollars, and at the lumberyard they sell fifty-pound bags of sand for nine dollars. Which store has the better deal?

This problem provided opportunities to focus on the multiplicative relationship between quantity and cost, and to consider common misconceptions students may have about these relationships (Lamon, 2007; Lobato & Ellis, 2010).

The elementary participants were shown a task in which students had to maximise the area of a rectangle given a fixed perimeter:

> Tina raises rabbits to sell for extra money. She has 16 feet of fencing with which to build a rectangular pen to keep the rabbits. If Tina wants her rabbits to have as much room as possible, how should she design her pen?

This problem provided opportunities to focus on the meaning of linear and area measurement, and consider common misconceptions that students may have about these measures and the relationship between them (e.g., Moyer, 2001; Battista, Clements, Battista & Van Auken Borrow, 1998). Both problems also provided opportunities to show students working to make sense of context or focusing on aspects of the context that were unconnected to the important mathematical relationships.

After this introduction, the participants engaged in a series of interactive slides similar to that shown in Figure 1:

- In the first, participants were asked to predict questions students might have and approaches they might try.
- Participants were then shown eight different initial statements made by students in one slide (see Figure 1 for the secondary version) and asked to think about them.

Participants were then shown slides zooming in on each of these initial eight reactions. For each slide, participants were asked two multiple-choice questions. The first question was, "What does this student's response tell you about their thinking about the problem?" Participants were then given five possible choices:

- The student is thinking about the important mathematics of the lesson in a way that will lead to a solution
- The student has a misconception that will get in the way of them creating a correct solution
- The student is thinking about the context in a way that obscures the mathematics of the lesson
- The student is confused about the context
- Other

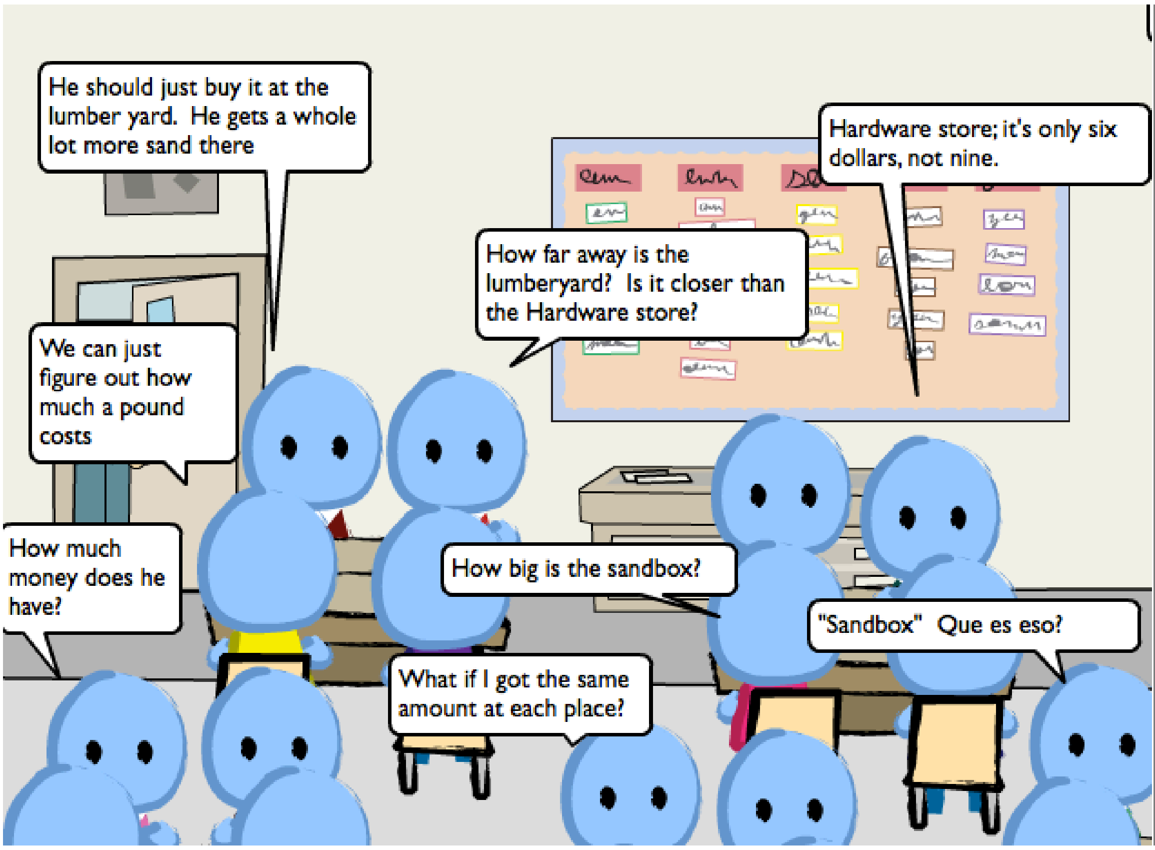


*Figure 1*. Initial student reactions - sandbox task.

These choices mirrored the types of initial student responses to rich tasks described above. The first choice ("the student is thinking about the mathematics in a way that will lead to a solution") reflects the idea that some students may quickly identify the important mathematical relationships and show readiness to think about those relationships productively. The second choice reflects the possibility that students might exhibit misconceptions that may prevent them from thinking about the important mathematics. For instance, in the secondary task, if they simply look at the price of the two bags of sand, and conclude that bag with the smaller price is cheaper, they will not work to make sense of how price and amount of sand are related (Lobato, 2007).

The final two choices reflect the importance of context as a potential support or barrier to student learning (Lubienski, 2000; Jackson et al., 2013). Students may simply not understand the context, or they may understand the context well, but pay attention to aspects of the context that distract them from attending to the important mathematical relationships.

To explore possible instructional responses, after making a selection participants were presented with a second question: "Given this possible [student] response, how would you plan to deal with it?" Participants were then given six choices:

- Discuss during the launch with the whole class
- Discuss during the launch with the individual student

- Mention during the launch, but do not discuss at length
- Do not address at all during the launch
- Revise the problem before the launch
- Other

These choices created variance along two different dimensions. The first dimension was audience: would the teacher choose to address everyone, the individual, or nobody? The second dimension was emphasis: would teachers not discuss the issue, discuss it briefly, or discuss it extensively? Finally, we included the option of changing the task as an alternative to discussion. These categories allowed teachers to clarify contextual confusions, address misconceptions through reasoning and dialog, and provide access to important student thinking.

The examples of student thinking provided in the experiences (e.g., Figure 1) were designed to elicit a range of different interpretations. For example, the speech bubble reading "how big is the sandbox" might be likely to elicit an interpretation related to the context, while "we can just figure out how much a pound costs" might be more likely to be interpreted as a student thinking about important mathematics. Although we had particular categories in mind when we created the initial student reactions, the goal of this research study was not to determine whether or not participants chose the "correct" interpretation. We were studying how they decided to respond once they selected a particular interpretation. By providing a range of initial student reactions, we hoped to elicit a range of initial interpretations, as well as a range of decisions about how to respond.

Overall, the experiences were designed specifically to align with two of the three elements of the professional noticing framework, interpreting and deciding how to respond. The first question, "What does this student's response tell you about their thinking about the problem?" prompted participants to engage in interpretation. The second question, "If you could anticipate this response, what would you plan to do?" prompted participants to engage in deciding how to respond based on their interpretation of student thinking. Our data does not address attending, the first sub-skill of noticing. By using *LessonSketch* to zoom in on student comments one at a time, we were focusing participants' attention on specific examples of students' mathematical thinking. Furthermore, we did not ask participants to describe what the students *did*, as did Jacobs and colleagues (2010) when they discussed attending.

Our decision to use multiple-choice questions was based specifically on the alternative conceptualization of noticing described in the introduction, as well as research on how students initially make sense of complex tasks, and what may be required during a launch to prepare students for productive struggle. As noted previously, we hypothesised that by experiencing these categories as multiple-choice options, teacher candidates would have the opportunity to make sense of these categories and expand their own schema for interpreting and responding to student thinking. Ultimately, we hoped that this would lead to more nuanced interpretations and responses (Wieman, Perry & MacAneny, 2015; Wieman et al., 2016; Wieman & Jansen, 2016).

We acknowledge our choices do not capture all of the nuances of decision-making in which a teacher launching a task might engage, such as what specific questions they would ask or how they would represent a student's ideas. But our choices do represent substantially different approaches to handling a particular student's idea. For instance, a participant choosing to address an individual student who wants to know how much sand is needed might simply tell them that they need over 1000 pounds, or might ask them whether it makes a difference.

However, whatever question they ask, choosing to address an individual is considerably different from addressing the whole class, or ignoring the issue altogether. If there are patterns between participants' interpretations of student thinking and their responses, even coarsely defined, these seem worth documenting.

## *Data analysis*

To analyse the data, we tallied the number of times participants gave each specific response for each item. We also kept track of this information by grade level (elementary vs. secondary).[1] We made similar counts for each example of student thinking in both the elementary and secondary versions, and combined answers across items. This allowed us to look at patterns between types of interpretations and types of responses. For instance, we could look at all the instances of participants choosing "the student is thinking about the context in a way that obscures the mathematics of the lesson" and count how many times participants decided to respond to this specific interpretation by discussing it with the whole group.

We sought to identify general patterns by looking for *which* responses were selected by the larger group, given a particular interpretation. We counted the total number of each interpretation type (e.g., how many participants selected "the student is confused about context" across all examples of student thinking), and within each of these, the total number of each response type. This allowed us to see if participants were more likely to decide to respond in a particular way given a specific interpretation.

We also looked for which interpretations elicited each of the responses, looking at the total number of each response type (e.g. the "do not address" response across all examples of student thinking), and within each of these, the total number of each interpretation type. This allowed us to see whether, given a specific response, participants were more likely to have interpreted student thinking in a particular way.

In both of these analyses, we retained the ability to separate the sample between secondary and elementary teacher-participants, so that we could look at whether patterns were different between the two groups. The charts in the results section provide examples of these patterns.

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[1] For instance, we knew that for Item 1 in the secondary version (the student said, "How big is the sandbox?")[1] participants made the following choices when asked, "What does this student's response tell you about their thinking about the problem?"

- The student is thinking about the important mathematics of the lesson in a way that will lead to a solution (0)
- The student has a misconception that will get in the way of them creating a correct solution (18)
- The student is thinking about the context in a way that obscures the mathematics of the lesson (13)
- The student is confused about the context (2)
- Other (1)

# Results

## *Are there patterns linking participants' interpretations with their decisions for how to respond to student thinking?*

Overall patterns in how participants chose to respond to specific interpretations are shown in Figure 2. These show that when participants selected the interpretation that students were *confused about the context* of the task, they were more likely to choose to respond by *discussing with the individual student* (42% of the time) than with any of the other interpretations. This may indicate that participants see contextual confusion as a barrier to productive struggle. By addressing it individually, participants may hope to remove this barrier without distracting students who are ready to address the important mathematics of the problem. However, 49% chose to address the whole class, either by mentioning it (17%) or having a longer whole class discussion (32%). These participants may see contextual confusion as a general problem rather than an individual one.

When participants thought that students were "*thinking about the important mathematics of the task in a way that would lead to a solution*", they were more likely to elect *not* to respond at all (48% of the time) than to choose other responses. This may indicate that participants did not want to move towards the solution during the launch. Alternatively, they may see students who are already thinking about the important relationships as not needing additional support. Revising the task was not often chosen for any interpretation.

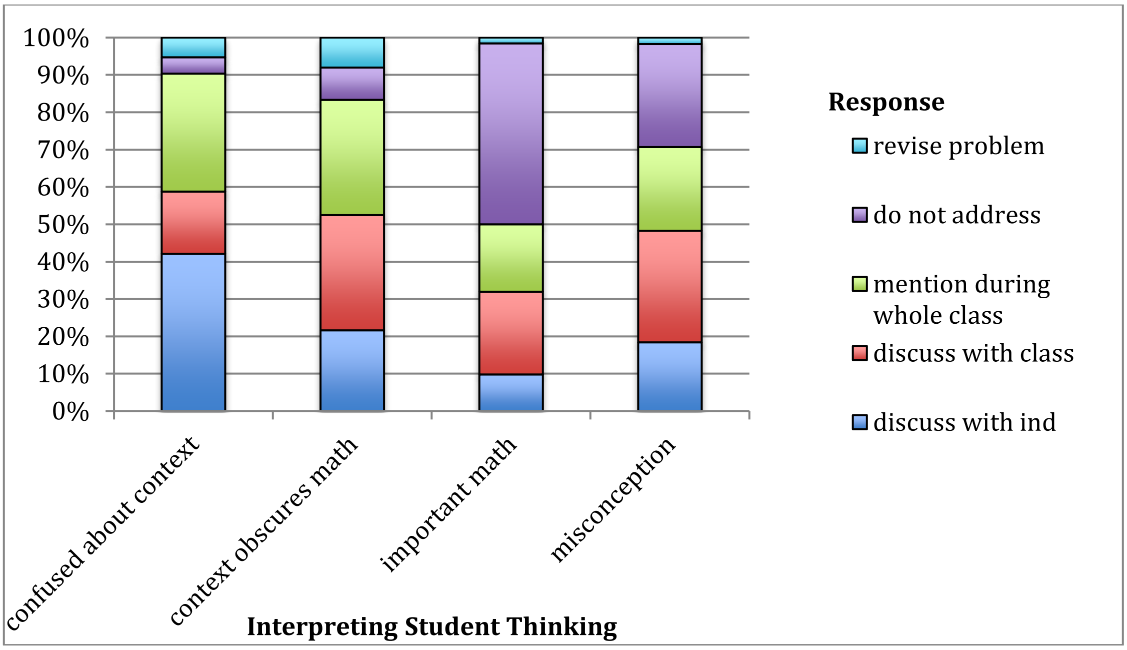


*Figure 2:* Relationships between selected interpretation and responses.

Looking at the data from the other direction reveals how certain responses were precipitated by particular interpretations (see Figure 3). When participants decided to not address student thinking, it was most often in response to what they interpreted as correct mathematical thinking (58%), However, nearly a third of the time that they chose not to address student thinking, it was in response to a misconception (30%). Only 12% of the time, when they chose not to address student thinking, was it in response to issues with context. Overall, when participants chose not to address student thinking, they did so when they were focused on the students' mathematical understanding, and generally did not choose this response when they interpreted students as struggling with context.

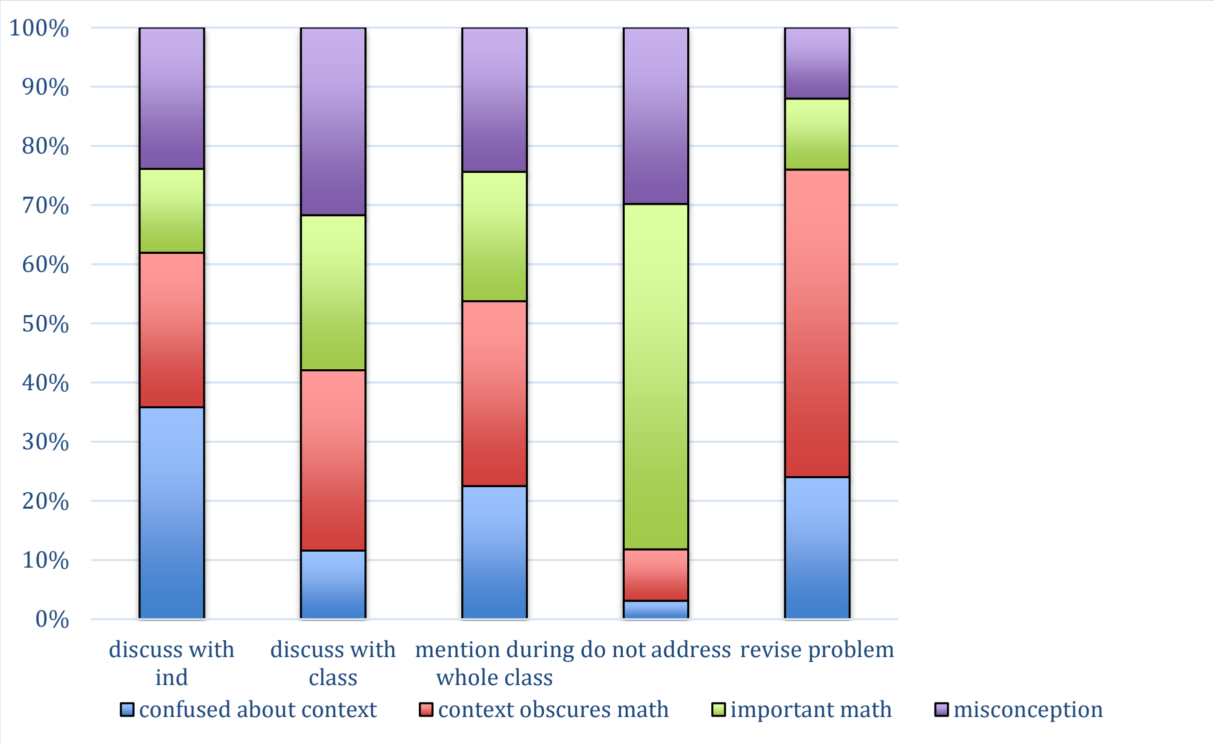


*Figure 3:* Relationship between selected responses and underlying interpretations.

Another pattern we can see in this data is that participants chose to revise the problem in response to problems with context. In 52% of the cases in which they chose to revise the task, it was in response to students who were attending to the context in ways that were not connected to the important mathematics of the lesson. In 24% of the cases, it was in response to students being confused by the context.

In looking at which interpretations elicited discussion, it is interesting to note that there were fewer pronounced differences between what elicited "discuss with the individual" and "discuss with the whole class." In general, when participants decided to respond by discussing with individual or with the class, there was not a single interpretation that was likely to have led to this response. However, participants seemed more likely to be prompted to discuss with the individual in cases of students confused by context, and discuss with the whole class if students were showing evidence of moving towards a solution.

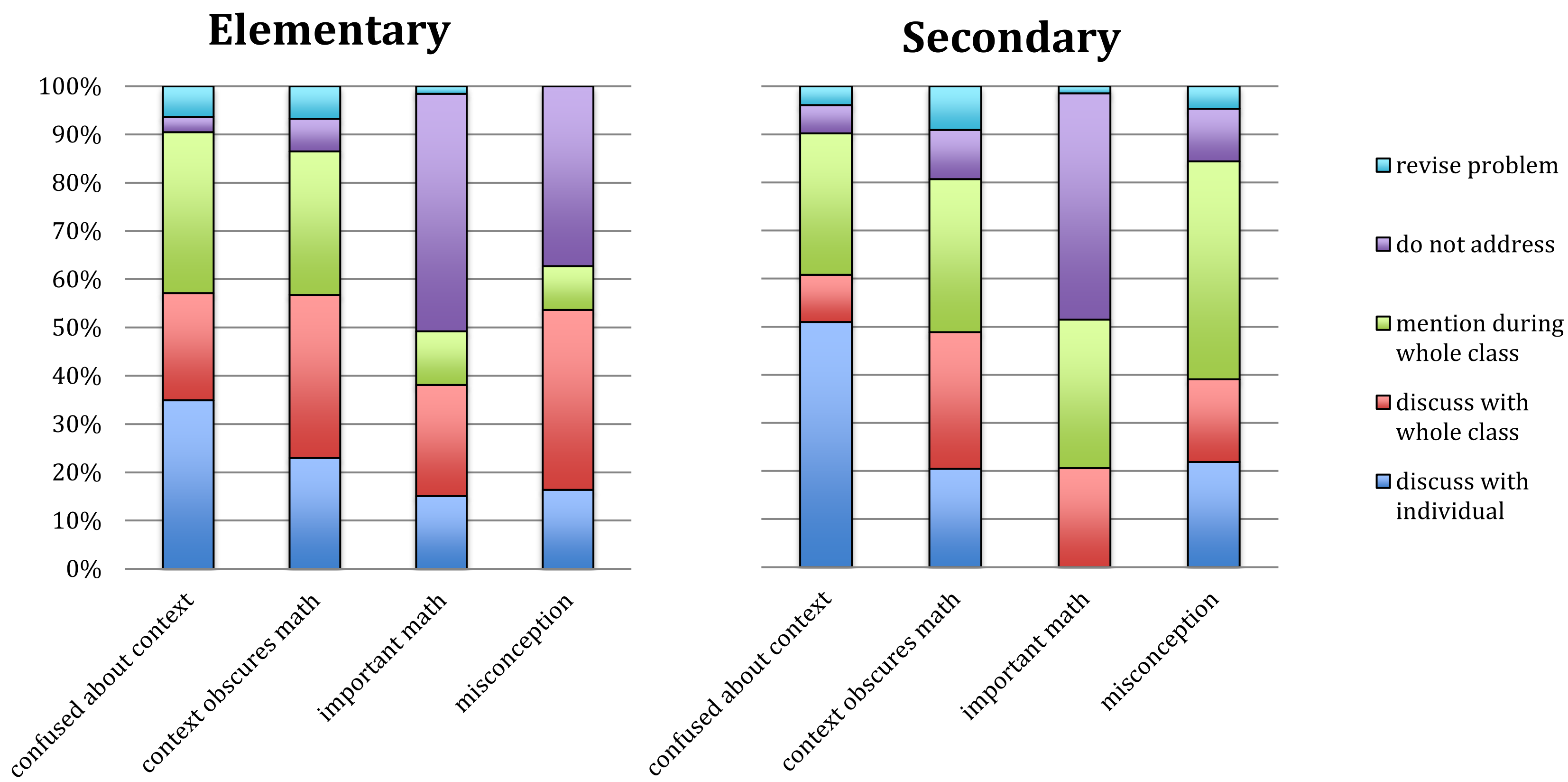


*Figure 4:* Distribution of all interpretations/responses for elementary and secondary participants.

## *Do elementary and secondary teacher candidates exhibit different patterns?*

In Figure 4, the relationships between selected interpretations and responses are broken out elementary and secondary participants. For some interpretations, elementary and secondary teachers respond to student thinking similarly; for others, they gave different responses. For example, when participants believed that context obscured the mathematics, their selected responses where remarkably similar. Figure 5 shows this pattern clearly. When they selected "context obscures the mathematics," participants generally chose to respond by discussing or mentioning the student's idea to the whole class, or by having a discussion with the individual student. Participants seemed to think that paying attention to aspects of the context that were not connected to the math of the lesson presented a barrier to productive work that needed to be addressed during the launch. Unlike being confused by the context, participants were less likely to discuss this with the individual student, and more likely to engage the whole class, either with a short mention, or a longer discussion.

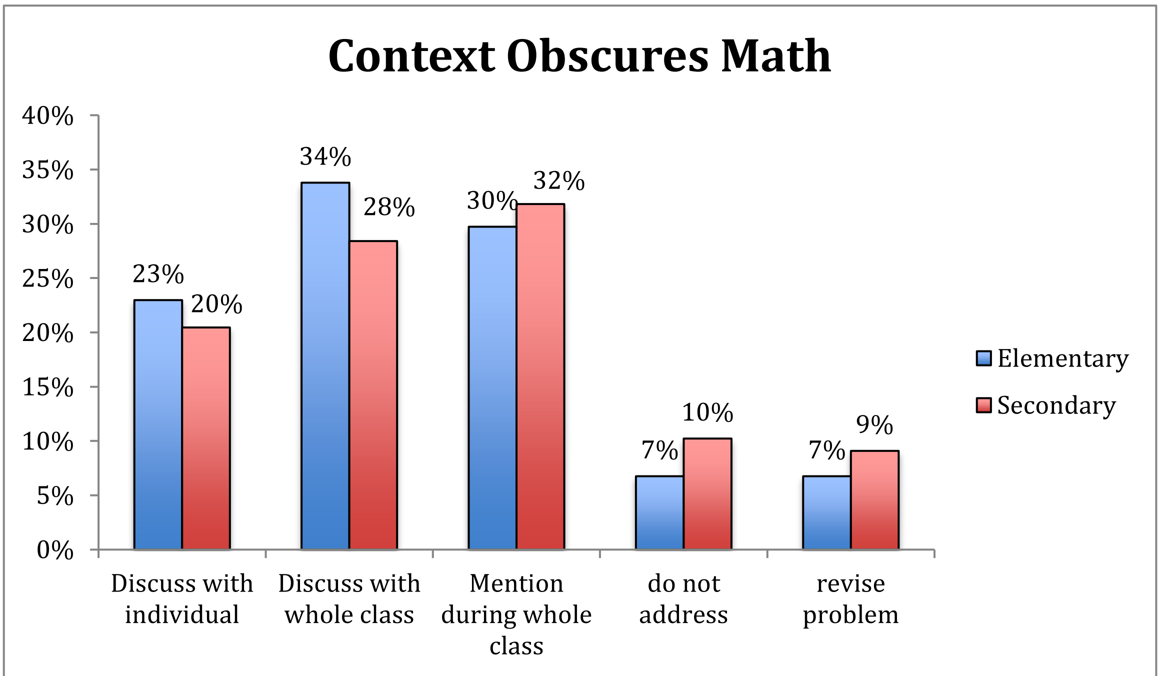


*Figure 5:* Elementary and secondary responses for the "student is thinking about the context in a way that obscures the mathematics of the lesson" interpretation.

Another notable pattern was that when participants believed that a student's idea represented important mathematics that would lead to a correct answer, many chose the "do not address" response (see Figure 6). This response is one that we might expect - the goal of the launch is to make sure students have access to the important mathematics, and these interpretations indicate that participants believe that the students do indeed have access. However, a substantial number of participants wanted to address the idea with the class (34%

for elementary, 52% for secondary). Perhaps participants wanted *other* students in the class to have access to a valid solution strategy.

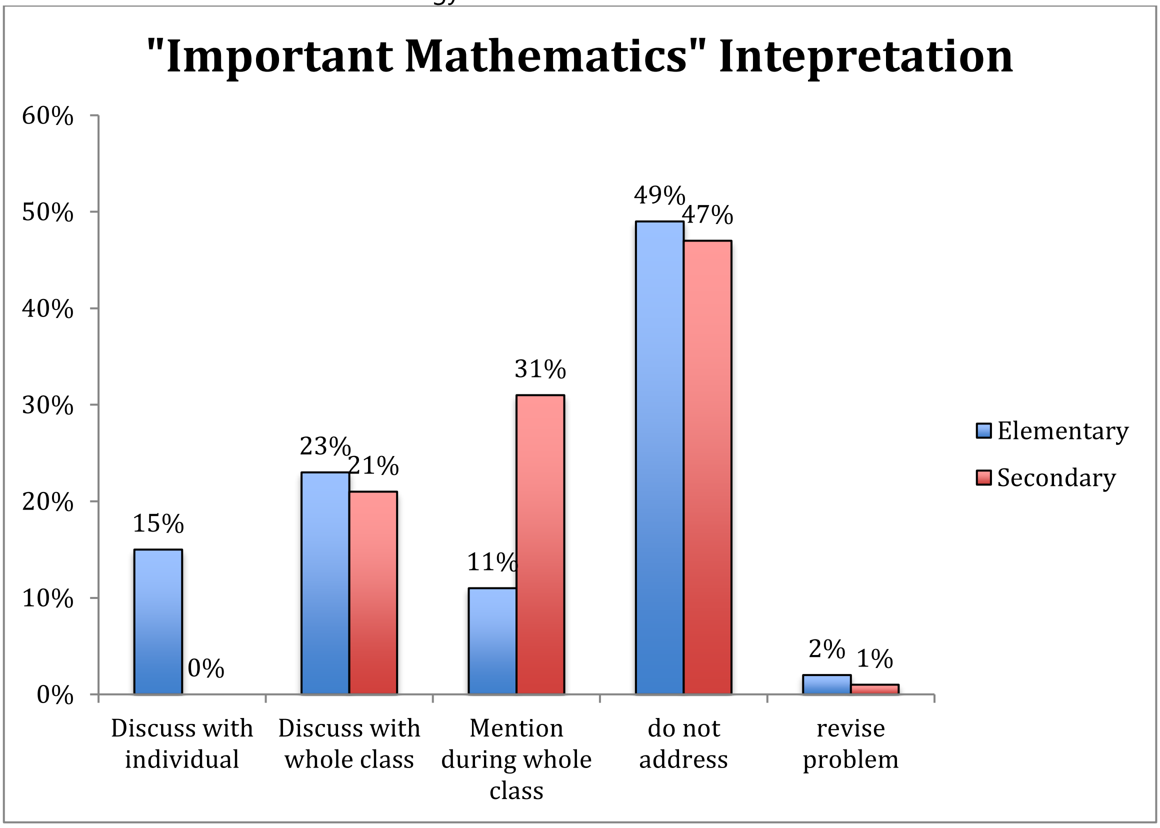


*Figure 6:* Elementary and secondary responses for the "student is thinking about the important mathematics of the lesson in a way that will lead to a solution" interpretation.

Another pattern that we noticed in the elementary group is that when participants believed that a student's idea represented a misconception, 37% of the time they decided not to address it at all (see Figure 4). This percentage was only 15% for the secondary participants, a significant difference ($\chi 2$ [1] = 14.05, $p < 0.001$). In one Item in the elementary task (Rabbit Pen), a student response was, "Let's try a 10 by 6," followed by another student who said, "Yeah, and then an 11 by 5". The majority of participants (32 out of 46) interpreted this as a misconception (see Figure 7). While many chose to discuss the students' idea during whole class (12, or 38% of the 32) or with the individual students (13%), several chose the option of not discussing it during the launch (31%).

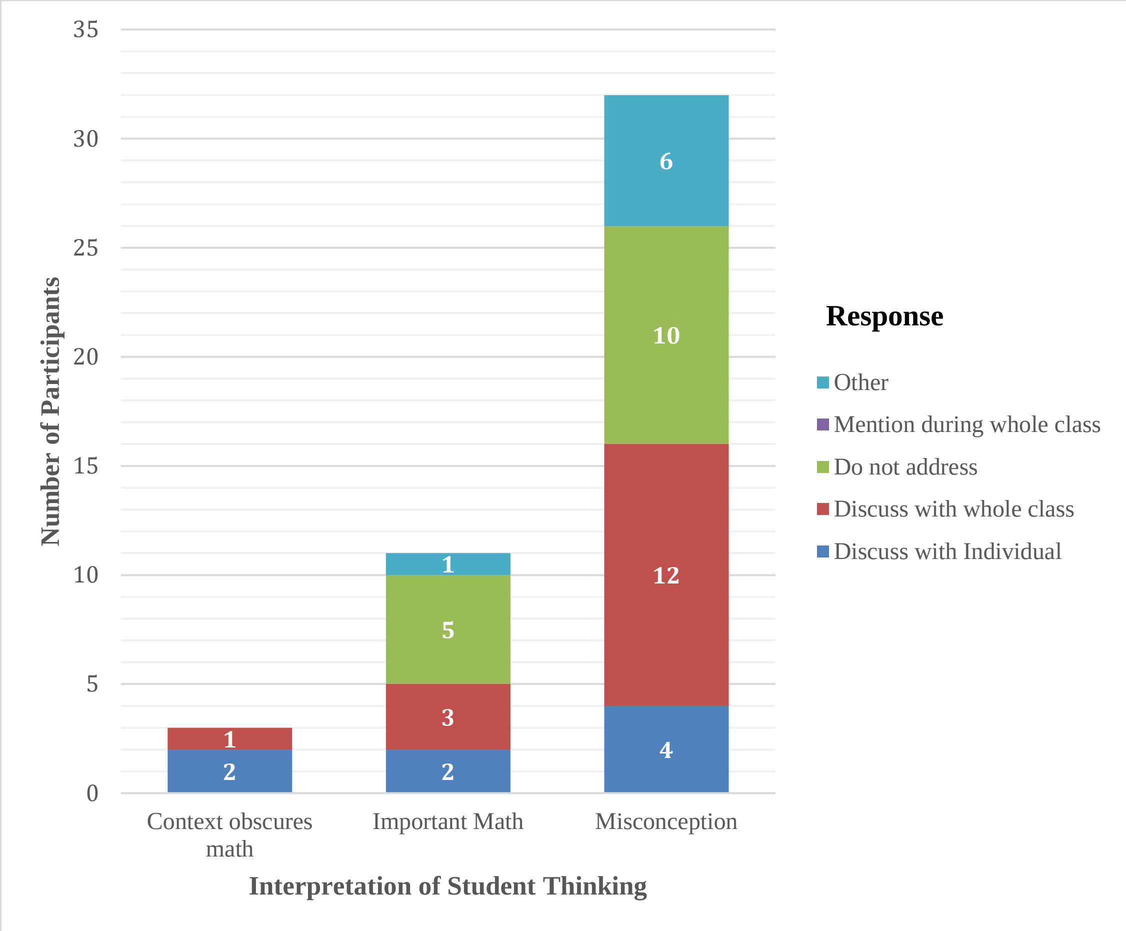


*Figure 7:* Elementary participants' interpretations/responses for item 3 ("Let's try a 10 by 6").

This was a surprising result; we anticipated that if participants identified a misconception, they would choose to address it in some way. Nevertheless, the elementary group exhibited this pattern consistently, responding similarly in the example in which one student said, "I think it is a 4 by 4," followed by a student who disagreed, saying, "That's a square, not a rectangle!"[2] Again, a substantial proportion of participants who believed this was a significant misconception chose not to address it during the launch (9 out of 25, or 36%).

In contrast, in examples where a substantial number of secondary participants believed that students showed misconceptions, they were more likely to address the issue during the launch. For example, one secondary student reaction was, "[buy from the] hardware store. It is only six dollars, not nine." While 20 out of 34 (59%) of the participants identified this as a misconception,

[2] We designed this example as a student misconception that limits access to the mathematics in the task, because if students do not believe that a square is a rectangle, they will not test the example that is in fact the solution to the problem—the case in which all four sides of the pen are the same.

only 4 (or 20% of those that identified it as a misconception) elected not to address this during the launch. In general, elementary participants were more likely to respond by electing not to address the misconception at all when compared to secondary participants. We hypothesise that this might be for one of three reasons:

- Elementary participants do not want to embarrass students whose response may be incorrect.
- Elementary participants do not want to "confuse" students by sharing incorrect thinking or answers.
- Elementary participants may worry that sharing implies validation, and they do not wish to validate a misconception.

Some elementary participants seem to believe that misconceptions are mistakes to be avoided, not opportunities to develop reasoning. This may be related to their own sense of mathematical efficacy. The secondary participants in this study were dual math and education majors, and may have been more confident of their ability to navigate misconceptions while facilitating a discussion.

## Discussion

At the beginning of this paper, we argued for operationalising noticing through non-hierarchical categories which would allow researchers to identify patterns between different ways of attending, interpreting and deciding how to respond. Our findings provide evidence that there are patterns connecting interpreting and deciding how to respond. For example, when participants interpreted student thinking as leading to a correct answer, a substantial portion of them chose to simply not address this during the launch, although many secondary teachers chose to address it with the whole class. Other data showed that there were different patterns with different groups of participants. Elementary participants, for example, were much more likely than secondary participants to respond to a perceived student misconception by not mentioning it.

### *Implications for teacher education*

The development of non-hierarchical categories for interpreting and deciding how to respond to student thinking can help teacher-educators understand how teachers actually make classroom decisions. In past studies of noticing, teachers have been given time to describe student thinking in detail, interpret this thinking and justify that interpretation using evidence, and then decide how to respond and justify those decisions. However, during teaching, teachers do not have time to describe, analyse, interpret and then choose from a variety of options, carefully weighing each one. Indeed, research on patterns of interaction, including IRE and funnelling, indicate that teachers make sense of, and respond to, student thinking by engaging in pre-determined approaches based on a specific schema (Hoetker & Ahlbrand, 1969; Fey, 1978; Hiebert et al., 2005; Wood, 1998). Although the participants in this study were also asked to notice outside of their actual practice, having them choose from a specific set of choices reflects this idea of decision making as schema-driven.

Non-hierarchical categories can also act as a tool for teachers working to improve their noticing. When faced with complex decisions, experts "chunk" large amounts of information according to conceptual frameworks (Bransford, Brown & Cocking, 2000). IRE and funneling represent non-optimal chunking - categories of interpretation and response that foreclose opportunities for student sense-making and discourse. The non-hierarchical categories that we describe could serve as accessible, expandable frameworks that help support teachers in moving beyond those that they currently employ. For example, having a category for being confused about the context may help teachers move beyond a general sense of student confusion, differentiating contextual confusion from mathematical misconceptions. Chunking also can apply to the connections between different categories. For instance, teachers can plan ahead of time to facilitate a discussion with the whole class about an important misconception during the launch. This then turns two processes (interpret misconceptions and then decide to discuss as a group) into one.

In order to develop more effective schemas for noticing that allow teachers to chunk information while teaching, teacher educators need to provide opportunities for teachers to make sense of and use new categories for interpreting and deciding how to respond to student thinking. Teachers also need opportunities to examine the patterns linking interpreting and responding that they already use, identifying effective patterns of instruction, and rooting out patterns that are ineffective or do not align with their stated goals and values.

## *Limitations*

In this paper, our data suggest that describing interpreting and deciding how to respond in terms of descriptive, non-hierarchical categories enables us to identify patterns between these different sub-skills of professional noticing. Furthermore, we argue that seeing professional noticing in this way will enable teacher educators to identify more focused learning goals in the service of improving teacher noticing. However, our data are limited. Our multiple-choice categories may mask differences in how teachers might enact specific moves, and we do not know teachers' rationales for choosing specific responses. Two participants that choose to revise the problem, for instance, may have very different reasons for doing so, and consequently revise the problem in very different ways. We do hope that despite the potential differences *within* any give response or interpretation, we have captured important and clear differences *between* different interpretations and responses. For instance, there are many ways to discuss something with the whole class, but they all differ substantially from not mentioning at all, or revising the problem.

In this paper, we do not describe how to make people better at choosing the "correct" category, nor do we study how to get people to "link" specific interpretations with specific teacher responses. We are, instead, trying to document and understand the links participants make when presented with options for interpreting and responding to student thinking. Furthermore, although we do provide rationale for the categories we created for our instrument, we do not claim that these are the best or most effective ones for teachers launching tasks. Finally, we do not specifically address the subskill of attending. These are all worthy subjects for inquiry, but beyond the scope of this paper.

## *Further Research*

If our theory is correct, teachers already use categories to help them notice, and engage in identifiable patterns linking categories between attending, interpreting and deciding how to respond. Research could verify that this is the case, describe categories of attending, interpreting and teacher response that teachers already use expand our understanding of the patterns connecting these different aspects of noticing.

Another area for research might be describing or determining how different groups of teachers compare regarding categories they use when noticing. For instance, do experienced teachers use different categories than novice teachers?

Similarly, researchers might examine how different contexts affect the categories that teachers use. For instance, teachers who give only "procedures without connections" tasks (Stein & Lane 1996) may have fewer opportunities to attend to student understanding of context.

Another area for research is to determine which categories and which patterns might be most effective for specific goals. Examining teachers' rationale for their choices would also help researchers' understand the connection between teacher knowledge and belief and teacher action, and will help those looking to develop a theoretical framework that can explain teacher actions and guide teacher educators in supporting more effective noticing.

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