Pre-Service Mathematics Teachers' Self-Perceptions of Readiness to Teach Secondary School Mathematics

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It has been argued that pre-service mathematics teachers (PSMTs) must possess a substantial level of both mathematical content knowledge and mathematical pedagogical knowledge to teach mathematics effectively. Therefore, studies have often evaluated teachers' classroom readiness against these factors. However, to-date, only a few studies have sought PSMTs' perceptions of their own readiness to teach. In this study, we evaluated PSMTs' self-perceptions of readiness to teach secondary mathematics. Specifically, PSMTs' self-perceptions of classroom readiness were explored in terms of mathematical content knowledge, pedagogical content knowledge, and mathematical knowledge for teaching. The study was conducted at an Australian university with campuses in different states, and includes PSMTs in both undergraduate and postgraduate programmes. Our results indicate that the majority of participants feel adequately prepared to teach lower secondary school mathematics. However, further training is required to develop both their content and pedagogical knowledge to confidently teach upper secondary mathematics.

Keywords Secondary mathematics teachers · Pre-service teachers · Mathematical content knowledge · Mathematical pedagogical knowledge· Self perceptions

Introduction

To effectively teach mathematics, teachers must possess a substantial level of mathematical content knowledge (MCK) and mathematical pedagogical knowledge (MPK). Adequate content knowledge in mathematics requires knowing the mathematical procedures, and also having a deep understanding and knowledge of connections between mathematical principles and concepts (Ball, Thames, & Phelps, 2008). Similarly, teachers require a sufficient level of MPK in order to support students in learning the mathematical concepts (Ball, Hill & Bass, 2005; Harris & Jensz, 2006). Additionally, while the professional experience (or practicum) is commonly regarded as pivotal for pre-service teachers' learning and development, few studies have sought to qualify the extent of the practicum on improving pre-service teachers' readiness to teach secondary mathematics (Hine, 2018). As such, it is critical to discern pre-service secondary mathematics teachers' (PSMTs) self-perceptions of readiness to teach secondary school mathematics, and to determine if there are any areas of learning that they might require further training.

Research Aims and Significance

This research project had two specific aims. The first aim was to explore how PSMTs understood and perceived their own readiness, in terms of MCK and MPK, before and after they undertake their first practicum and hence teach mathematics for the first time. This aim was achieved through an analysis of their self-perceptions against key themes presented in the theoretical framework. The second aim was to investigate if there were any additional areas of learning or professional development that these PSMTs required to undertake this role. The significance of this research lies in the assumption that current tertiary education programmes adequately prepare students for a secondary mathematics teaching role, and that research into this area can strengthen future efforts in preparing PSMTs. The research itself extends on the findings on a similar study (Hine, 2018), which was conducted with Graduate Diploma of Education (GDE) students at an Australian university. In this study, we evaluated the perceptions of all undergraduate and postgraduate PSMTs at a university with two campuses in different states of Australia. In addition to an increased participant base, the survey data for this study were supplemented with testimony gleaned from semi-structured, face-to-face interviews.

Literature Review

PSMTs' Self-Perceptions of Readiness

In all teacher education programmes, there is an assumption that teachers who complete the requisite mathematics courses, pedagogical courses, and practicum should be ready to teach secondary school mathematics at a sufficient level for student learning. As such, the majority of studies have investigated teachers' MCK or MPK in association with the teachers' effectiveness in the classroom (Ball et al., 2008; Beswick & Goos, 2012; Norton, 2010). However, it is becoming increasingly apparent that performance, or course achievement during the teacher education programme, may not be directly correlated with classroom readiness (Burghes & Geach, 2011; Tatto et al., 2008). To address this issue, recent studies have assessed PSMTs' classroom readiness by evaluating teachers' self-perceptions of their readiness to teach secondary school mathematics (Hine, 2015; Hine, 2018). These studies reported that the majority of participants feel ready to teach secondary mathematics at a sufficient level for student learning. Despite this assertion, as many as half the participants in one of these studies self-reported a need for further training in MCK and MPK required to teach lower secondary mathematics, and a higher percentage required further support for teaching upper secondary mathematics (Hine, 2018). Interestingly, participation in the practicum appears to positively influence PSMTs' self-perceptions of readiness (Hine, 2018).

The Role of the Practicum

Numerous studies have reported that pre-service teachers consider the professional experience component to be a major influence in their teacher education and training (Allen & Wright, 2014; Goos, 2006; Smith & Lev-Ari, 2005). The opportunity to enact theory in the classroom makes the practicum an integral part of pre-service teachers' learning and professional development. However, the multi-faceted nature of practica, including the school and its dynamics, the

experience and teaching style of the mentoring teacher, as well as the student cohort, means not all pre-service teachers will receive the same training and opportunities for personal development during the practicum. Specifically, the practicum can sometimes conflict with what is taught at university, especially when the mentoring teacher takes a traditional approach to teaching (Shane, 2002). In addition, the practicum may not afford PSMTs the opportunity to observe or teach secondary mathematics using current pedagogical approaches (Cavanagh & Garvey, 2012). Such hindrances could influence PSMTs to default to the type of teaching they experienced in their own schooling and hence do not broaden their pedagogical knowledge and practice (Eames & Coll, 2010). With these factors in mind, it is therefore important to explore PSMTs' self-perceptions of readiness before and after their first practicum.

Theoretical Framework

Three interrelated themes form the theoretical framework for this research, namely: MCK, MPK, and the domains of Mathematical Knowledge for Teaching (MKT). These themes are now explored within the context of preparing PSMTs for the teaching profession.

Mathematical Content Knowledge (MCK)

There is a substantive literature base to support the claim that knowledge of mathematical content is central to its teaching (Norton, 2010). Ma (1999) contended that teachers require a *Profound Understanding of Fundamental Mathematics*, which she described as a knowledge base concerned with the depth, breadth, connectedness, and thoroughness of mathematical concepts and theory. Additionally, Schoenfeld and Kilpatrick (2008) asserted that proficient mathematics teachers possess a broad and deep knowledge of the mathematics taught at school level, as well as knowing multiple methods of representation and how ideas develop from conceptual understanding. Various empirical studies have suggested strongly that the knowledge of mathematics teachers positively affects student achievement (Baumert et al., 2010; Campbell et al., 2014). Furthermore, there is evidence to suggest that the consolidation of PSMTs' MCK during initial teacher training contributes positively to their MKT, MPK and confidence to teach mathematics successfully (Hine, 2015). Given that the participants in this study are training to be secondary mathematics teachers, MCK is defined as knowledge related to or underlying the secondary school mathematics content assessed at Years 7-12.

Mathematical Pedagogical Knowledge (MPK)

Following extensive research on the relationship between teachers' MCK and their ability to teach, there is clear and growing evidence to support a positive association on this relationship (Ball et al., 2005; Ma, 1999; Norton, 2010). Scholars have suggested that teachers require a development of MPK, which has been described as an intersection of subject knowledge and pedagogical knowledge (Delaney et al., 2008). In consideration of MPK development, Baumert et al. (2010) have identified MPK as a stronger predictor of student learning than MCK, presuming teachers' adequate content knowledge. For this study, MPK can be understood as knowing a variety of ways to present mathematical content and to assist students in deepening their understanding of mathematics (Ma, 1999). More recently, the profound knowledge of mathematics and methods

of representing it to students has been described as MKT (Delaney et al., 2008). These authors have maintained that in addition to possessing a deep knowledge of the content (i.e. the 'what' of mathematics), teachers must also know 'how' to teach mathematics.

Mathematical Knowledge for Teaching (MKT)

In light of Shulman's (1999) proposal that teaching knowledge is a complex, multi-dimensional construct, Ball et al. (2008) analysed extensively the work of mathematics teachers and hypothesised a conceptual framework for MKT. As represented in Table 1, this framework comprises two overarching domains, Subject Matter Knowledge and Pedagogical Content Knowledge, each of which are comprised of three sub-domains. Subject Matter Knowledge comprises the sub-domains: Common Content Knowledge (CCK), Specialised Content Knowledge (SCK), and Horizon Content Knowledge (HCK). Pedagogical Content Knowledge consists of the sub-domains: Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Curriculum (KCC).

Table 1
Domains of MKT. Adapted from Ball et al. (2008, p. 403)

Subject Matter Knowledge	Pedagogical Content Knowledge
Common Content Knowledge (CCK)	Knowledge of Content and Students (KCS)
Specialised Content Knowledge (SCK)	Knowledge of Content and Teaching (KCT)
Horizon Content Knowledge (HCK)	Knowledge of Content and Curriculum (KCC)

For the purposes of this research, each of the six domains of mathematical knowledge for teaching is described and contextualised with an example in Table 2.

Table 2
Domains of mathematical knowledge for teaching defined. Adapted from Ball et al. (2008), pp. 389-407.

Domain	Definition	Example
CCK	The mathematical knowledge and skill used in settings other than teaching.	Knowing the algorithm to multiply together two numbers.
SCK	The mathematical skill and knowledge unique to teaching.	Knowing the algorithm to multiply together two numbers connects to place value and the distributive property.
HCK	An awareness of how mathematical topics are related over the span of mathematics included in the curriculum.	Knowing how the algorithm to multiply together two numbers is related to multiplying together two polynomials.
KCS	Knowledge that combines knowing about students and knowing about mathematics. Teachers must anticipate what students are likely to think and what they will find confusing.	Knowing that when multiplying two numbers students may make the error of appropriately 'shifting' the terms to be added.
KCT	Combines knowing about teaching and knowing about mathematics. Many of the mathematical tasks of teaching require a mathematical knowledge of the design of instruction.	Knowing what teaching strategies to employ so that students, when multiplying two numbers, learn how and why to appropriately 'shift' the terms to be added.
KCC	Represented by the full range of programmes designed for the teaching of particular subjects and topics at a given level. The variety of instructional materials available in relation to these programmes, and the set of contradictions for the use of particular curriculum or programme materials in particular circumstances.	Knowing what instructional materials are available for teaching and learning multiplication of two numbers, what approach these materials take, and how effective they are.

Methodology

Methods

This study was interpretive in nature and used qualitative research methods to collect and analyse data about how PSMTs perceived their readiness to teach mathematics. Drawing upon the theoretical perspective of symbolic interactionism (Crotty, 1998), the researchers placed themselves in the setting of those being studied, and to consider situations from the perspective of 'the actor'. Methodologically, symbolic interactionism requires researchers to take, to the best of their ability, the standpoint of the research participants (Crotty, 1998). In doing so, researchers are able to uncover how research participants devised and attribute meanings to objects, events

and phenomena (Berg, 2007). When uncovering these meanings, Blumer (1969) posited three interactionist assumptions. First, human beings act toward things on the basis of the meanings that these things have for them. Second, the meaning of such things is derived from, and arises out of, the social interaction that one has with one's fellows. Third, these meanings are handled in, and modified through, an interpretive process used by the person in dealing with the things encountered. Essentially, the central tenet underpinning symbolic interactionism is that objects, phenomena, situations, and people do not in themselves possess meaning. Rather, meaning is conferred on these elements by and through human interaction (Berg, 2007). For this study, the researchers sought to uncover the meanings PSMTs conferred upon their perceived readiness to teach secondary mathematics (in terms of MCK & MPK), before and after they undertake their first practicum.

For this study, the researchers developed and used two online, anonymous, qualitative surveys and semi-structured qualitative interviews to collect data from participants. Participants were asked to respond to a 10-item survey prior to commencing their first 10-week teaching practicum experience. The qualitative survey and interview questions comprised predominantly of open-ended items (See section, Survey and Interview Items). Immediately following the teaching practicum experience, the participants were asked to respond once more to the same survey. Then, as a point of difference from the original study, both researchers invited all participants to participate in a semi-structured interview. In this manner, the researchers were able to determine at greater depth the extent to which any of the participants' self-perceptions of readiness had changed following their 10-week experience in the classroom. The interview also afforded participants the opportunity to provide detailed responses to various questions asked. The survey items and interview questions are included within this section.

Research Context

This research was conducted on site across two university campuses, situated in different states in Australia. At Campus A, PSMTs undertake a course that covers secondary mathematical pedagogy (both for lower school and upper school students), which examines key curriculum and educational policy documents, and investigates best practice approaches regarding planning, instructional, and assessment resources. At Campus B, PSMTs undertake two courses, first covering lower secondary school pedagogy and then later upper secondary school pedagogy. These courses meet the requirements of the Australian Qualifications Framework (AQF) for secondary teachers, are nationally accredited for initial teacher education programmes, and address a variety of Australian Institute for Teaching and School Leadership (AITSL) standards (AITSL, 2015).

Research Participants

The entire student cohort enrolled in courses for secondary mathematics pedagogy was invited to participate in the research. Specifically, of the 53 students enrolled in these courses across the two campuses, 20 elected to participate in the pre-practicum survey and 14 in the post-practicum survey. A total of six students participated in a post-practicum face-to-face interview. The demographic details of the survey and interview participants are included in Table 3. The demographic details of the survey and interview participants are listed in Table 3. Within the GDE

and Master of Teaching (MTeach) degrees (Campus A only), PSMTs with a major teaching area are trained to teach secondary students from Years 7 – 12 (typically aged 13 – 18 years); those with a specialisation teaching area are trained to teach secondary students from Years 7 – 10 (13 – 16 years). Across a four-year degree, BEd students complete eight mathematics content courses and a mathematics pedagogy course (Campus A) or six mathematics content courses and two mathematics pedagogy courses (Campus B) and undertake four practicum experiences, totalling 32 weeks in schools. For this study, participants undertaking a GDE or MTeach qualification completed the mathematics pedagogy course in their first year (and first semester) of study, while those enrolled in a BEd completed the course in their second year (first semester).

Table 3
Summary of Participants' Demographic Data

	Gender	Age	Degree	Major	Specialisation
Pre-Practicum Survey Participants [n=20]	13 Female	17-25 = 14	Grad. Dip. = 8	Math = 11	Math = 9
	7 Male	26-35 = 4	MTeach = 2	Science = 5	Science = 7
		36-45 = 2	BEd(Sec) = 10	Other = 4	Other = 4
Post-Practicum Survey Participants [n=14]	7 Female	17-25 = 9	Grad. Dip. = 7	Math = 7	Math = 7
	7 Male	26-35 = 5	MTeach = 1	Science = 3	Science = 5
			BEd(Sec) = 6	Other = 4	Other = 2
Interview Participants [n=6]	5 Female	17-25 = 6	Grad. Dip. = 4	Math = 4	Science = 2
	1 Male		MTeach = 2	Science = 2	Math = 4

Survey and Interview Items

Ten items comprised the pre-practicum and post-practicum surveys of this research. Survey items 1-4 were for participants to indicate specific background information regarding their age, gender, and prior tertiary studies. Survey items 5-10 directly assisted the researchers in pursuing the specific aims of the research. The research participants had been furnished with the terms MCK and MPK in the secondary mathematics courses they were enrolled in during Semester 1, 2017. These items required participants to adopt a critically reflective stance towards their perceived readiness (before and after the practicum) in teaching secondary mathematics. The interview schedule was comprised of survey items 5-10.

1. What is your gender?
2. What is your major teaching area (i.e. for Years 7 - 12)?
3. What is your minor teaching area (i.e. for Years 7 - 10)?
4. Which category below includes your age? 20-29 30-39 40-49 50-59
5. Describe your readiness to teach secondary mathematics students in terms of the

mathematical content knowledge and skills you currently possess.

6. In what area(s) of mathematical content knowledge do you feel you require further training?

7. Describe your readiness to teach secondary mathematics students in terms of the mathematical pedagogical knowledge and skills you currently possess.

8. In what area(s) of mathematical pedagogical knowledge do you feel you require further training?

9. As a pre-service, secondary mathematics educator, are there any other areas you would like to receive professional training and development in?

10. Overall, describe your readiness to teach mathematics to secondary students.

Data Analysis Process

The researchers analysed qualitative data collected from the pre-practicum and post-practicum surveys (items 5 - 10) and interviews according to a framework offered by Miles and Huberman (1994) that comprises three components: data reduction, data display, and drawing and verifying conclusions. Within each of these components the researchers executed the following operations: coding, memoing, and developing propositions. According to Miles and Huberman (1994), codes are "tags or labels for assigning units of meaning to the descriptive or inferential information compiled during a study" (p. 56). Codes developed by the researchers were attached to data gathered via pre-practicum surveys, post-practicum surveys, and interviews, and were selected from those data based on their meaning. In particular, the codes were developed according to the domains of MKT (Ball et al., 2008), delineated in Table 1. After the first pass of coding, researchers met virtually to discuss themes that arose throughout the analysis. These discussions allowed the researchers to ensure that the *a priori* codes were applied consistently, to incorporate any additional themes and to remove themes inapplicable to certain questions. Following these discussions, the researchers analysed the data once more using the same *a priori* codes. Once completed, they met virtually once more to discuss similarities and differences in their analyses and came to consensus on codes for each PSMT response to each question. The inter-rater reliability (IRR) was 91%, and was calculated as the number of PSMT responses for which there was initial agreement on one or more codes (as more than one code could be used per response), divided by the total number of PSMT responses. Memoing was then used to synthesise coded data so that they formed a recognisable cluster of information anchored in one general concept, for example, Common Content Knowledge (CCK). Additionally, memoing helped to capture the ongoing, salient thoughts of the researchers as the coding process proceeded. Finally, the researchers generated propositions about connected sets of statements, reflected on the findings, and drew conclusions about the study.

Results

The key results of this research have been generated exclusively by participant responses from the surveys and interviews. Overall, PSMTs' responses suggested a self-perceived degree of readiness within the themes of MCK and MPK. These results have been summarised in tabulated and discursive formats, and in alignment with the six domains of MKT. Within this section and in subsequent sections the titles of various secondary mathematics courses in Western Australia and New South Wales have been mentioned. To assist in familiarising the readership with these mathematics courses, tabulated summaries of those courses taught in Western Australia and New South Wales have been provided in the appendices (Appendix 1 & Appendix 2, respectively). Results from post-practicum interviews are also included.

Survey Findings

Mathematical content knowledge: Readiness

Most of the PSMTs' (17 of 20) statements were coded as them feeling ready to teach mathematics before their first practicum experience (see Table 4). For example, one participant (who was coded as having CCK, SCK & HCK) stated:

> I feel confident to teach the content of secondary mathematics. I have recently completed mathematics content units which I did not find difficult. I feel I have a good conceptual understanding of the different mathematical concepts I will be required to teach and feel confident that I will easily be able to "brush up" on any topics (if need be) before I am required to teach them.

Another (who had CCK) stated:

> I feel very ready to teach lower school mathematics (Years 7-9). I haven't had any experience with Year 10s but would enjoy the challenge. I feel I could happily teach up to ATAR[1] Mathematics Methods; however, specialist would be a stretch at the current time.

Following the practicum, all PSMTs (14 of 14) declared they were ready to teach in terms of their MCK. Specifically, all participants' statements were coded as having appropriate CCK, and many of these expressed feeling confident in teaching lower school classes (i.e.. Years 7-10) only. Herein one participant (who was coded as having CCK) described:

> [I feel] good overall, although there were some topics in Year 11 and Year 12 classes that I had not seen for a long time. I think that I'll need to take the time to learn this content properly and master it. Things like matrices, some parts of vectors, proofs and pieces of calculus. I'm ready overall, and really ready for lower school classes.

In a consistent manner to this claim, another participant offered how he felt:

[1] The Australian Tertiary Admissions Ranking (ATAR) is a percentile score which denotes an Australian student's academic ranking relative to his or her peers upon completion of secondary education. This score is used to predict a student's suitability for particular university courses, and ultimately, for university entrance.

> Fairly ready. The practicum was an eye opener for me for the maths I still need to consolidate. I had a Year 10 Extension class and some of the algebra and trigonometry I hadn't seen or done for a long time. I found that I really had to put in a lot of hours to make sure I was on top of things each day. I also had an upper school general class, but this content was fairly basic, so not much revision was needed there.

The reported self-perceptions of PSMTs' readiness in MCK are displayed in Table 4.

Table 4
Mathematical Content Knowledge: Perceived Readiness

Pre-Practicum	Relative Frequency	Post-Practicum	Relative Frequency
I Feel Prepared	17 of 20	I Feel Prepared	14 of 14
I Have CCK	19 of 20	I Have CCK	14 of 14
I Have SCK	7 of 20	I Have SCK	3 of 14
I Have HCK	1 of 20	I Have HCK	0 of 14

Mathematical content knowledge: Further training needed

Before the practicum, all PSMTs identified an aspect of their MCK that they required further training in (see Table 5). In particular, our coding showed that most PSMTs identified these aspects as HCK (20) and SCK (17). One participant (who was coded as needing SCK & HCK) reflected how she felt that her lower school MCK required consolidation in various years and topics:

> I need to consolidate my content knowledge especially for the advanced classes. Year 8 content knowledge I'm fine, it's probably everything for Year 9 and Year 10 advanced classes that I need to practise. Things like algebra, probability, trig[onometry], indices and especially the harder examples.

Another participant was coded as needing HCK, and more specifically, this knowledge encompassed self-directed learning:

> [Mathematics] Extension 1 and 2; I will have to teach myself these topics. Maybe a brief overview of the general content for seniors when I was teaching it in my first maths practice there were content I never covered before. It wasn't difficult to learn but still I had no preparation for it.

In a similar vein to pre-practicum responses, the PSMTs continued to focus on HCK and SCK as areas for further training post-practicum. For example, one PSMT (who felt the need to develop HCK) stated "I feel as though I only need further training with Extension content as I have never taught an Extension class, and only had the opportunity to observe one." Similar to this comment, but with a focus on senior secondary MCK, one participant observed "As with the pre-service survey I need further development in ATAR maths but that will come with time". Similarly, another PSMT listed various senior secondary curriculum topics he required further training in: "I will need to refresh the higher skills of calculus, trig[onometric] relationships,

geometry, matrices, and linear algebra." A summary of PSMTs' needs for further MCK training is offered in Table 5.

Table 5
Mathematical Content Knowledge: Further Training Needed

Pre-Practicum	Relative Frequency	Post-Practicum	Relative Frequency
I Need HCK	20 of 20	I Need HCK	13 of 14
I Need SCK	17 of 20	I Need SCK	11 of 14
I Need CCK	1 of 20	I Need CCK	5 of 14
I Need None	0 of 20	I Need None	1 of 14

Mathematical pedagogical knowledge: Readiness

A majority of PSMTs (17 of 20) claimed they felt ready to teach in terms of their MPK, particularly with regards to KCS (see Table 6). From those who expressed that they felt prepared, one participant (who was coded as having KCS) stated:

> Coming from a high school education where it was majorly based off the 'chalk and talk' style of teaching, I felt I did not have as much knowledge on different pedagogical skills and knowledge that can be used to engage students in mathematics. Coming to university … taught me there are many different ways that mathematics should be taught to students … I feel much more ready after doing some units.

Moreover, four participants emphasised how they only felt ready to teach lower school classes. To illustrate, one of these four noted "I am confident in my ability in my pedagogical knowledge when it comes to junior years, but I feel once again when it comes to the harder concepts there is less variety and more difficulty." From the three participants who did not feel ready, one stated that:

> I feel like I am still learning what my pedagogy is. Through practicums I am learning the pedagogies of other teachers, and it is through that, that I am finding what I truly value. I believe that my pedagogy is changing as I go to each practicum…it will be a few more years until I feel like I have a solid pedagogy.

After the practicum experience, 13 of 14 PSMTs expressed feeling ready to teach mathematics, and particularly in terms of their KCS. One participant (who was coded as having KCS) stated:

> I'm pretty happy with my teaching so far. I felt I was learning new things each week with my classes, like how to break down concepts so that the younger school students can understand better. My mentor was really helpful in showing me how to make a lesson engaging for younger students, like splitting up the activities, getting students involved, and checking work.

Despite communicating feeling ready to teach mathematics, another participant expressed frustration at a lack of MPK development following his practicum experience. Herein, he outlined:

> As my mentor was a 70-year old teacher, she was very much of the opinion [that the] textbook and her way of teaching was correct which meant I was unable to extend my skills and attempt new things without being marked down. My pedagogy is relatively limited because of this.

A summary of PSMTs' self-perceptions of readiness in MPK in presented in Table 6.

Table 6
Mathematical Pedagogical Knowledge: Perceived Readiness

Pre-Practicum	Relative Frequency	Post-Practicum	Relative Frequency
I Feel Prepared	17 of 20	I Feel Prepared	13 of 14
I Have KCS	17 of 20	I Have KCS	13 of 14
I Have KCT	2 of 20	I Have KCT	1 of 14
I Have KCC	0 of 20	I Have KCC	0 of 14

Mathematical pedagogical knowledge: Further training needed

Prior to the practicum experience, 17 PSMTs identified a need for further MPK training (see Table 7). Moreover, a majority of these were coded as requiring KCS, KCT or KCC (or any combination of these domains). Two PSMTs (who were both coded as needing KCS, KCT & KCC) offered specific areas they wished to become more proficient in:

> Diversifying the teaching of the content. If it is explained one way and students do not understand, how do you change your thought process to adapt and meet their requirements?

Breaking down Year 11 and Year 12 content and low learning ability content

Following the practicum, all participants nominated something to work on, pedagogically speaking. One PSMT (who was coded as needing KCC & KCT) stated:

> I think that [I need help in] learning how to be more creative with lessons so it's not the same kind of lesson each time. I did try to avoid this so the students wouldn't get too bored, but planning huge and exciting lessons takes so much time! Finding new or different ways to help students connect their knowledge to new ideas would also be helpful.

In a similar way, another participant (who was coded as needing KCC & KCT) echoed this comment:

> I found that I could teach the theory quite well but a lot of the students I taught learnt through visual and practical strategies. I was challenged in not only my ability to come up with creative ways [for students] to learn, but to explain formulas and mathematical theory in a more practical way.

A summary of PSMTs' responses for further MPK training is presented in Table 7.

Table 7
Mathematical Pedagogical Knowledge: Further Training Needed

Pre-Practicum	Relative Frequency	Post-Practicum	Relative Frequency
I Need KCC	17 of 20	I Need KCC	14 of 14
I Need KCT	17 of 20	I Need KCT	14 of 14
I Need KCS	11 of 20	I Need KCS	1 of 14
I Am Unsure	2 of 20	I Need None	0 of 14

Further professional development

Before the practicum experience, most participants were able to identify at least one area of professional development (PD) to receive support in. In a similar vein to previous findings, common responses included further training in MPK and MCK (see Table 8). To illustrate, one participant (needing MPK & Learner Diversity training) expressed he needed to know:

> How to teach students who still do not have the basic knowledge that they should have gained in primary school. For example, having a student in your class who cannot add one-digit numbers but is expected to [solve] trigonometric equations.

Two participants were unable to suggest any areas for PD. Post-practicum, nearly all participants (13 of 14) were able to identify at least one area they wished to receive PD in. This time, the use of technology (especially graphics calculators) and MPK were the most commonly proffered needs. For the former theme, one participant described how she needed to know "[How to] use technology in each lesson. All upper school students use [Casio] Class Pads and even some lower school classes too, so this would be good." A summary of PSMTs' responses regarding further PD is offered in Table 8 below.

Table 8
Further Professional Development

Pre-Practicum	Relative Frequency	Post-Practicum	Relative Frequency
MPK	17 of 20	Technology	14 of 14
MCK	17 of 20	MPK	14 of 14
Learner Diversity	11 of 20	Learner Diversity	1 of 14
Technology	2 of 20	Assessments	0 of 14

Overall readiness to teach secondary mathematics

Nearly all PSMTs (18 of 20) stated that they felt ready to teach secondary mathematics prior to their first practicum experience (see Table 9). Whereas such assertions of readiness were conditional, over half of those PSMTs stated they needed to develop elements of their MCK, MPK, or both of these knowledge domains. For instance, one PSMT (coded as needing SCK & HCK) qualified her self-perception of readiness with "Lower secondary I feel 90% confident. Upper secondary I do not feel confident at all, maybe 40% at that. I could learn the content the night before the lesson. I am aware that this is not good going into prac[ticum]". Another PSMT (coded as needing SCK, HCK, KCT, & KCC) stated how he "would feel confident delivering certain blocks of content, although I'd prefer to have a more solid understanding of that content and of teaching methods."

Following the practicum, an overwhelming proportion of PSMTs averred feeling prepared to teach (13 of 14). Again, all of these responses were qualified with an expressed need for PSMTs to develop professionally in MCK and MPK domains. While one participant shared how he was "Itching to get started", another (who was coded as needing SCK & HCK) stated "Overall, I feel as though I am quite ready to teach in secondary schools. There are definitely a few gaps [in my content knowledge] but nothing that I don't think won't be sorted out after a year or two of teaching in my own classroom". Approximately half of the pre- and post-practicum cohorts reported feeling ready to teach lower school classes, but conceded that elements of their MCK and MPK for upper school courses required improvement. For instance, one PSMT explained how she felt:

> …very ready to teach the lower secondary as a graduate teacher. I did pretty much this during my prac[ticum] as I lacked a mentor. However, upper secondary is going to be a challenge for me. I feel that I am capable of handling the behavioural management and have the ability to develop rapport and relationships. However, it is going to take extra time and effort for me to build my confidence in the maths content.

Participant responses regarding an overall readiness to teach secondary mathematics are summarised in Table 9.

Table 9
Overall Readiness to Teach Secondary Mathematics

Pre-Practicum	Relative Frequency	Post-Practicum	Relative Frequency
I Feel Prepared	18 of 20	I Feel Prepared	13 of 14
I Need KCT	14 of 20	I Need SCK	7 of 14
I Need KCC	13 of 20	I Need HCK	7 of 14
I Need HCK	12 of 20	I Need KCC	3 of 14
I Need SCK	11 of 20	I Need KCS	2 of 14

Interview Findings

Overall, the testimony offered by interviewees closely reflected those findings gathered through anonymous pre-practicum and post-practicum surveys. As per the interview schedule (see Methodology), interviewees' comments were focused exclusively on their self-perceptions of readiness to teach secondary mathematics in terms of MCK, MPK, and overall. In this section, pseudonyms have been used for the five female interviewees (Abigail, Beatrice, Candyce, Demetra & Eloise) and the one male interviewee (Francis). To commence, most interviewees (4 of 6) shared how they felt ready to teach lower school classes only with regards to their MCK. For instance, Francis emphasised that his own learning and practicum experience at a middle school left him feeling confident to teach lower school classes only. He stated:

> … Year 7, Year 8, Year 9, Year 10 I feel quite confident with. But in Year 11 and Year 12 I would probably not feel very confident at all. I would have to look up what I was doing, although I have been tutoring in it. Still, I haven't had any experience in the classroom, and I think that that's very different when you're teaching in a classroom as opposed to one-on-one tutoring.

Candyce (who holds an undergraduate degree in engineering) shared a similar sentiment regarding her MCK, although her practicum placement was for Years 7-12.

> Well, I'd probably feel confident with anything Year 10 or below. I think that the time that's [passed] between me doing my degree and actually concentrating on mathematics, to going in and teaching upper school mathematics to Year 11 and Year 12 students; I got to experience a little bit of it on practicum with Specialist Year 11 classes, and it came back really quickly, but that's probably my only area where I'm not too certain. If students were to ask me extension question or to elaborate on things, where it's not so readily available for me to look in the textbook and see how we do this type of equation, I'd need to do some thinking.

The two interviewees who averred their confidence to teach all year levels and courses (Demetra & Abigail) indicated that, while their tertiary studies had provided them with adequate MCK, there remained several conceptual "gaps" in upper school mathematics courses that needed to be

addressed. Both interviewees confirmed that these perceived gaps had been discerned during their practicum experience.

With regards to MPK, most interviewees (5 of 6) described how they felt more ready to teach lower school classes than upper school classes. To illustrate, Beatrice stated that one challenge with upper school classes was to:

> Try and teach outside of the text book. I find that with Year 11 and 12, everything you need is there. So I think…it's hard to find support materials for that…if you do the extra research and try to find different ways to teach topics in senior maths, I think there is a way to make it more enjoyable for the students. But right now, I feel like it's still a bit hard because especially with senior maths, because most teachers just teach by the text book.

Demetra focused on learning how to deal with the complexity, time constraints and best pedagogical practice associated with upper school classes as challenges to overcome in her MPK.

> For upper school, I suppose the content is so much heavier and harder, and it's difficult to expand upon it with the time you have available. So, learning the different ways to do that is a bit more difficult, and once you get those ideas, you've just got to use trial and error. But I think at the same time, by that age the students have a bit more respect for initiative, so you're not as afraid to try it with them, as you would say, a lower school class where you would lose all of your students' respect, so I'm okay with being less confident in the upper school years.

The claims of feeling more ready to teach lower school students than upper school students were not made without equivocation, however. For instance, two interviewees (Candyce & Demetra) shared how they wished to learn ways to engage all ranges of learners within a mathematics classroom. Specifically, comments from Candyce and Demetra, respectively, included:

> I think after experiencing a lot in terms of a low ability class, and coming from someone who was able to grasp the concepts obviously with somewhat ease, so developing those low-level strategies to bring it right back and actually simplifying it to a level that is accessible…trying to judge whether you're oversimplifying it or whether it's at their level, is the hardest thing for me. So it's trying to break it right down so you think it's understandable, and yet it's not understandable to them, and so you have got to take it that extra step and that's what I want to work on the most pedagogically.

> When I was working with the Year 7s on fractions I even approached the Year 5 teacher to see how she had taught them because I knew she had these kids, so it could be a bit of consistency to go back that far…for the lower ability kids, it requires you to go back and to find out what they've been taught, how they've been taught, and to sometimes teach the concepts at a Primary level, instead of just assuming that they are all at the required level. Yeah, so breaking right down to basics.

Other interviewees' comments regarding lower school MPK included learning strategies for special needs students, and both the sequencing and teaching of particular mathematical topics.

Discussion

The results in this study support previous findings that PSMTs mostly consider themselves ready to teach secondary mathematics (Hine, 2018). Specifically, PSMTs are generally prepared to teach lower secondary mathematics (Years 7-10), but express feeling less prepared to teach upper secondary mathematics, especially Specialist/Extension courses (Tables 4 & 5). Our findings support the notion that participation in the pedagogical course and/or the practicum plays a significant role in preparing PSMTs for the profession. Moreover, these findings showed that all participants in the post-practicum survey feel they possess the MCK to teach lower secondary mathematics. Interestingly, we also observed a reduction in the proportion of participants who claimed to possess SCK and HCK after the practicum (Table 4). This change is surprising, given that all the participants have completed two or more tertiary level mathematics courses, covering topics such as algebra and calculus at a level equal to or higher than secondary mathematics. Although exploring this shift was not the initial intention of this study, our interview data suggests that this self-perceived lack of SCK and HCK post-practicum is due to PSMTs not yet mastering these mathematical skills and concepts, and therefore do not feel confident teaching them. This finding supports the work of both Monk (1994) and Burghes and Geach (2011), who showed that teachers' classroom effectiveness is not associated with the number of university mathematics courses completed nor their performances in these courses.

It is also possible that a shift in PSMTs' confidence to teach upper secondary mathematics is influenced by their self-perceived MPK, which has been made apparent through completing their first practicum. The data show that as many as 85% (17 of 20) of PSMTs claimed to possess the requisite MPK to teach Years 7-10 prior to the practicum, which increased to 93% (13 of 14) when surveyed after the practicum (Table 6). Furthermore, after the practicum, all 14 participants indicated that they lacked the MPK to effectively teach Years 11 and 12, especially the Specialist/Extension courses (Table 7). When probed on these indications during the post-practicum interview, common explanations for this self-perceived deficiency were offered. These explanations included: a perceived lack of MCK required for teaching upper secondary, limited exposure to senior classes during the practicum, or not seeing a direct link between university level mathematics and the senior secondary mathematics syllabus.

In 2006, Goos asserted that pre-service teachers often viewed the practicum as being far more effective than content covered in tertiary education programmes. However, a number of studies have reported that this is not always the case (Allen & Wright, 2014; Smith & Lev-Ari, 2005). Specifically, these studies found that pre-service teachers saw the practicum as an opportunity to observe and enact the integration of theory and practice. In line with these studies, this research has determined that PSMTs reported a need for further development in bridging theory and practice, especially with regards to linking MCK learnt at university to the secondary syllabus and to support MPK to effectively teach senior secondary classes. Indeed, it has been previously reported that, when such alignment exists, PSMTs not only deepen their MCK and MPK, but also experience increased confidence in teaching mathematics for the first time (Hine, 2015; 2018). Given that MCK is often taught outside the Faculty of Education (e.g., Faculty of Mathematics) by educators demonstrating different instructional approaches to those encouraged in the Faculty of Education, a potential solution is to have closer collaborations (e.g., team-teaching) between the different faculties. Another potential solution is to replace didactic-styled lectures with smaller

classes such as workshops and tutorials, which mimics the secondary classroom environment and therefore allows the educator to demonstrate best practice.

Conclusion

Overall, this study found that the majority of PSMTs, in both undergraduate and postgraduate programmes, perceived themselves to have adequate MCK and MPK and to be ready to teach Years 7-10. However, there was an expressed general lack of confidence in teaching upper secondary mathematics, which was caused by a self-perceived deficiency in MCK and MPK. These results are consistent with work on postgraduate PSMTs self-perception in a GDE programme (Hine, 2018). This work extends on previous work (Hine, 2018) by expanding the participant size and including both undergraduate and postgraduate PSMTs, as well as seeking to answer why PSMTs lack confidence in teaching upper secondary mathematics. Our findings suggest that the PSMTs required more support to develop their MPK (especially during the practicum) and to develop mastery of MCK that specifically relates to the upper secondary mathematics curriculum. Further research is required to explore approaches that would best address these areas of development.

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Appendix 1: Summary of ATAR Mathematics Courses (Western Australia)

Year 11 Mathematics Applications	Year 12 Mathematics Applications
Unit 1 Topic 1.1 Consumer arithmetic (20 hours) Topic 1.2 Algebra and matrices (15 hours) Topic 1.3 Shape and measurement (20 hours) **Unit 2** Topic 2.1 Univariate data analysis and the statistical investigation process (25 hours) Topic 2.2 Applications of trigonometry (10 hours) Topic 2.3 Linear equations and their graphs (20 hours)	**Unit 3** Topic 3.1 Bivariate data analysis (20 hours) Topic 3.2 Growth and decay in sequences (15 hours) Topic 3.3 Graphs and networks (20 hours) **Unit 4** Topic 4.1 Time series analysis (15 hours) Topic 4.2 Loans, investments and annuities (20 hours) Topic 4.3 Networks and decision mathematics (20 hours)

Year 11 Mathematics Methods	Year 12 Mathematics Methods
Unit 1 Topic 1.1 Functions and their graphs (22 hours) Topic 1.2 Trigonometric functions (15 hours) Topic 1.3 Counting and probability (18 hours) **Unit 2** Topic 2.1 Exponential functions (10 hours) Topic 2.2 Arithmetic and geometric sequences and series (15 hours) Topic 2.3 Introduction to differential calculus (30 hours)	**Unit 3** Topic 3.1 Further differentiation and applications (20 hours) Topic 3.2 Integrals (20 hours) Discrete random variables and networks (15 hours) **Unit 4** Topic 4.1 The logarithmic function (18 hours) Topic 4.2 Continuous random variables and the normal distribution (15 hours) Topic 4.3 Interval estimates for proportions (22 hours)

Year 11 Mathematics Specialist	Year 12 Mathematics Specialist
Unit 1 Topic 1.1 Combinatorics (11 hours) Topic 1.2 Vectors in the plane (22 hours) Topic 1.3 Geometry (22 hours) **Unit 2** Topic 2.1 Trigonometry (16 hours) Topic 2.2 Matrices (19 hours) Topic 2.3 Real and complex numbers (20 hours)	**Unit 3** Topic 3.1 Complex numbers (18 hours) Topic 3.2 Functions and sketching graphs (16 hours) Topic 3.3 Vectors in three dimensions (21 hours) **Unit 4** Topic 4.1 Integration and application of integration (20 hours) Topic 4.2 Rates of change and differential equations (20 hours) Topic 4.3 Statistical inference (15 hours)

Appendix 2: Summary of ATAR Mathematics Courses (New South Wales)

Year 11 Mathematics Standard (120 hours)	Year 12 Mathematics Standard 1 (120 hours)
Unit 1. Algebra Topic 1.1 Formulae and equations Topic 1.2 Linear relationships **Unit 2. Measurement** Topic 2.1 Applications of measurement Topic 2.2 Working with time **Unit 3. Financial Mathematics** Topic 3.1 Money matters **Unit 4. Statistical Analysis** Topic 4.1 Data analysis Topic 4.2 Relative frequency and probability	**Unit 5. Algebra** Topic 5.1 Types of relationships **Unit 6. Measurement** Topic 6.1 Right-angled triangles Topic 6.2 Rates Topic 6.3 Scale drawings **Unit 7. Financial Mathematics** Topic 7.1 Investment Topic 7.2 Depreciation and loans **Unit 8. Statistical Analysis** Topic 8.1 Further statistical analysis **Unit 9. Networks** Topic 9.1 Networks and paths

	Year 12 Mathematics Standard 2 (120 hours)
	Unit 5. Algebra Topic 5.1 Types of relationships **Unit 6. Measurement** Topic 6.1 Non-right-angled trigonometry Topic 6.2 Rates and ratios **Unit 7. Financial Mathematics** Topic 7.1 Investments and loans Topic 7.2 Annuities **Unit 8. Statistical Analysis** Topic 8.1 Bivariate data analysis Topic 8.2 The normal distribution **Unit 9. Networks** Topic 9.1 Networks concepts Topic 9.2 Critical path analysis

Year 11 Mathematics Advanced (120 hours)	Year 12 Mathematics Advanced
Unit 1. Functions Topic 1.1 Working with functions **Unit 2. Trigonometric Functions** Topic 2.1 Trigonometry and measure of angles Topic 2.2 Trigonometric functions and identities **Unit 3. Calculus** Topic 3.1 Introduction to differentiation **Unit 4. Exponential and Logarithmic Functions** Topic 4.1 Logarithms and exponentials **Unit 5. Statistical Analysis** Topic 5.1 Descriptive statistics Topic 5.2 Probability Topic 5.3 Discrete probability distributions	**Unit 6. Functions** Topic 6.1 Graphing techniques **Unit 7. Trigonometric Functions** Topic 7.1 Trigonometric functions and graphs **Unit 8. Calculus** Topic 8.1 Differential calculus Topic 8.2 The second derivative Topic 8.3 Integral calculus **Unit 9. Financial Mathematics** Topic 9.1 Modelling financial situations **Unit 10. Statistical Analysis** Topic 10.1 Bivariate data analysis Topic 10.2 Random variables

Year 11 Mathematics Extension 1 (60 hours, taken in addition to Mathematics Advanced)	Year 12 Mathematics Extension 1 (60 hours, taken in addition to Mathematics Advanced)
Unit 1. Functions Topic 1.1 Further work with functions Topic 1.2 Polynomials **Unit 2. Trigonometric Functions** Topic 2.1 Inverse trigonometric functions Topic 2.2 Further trigonometric identities **Unit 3. Calculus** Topic 3.1 Rates of change **Unit 4. Combinatorics** Topic 4.1 Working with combinatorics	**Unit 5. Proof** Topic 5.1 Introduction to proof by mathematical induction **Unit 6. Vectors** Topic 6.1 Introduction to vectors **Unit 7. Trigonometric Functions** Topic 7.1 Trigonometric equations **Unit 8. Calculus** Topic 8.1 Further calculus skills Topic 8.2 Applications of calculus **Unit 9. Statistical Analysis** Topic 10.1 The binomial distribution

	Year 12 Mathematics Extension 2 (60 hours, taken in addition to Mathematics Advanced and Mathematics Extension 1)
	Unit 1. Proof Topic 1.1 The nature of proof Topic 1.2 Further proof by mathematical induction **Unit 2. Vectors** Topic 2.1 Further work with vectors **Unit 3. Complex Numbers** Topic 3.1 Introduction to complex numbers Topic 3.2 Using complex numbers **Unit 4. Calculus** Topic 4.1 Advanced calculus skills **Unit 5. Mechanics** Topic 5.1 Application of calculus in mechanics