Pre-service Teachers' Learning from Significant Opportunities for Improvement in a Positive Error Culture

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Many teacher education models involve reflecting on teaching practice for the sake of improving it. Such reflection must be carefully structured to help practitioners identify and act upon significant opportunities for improvement (SOIs). Learning from SOIs requires the cognitive activities of noticing students' mathematical thinking and its connection to instructional practice, along with an affective disposition to view sub-optimal teaching practices as learning opportunities. We draw upon existing literature and theory related to the notion of developing positive error cultures to identify design principles for helping teachers learn from their own sub-optimal practices rather than becoming discouraged by them. The design principles include experience-based learning, low-stakes settings, collaboration, process reflection, and exploration of disagreements. We then describe a mathematics teacher education environment incorporating the design principles. Examples of pre-service teachers' work within the environment are analysed for possible patterns of learning from SOIs within a positive error culture. Based on these examples, a four-quadrant model to characterise teachers' learning from SOIs is proposed. The four quadrants describe various outcomes related to recognising and resolving SOIs.

Keywords · reflection · practice-based learning · errors · collaboration

Introduction

Reflecting on one's teaching practice for the sake of improving it is a prominent feature of many teacher education efforts. This type of reflection implicitly drives several models, such as Japanese Lesson Study (Lewis & Hurd, 2011), some types of action research (Leitch & Day, 2006), and multiple teacher coaching models (Woulfin, 2020). Such reflection goes beyond just analysing classroom incidents; it requires making and testing conjectures about improving practice. Ricks (2011) used the term *process reflection* to characterise this type of analysis, noting that it is resonant with the thinking of Dewey (1933) and Schön (1983), who spoke of the value of learning from empirical problems by developing ideas for improvement grounded in data and theory, testing the ideas through implementation, observing the results, making adjustments, and repeating the cycle.

In any given lesson, there are myriad events on which one may choose to reflect. Some foci are more valuable than others, as demonstrated by literature on professional noticing (Sherin et al., 2011). Noticing entails attending to students' mathematical reasoning, interpreting it, and making decisions about future instruction (Amador et al., 2016; Jacobs et al., 2010). Literature on noticing emphasises that teachers often need support focusing on students' learning and not just their own teaching actions (Leatham & Peterson, 2010; Santagata, 2011). Attending to students' mathematical thinking rather than just focusing on whether students' answers are correct or incorrect is another important area for teacher education (Barnhart & van Es 2015; Goldsmith & Seago 2011; Simpson & Haltiwanger, 2017). Tracing patterns of student thinking back to specific teaching strategies can reveal opportunities to restructure practices that did not produce optimal learning outcomes. In this essay, we refer to such opportunities to improve practice as *significant opportunities for improvement* (SOIs).

SOIs are conceptually related to critical incidents, though the two ideas are not precisely equivalent. Broadly speaking, critical incidents are life events that mark significant turning points; in education, these can include everyday classroom events (Tripp, 1993). However, critical incidents only become "critical" if a teacher recognises and acts upon them (Tripp, 1993). In contrast, we use the notion of SOI to encompass both recognised and unrecognised opportunities for improvement that occur in everyday classroom settings. Recognising opportunities to improve one's approach to develop students' mathematical thinking requires professional noticing (Finch, 2010). When teachers recognise such opportunities, teacher educators can work alongside them to help adjust and optimise instructional practices. When teachers do not notice such opportunities, teacher educators can intervene to attempt to draw their attention to them. Using opportunities to improve as learning sites can be powerful, as has been demonstrated in the literature on students' learning from their errors in mathematics and other subjects (Boaler, 2016; Borasi, 1994; Metcalfe, 2017). We aim to extend the exploration of such learning sites to the specific domain of mathematics teacher education in the present article.

Purpose

Noticing and reflecting on practices in one's lessons that have pronounced impact on students' mathematical learning is a challenging, and important, part of teacher learning. Teacher education that is grounded in process reflection provides opportunities for such noticing but does not guarantee that it will occur. It can still be challenging, and uncomfortable, for teachers to notice and act upon SOIs. As noted earlier, teachers' foci for noticing may or may not connect to the most significant events in a lesson. Further, identifying opportunities to improve requires acknowledging that some portions of one's teaching need revision; this requires a level of critical self-reflection that is non-trivial to attain, for various cognitive and affective reasons. In this report, we draw upon existing theory to conceptualise a teacher education environment to support pre-service teachers' learning from SOIs, provide empirical examples of pre-service teachers' work in the environment, and propose a framework to characterise teacher learning in relation to SOIs.

Design Principles for a Positive Error Culture to Support Teachers' Learning from SOIs

Identifying SOIs requires acknowledging that some teaching practices implemented during a lesson were sub-optimal. Such acknowledgement is affectively challenging because it involves confronting possible errors in planning, implementing, or assessing instruction. Here, we use the word "error" to refer to potential areas for continuous, progressive, iterative improvement rather than to signify dichotomies such as right/wrong and correct/incorrect. We use the notion of error not to paint a deficit portrait of teachers, but to leverage theoretical resources in the growing body of literature from various fields on using errors as sites for learning. Analysing one's errors and receiving feedback from others about how to address them is a difficult, yet powerful, way to learn (Boaler, 2016; Borasi, 1994; Metcalfe, 2017). Individuals must feel a degree of psychological safety to openly acknowledge errors and learn from them (Edmondson, 1999). Helping teachers learn from SOIs requires teacher educators to structure environments in ways that honour possible errors as sites for learning rather than stigmatising them.

Literature pertaining to the notion of "positive error culture" (Minnameier, 2006; Tulis, 2013) contains design principles that support the process of learning from errors. Positive error cultures stand in contrast to negative error cultures, in which communication about errors is avoided, mainly due to individuals' fears of being penalised (Tulis, 2013). In a negative error culture, teachers would avoid focusing on SOIs when analysing their lessons because doing so would require acknowledging sub-optimal practices in their teaching. In a positive error culture, such analyses are explicitly encouraged as a natural part of learning. Several principles from the literature on establishing a positive error culture can inform teacher education models that are driven by learning from SOIs. Relevant design principles from the literature include: experience-based learning, a low-stakes environment, collaboration, process reflection, and learning from disagreements.

Experience-based Learning

Harteis and colleagues (2008) noted that literature on learning from errors emphasises the importance of experience-based learning. In the specific domain of teacher education, Wieman and Hiebert (2018) observed that teachers need experiences trying new teaching methods, stating, "All of the best current evidence on learning says that this kind of planned experience, which inevitably leads to mistakes, is the best way for teachers to learn what they need to know to teach in more ambitious ways" (n. p.). Identifying SOIs becomes possible when engaging in teaching practice because firsthand observations of students' reactions to one's teaching can be done. Linking information about students' mathematical learning to specific teaching actions leads to the identification of SOIs. This kind of experiential learning is inherent to models such as lesson study; after a group of educators collaboratively plans a lesson, at least one member of the group teaches it to students while others look for evidence of its impact on student learning (Lewis et al., 2009). The ability to gather classroom data linking students' mathematical learning to specific teaching actions is foundational to the process of identifying SOIs.

Low-stakes Environment

Suboptimal practices that occur in a high-stakes environment, by nature, are more heavily laden with negative affect than those made in a low-stakes environment. Creating a low-stakes learning environment for teachers is related to, but more challenging than, designing such environments for students. Metcalfe (2017) observed, "If the goal is optimal performance in high-stakes situations, it may be worthwhile to allow and even encourage students to commit and correct errors while they are in low-stakes learning situations rather than to assiduously avoid errors at all costs" (pp. 465). Several mathematics education researchers have, in fact, sought to help students become comfortable making and correcting mathematical errors by establishing low-stakes settings and celebrating their errors (Boaler, 2016; Borasi, 1994). In such settings, low-stakes environments can be established by not tying students' performance to grades or test scores and by reinforcing the idea that errors are valued as learning sites. It is less clear, however, what might constitute a low-stakes environment for mathematics teachers, because teachers' actions have consequences not just for themselves, but also for students' learning.

One way to create a lower-stakes environment for teacher learning is to have teachers engage in micro-teaching a lesson to peers and teacher educators before conducting the lesson with children. Doing so can provide opportunities to discuss strengths and weaknesses of a lesson to be implemented and help teachers experience a greater degree of success when teaching the lesson to children (He & Yan, 2011). Although micro-teaching can help in this manner, it is somewhat artificial when compared to teaching in an actual classroom (He & Yan, 2011). So, not all potential teacher errors can be anticipated and addressed in advance. Nonetheless, making, anticipating, and addressing as many as possible lowers the stakes by helping teachers avoid some practices that may lead to undesired outcomes. This can leave a smaller set of sub-optimal practices to adjust when reflecting on the lesson after actual classroom instruction.

Collaboration

Benefits of collaborating with other professionals to learn from opportunities for improvement are well-documented in the literature (Cannon & Edmondson, 2001; Steuer et al., 2013; Tjosvold et al., 2004). Professional collaboration can contribute to a low-stakes environment if structured to attribute observed successes and failures to teams rather than individuals. Collaboratively planning lessons distributes responsibility for lesson planning decisions and consequences across multiple individuals; successes and failures can be attributed to the entire group rather than just the teacher(s) implementing the lesson. Teacher education models that incorporate this type of collaborative planning can build teachers' self-efficacy (Sibbald, 2009) and help them avoid becoming discouraged by setbacks in practice by reframing them as collective learning opportunities.

Fostering collaboration can be difficult because of the isolated nature of the teaching profession; teachers spend much of the day interacting with children rather than with peers (Sutton & Shouse, 2016). Mathematics education literature does, however, suggest strategies to address this problem. Lesson study is one such strategy, and it is offered as an example here because the positive error culture we describe later builds upon its tenets. During lesson study, teachers collaboratively plan a lesson, observe its implementation, and study its impact on students' learning (Fernandez & Yoshida, 2004; Lewis et al., 2009). The debriefing portion of lesson study is especially valuable as a site for learning from SOIs, as its purpose is to identify not only the successes of the demonstration lesson but also its weaknesses and where it can be improved (Groth, 2011). In cases where teachers cannot all gather at the same time to observe lessons, classroom video can provide a viable means for sharing and discussing practice with one another (Star et al., 2011). Teachers can also benefit from inviting outside observers to debriefing sessions; they often notice important aspects of content and pedagogy that may otherwise go unaddressed (Lewis, 2016). The growing prevalence of collaborative structures for teacher education may provide increased opportunities to establish professional communities among teachers.

Process Reflection

As individuals collaboratively discuss practice with one another, they benefit from working structures that provide metacognitive support (Keith & Frese, 2005; Shilo & Kramarski, 2019; Westermann & Rummel, 2012). Mathematics teacher educators often provide this type of support by having teachers analyse video of their own teaching and interactions with students. For example, when Moyer and Milewicz (2002) had preservice teachers reflect on the quality of the mathematical questions they asked children during video recorded clinical interviews, many identified missed opportunities to probe children's thinking. Given this experience, the pre-service teachers were able to make adjustments to improve the quality of their classroom discourse. Similarly, teachers participating in a video club (van Es & Sherin, 2010) were able to improve their abilities to attend to children's mathematical thinking, interpret it, and respond to it as they analysed their classroom videos together.

In general, metacognitive support in mathematics teacher education often comes about as a result of structures that encourage process reflection, which Ricks (2011) defined as "an active form of reflection that extends and links together separate reflective incidents into cohesive mental continuums as ideas through action" (p. 252). Ricks (2011) noted that this form of reflection underlies models such as lesson study because it requires analysing data on children's learning during a lesson, identifying areas for improvement in teaching, and re-designing subsequent lessons accordingly. Process reflection provides opportunities for the identification of SOIs by prompting teachers to examine how students' mathematical thinking developed in response to specific teaching actions. Teaching actions can then be revised in accord with the goal of improving students' future mathematical learning.

Learning From Disagreements

Prompting teachers to identify SOIs as they analyse their practice with one another is important, but not sufficient. As noted earlier, professional noticing can vary a great deal among teachers (Santagata, 2011). Preservice teachers at times over-estimate their abilities to notice students' mathematical thinking (Simpson & Haltiwanger, 2017). Even experienced teachers at times disagree about the impact of a specific teaching action on children's mathematical learning. If learning from SOIs is to occur, there should be mechanisms in place to encourage the expression and exploration of disagreements that occur among teachers and between teachers and teacher educators as they analyse classroom data.

Facilitators of teacher conversations like those that take place during lesson study debriefing can help group members express and explore differences of opinion. One of the most important actions facilitators can take is to focus teachers' attention on specific evidence of students' thinking and the impact of teachers' actions on learning (Santagata, 2011). For example, facilitated video discussions can help novice teachers make sense of students' thinking in relation to specific teacher moves (Stockero, 2013). Such discussions are needed if disagreements in interpretation are to be expressed and explored. Although such discussions sometimes help teachers reach consensus, there is value in engaging differences of opinion even if agreement is not reached. Interpretations of data will sometimes differ even when all parties are focused on student learning in relation to specific teaching actions. Matusov (1996) noted that such disagreements need not be viewed only as nuisances or obstacles. Different paradigms and goals will, at times, lead to lack of consensus, but openly expressing disagreements can strengthen a group rather than weaken it, even if consensus is not ultimately reached.

Summary of Applying Positive Error Culture Principles Teacher Learning

A summary of how the design principles discussed above can serve to help translate SOIs to teacher learning is provided in Figure 1. Affective factors are shown in the top half of Figure 1 and cognitive factors in the bottom half. Affective factors that help SOIs translate to teacher learning include reducing anxiety associated with high-stakes situations, distributing responsibility for consequences of teaching actions through collaboration, and de-stigmatising disagreements in interpretations of classroom data. Cognitive factors illustrated in Figure 1 pertain to developing teachers' professional noticing. Literature suggests such noticing can be fostered through techniques that provide metacognitive support, such as collaboratively engaging in process reflection and exploring disagreements in interpretation of classroom data. Experience-based learning is foundational to all of the cognitive and affective factors illustrated, because grounding teacher learning in their actual practice allows for the generation of SOIs.

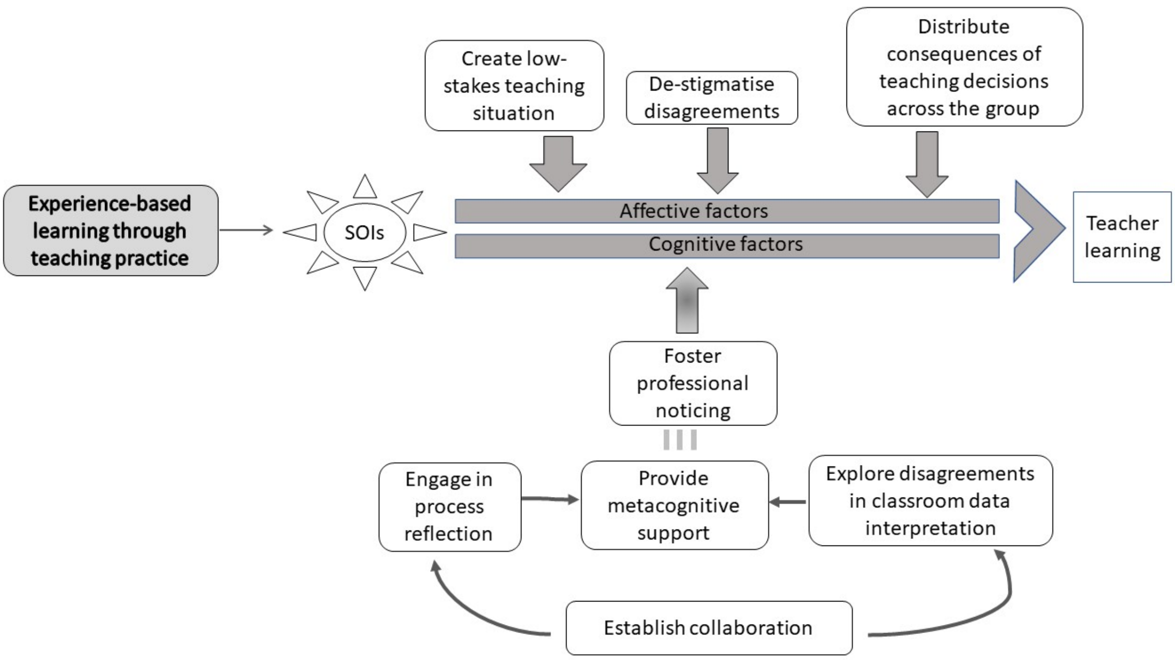

Figure 1. Hypothesised affective and cognitive factors in translating SOIs to teacher learning.

Method

Next, we explain how we used the design principles for encouraging a positive error culture discussed above to support mathematics teacher learning from SOIs. Connections between the design principles and the learning environment to be described are summarised in Table 1. The environment has many similarities to lesson study in that it is driven by process reflection. It goes beyond lesson study, however, by introducing elements of design-based research such as more intensive qualitative data analysis and design of a sequence of lessons rather than a single lesson (Bakker & van Eerde, 2015), We also incorporated micro-teaching of each lesson (He & Yan, 2011) before it was taught to children. We will describe the work of three pairs of pre-service teachers who worked in the environment over the same 10-week time period, focusing on empirical episodes to illustrate variation that can occur in the process of learning from SOIs.

Table 1
Summary of our enactment of five positive error culture design principles.

Design Principle	Enactment in Sample SOI Learning Environment
Experience-based learning	Participants experiment with new methods in the context of teaching children.
Low-stakes setting	Micro-teaching of the lesson is carried out with an audience of peers to receive feedback before the lesson is taught to children.
Collaboration	Participants work with a co-teacher, a university faculty mentor, and other pairs of pre-service teachers in the process of designing and analysing instruction.
Process reflection	Empirical data from each interaction with children are analysed for the purpose of improving children's learning during subsequent interactions.
Exploration of disagreements	Individuals in mentor/pre-service teacher triads compare interpretations of classroom data with one another during qualitative classroom data analysis.

Participants

The six pre-service teachers whose experiences are described in the empirical episodes to follow have the pseudonyms Ava, Clara, Jade, Eliza, Oliver, and Molly. Oliver was the sole male pre-service teacher in the group; the other five participants were female. Oliver and Clara were studying to be secondary school mathematics teachers; the other four participants were in primary school teacher preparation programs. All six participants were selected for a paid 10-week extracurricular research experience during the summer, when they were not taking coursework (monetary compensation for participants was approved by the institutional review board, resonant with common conventions for research in the home country). To be selected, they needed a minimum 3.0 grade point average and two letters of recommendation from university faculty members who had taught them in mathematics or education courses. Because the research experience involved working with children, all participants also had to pass criminal background checks and agree to abide by guidelines for gathering, analysing, and handling data specified by the human subjects institutional review board of the institution hosting the research. All participants were treated in accord with institutional review board guidelines, volunteering to participate rather than being coerced or required to do so to satisfy degree requirements.

Procedure and Data Analysis

Participants were paired to plan, carry out, and analyse instruction over a 10-week timespan. Each pair worked with an assigned group of four children. Ava and Clara were given the task of helping a group of 9-10-year-old children understand multiple representations for multiplication and connections among them. Jade and Eliza had the objective of helping 8-9-year-old children make sense of mathematics word problems. Oliver and Molly were to help 12-13-year-old children understand compound probability. Each pair was supervised by a university faculty mentor whose research interests matched the given project objectives. Each triad engaged in the process reflection cycle depicted in Figure 2 to design and analyse seven weekly one-hour lessons. Next, we describe the stages in the cycle.

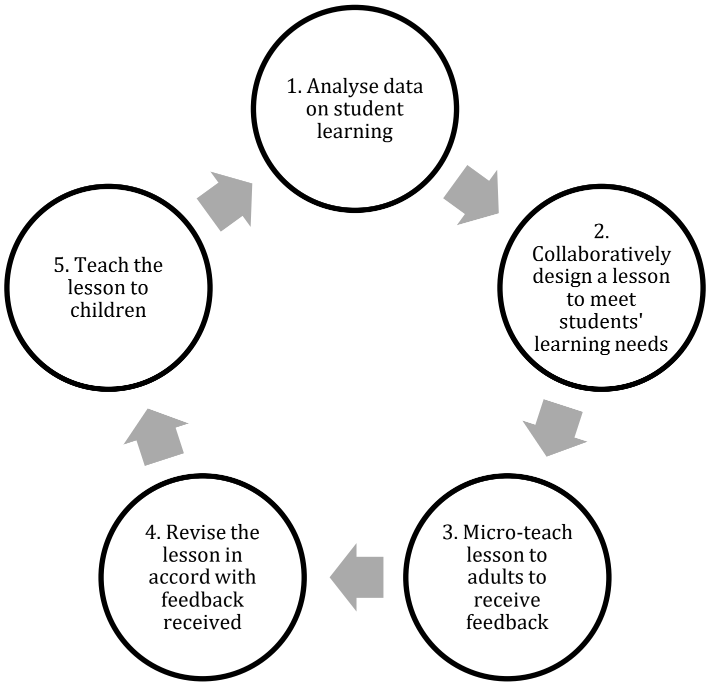

Figure 2. Teaching and research process reflection cycle used to guide participants' work.

Stages 1 and 2: Analyse Data on Student Learning to Collaboratively Design a Lesson

Student learning data to initiate Stage 1 (Figure 2) were gathered during the first week of the project, when pre-service teachers interviewed the children to learn what they knew about the mathematics they would be responsible for teaching them in subsequent weeks. Faculty mentors drew interview items from journals and websites of professional organisations. Items were selected with the intent of revealing children's thinking in regard to the mathematical ideas pre-service teachers were to address in their lessons. Pre-service teachers interviewed children in their group, video recording and transcribing the interactions. Additional details about the process used to prepare pre-service teachers to conduct interviews can be found in Groth et al. (2016).

Interview transcripts and video were viewed and coded in collaboration with a faculty mentor. Each triad began analysis with open coding (Corbin & Strauss, 2008), devising and attaching labels to capture salient student thinking patterns. Mentors guided the labeling process by introducing terms generally used in the mathematics education literature to describe observed patterns in students' thinking. For example, Ava and Clara's mentor introduced the distinction between additive and multiplicative reasoning (Tobias & Andreasen, 2013) to help pre-service teachers attach descriptive labels to their interview transcripts. This type of guidance allowed faculty mentors to introduce relevant theoretical constructs to guide the pre-service teachers' reflective conversations and subsequent lesson design, as would be done by an outside expert during lesson study (Lewis, 2016). The end product of this stage was a lesson plan designed to address the specific learning needs of children in each group. Additional details about procedures used to help pre-service teachers design lessons based on their qualitative data analyses can be found in Groth et al. (2020).

Stages 3 and 4: Micro-Teach the Lesson to Adults to Receive Feedback and Revise

After collaboratively designing each lesson, the pre-service teachers micro-taught it to an audience of adults who provided critical feedback before lesson implementation. The audience was comprised of faculty mentors and peer pre-service teachers. Pre-service teachers prefaced each micro-teaching episode by telling the audience what they knew of children's thinking about the mathematics objectives for the lesson. As pre-service teachers micro-taught the lesson, the audience commented on elements of the lesson they believed needed to be revised to better address children's thinking. The audience feedback was summarised in writing and given to each pre-service teacher/mentor triad. Each triad met after micro-teaching to discuss the feedback and make final adjustments to the lesson before teaching it to their assigned group of children.

Stages 5 and 1: Teach the Lesson to Children and Gather Data to Analyse

As each lesson was implemented with children, it was video recorded. Parental consent was obtained for video recording children and retaining children's written work. Lesson video, transcripts, and written work were then analysed collaboratively by each pre-service teacher/mentor triad. During lesson analysis, codes were assigned to describe student thinking patterns observed during each lesson segment, as was done during pre-interviews, in order to connect data on students' thinking to specific teaching actions. Student thinking patterns that revealed lack of intended mathematical learning provided starting points for revising teaching actions. The analysis of each lesson led to the design and testing of the next lesson during the subsequent cycle.

Retrospective Analysis after Completion of all Cycles

Post-interviews with children were conducted and analysed during the ninth week of the project, and retrospective analyses of the entire nine weeks of interaction were done during the tenth and final week of the experience. Post-interviews contained the same tasks as pre-interviews in order to provide evidence of how students' thinking may have developed over the course of the entire project. Participants constructed posters that compared student thinking patterns from pre-interviews to those observed in post-interviews. They also summarised their observations about student thinking during lessons into three clusters to identify salient events from the beginning, middle, and end of the instructional sequence they had designed and implemented. These retrospective analyses were to help participants further identify connections between the teaching practices they had used and the accompanying student thinking patterns they observed. Each group gave an oral presentation of their poster at the end of the summer, which provided additional information on pre-service teachers' learning.

As the project was carried out, the two authors of this report gathered and analysed data about pre-service teachers' learning on a continuous basis. We participated in each micro-teaching session and provided feedback to each pair on possible improvements to each week's lesson. We also reviewed video of participants' teaching sessions to provide additional constructive feedback. After each meeting with the participants and after each lesson the participants taught, we debriefed about the strengths and weaknesses of the participants' planned and implemented instruction. As the project progressed, we also noted the extent to which participants revised their teaching practices in accord with the feedback they received. At the end of the experience, we provided feedback on participants' posters and final presentations. Constant analysis of the data throughout the 10-week span of the project allowed us to perform the functions of supporting and examining the pre-service teachers' learning simultaneously. At the conclusion of the project, we summarised our observations about participants' growth by focusing specifically on opportunities for participants to learn from SOIs embedded in the lessons they taught. While analysing data, we noticed that pre-service teachers' experiences could be characterised according to whether or not they recognised and/or resolved SOIs. Next, we use this observation to organise our narrative about participants' experiences.

Results

During the project, we noticed that participants sometimes recognised SOIs, and at other times they did not. Likewise, they sometimes resolved SOIs by making improvements to their instructional practices and at other times left them unresolved. In this section, we offer three contrasting accounts of participants' work. The accounts illustrate different learning outcomes and pathways within the positive error culture environment we sought to establish. The illustrative accounts are drawn from our experiences working with three pairs of pre-service teachers, so they are not representative of any larger population or intended to be exhaustive; however, they do exemplify some possible patterns of teacher learning from SOIs within a positive error culture.

Ava and Clara: Progressing from Unrecognised, Unresolved SOIs to Recognised, Resolved SOIs

Early in the experience, Ava and Clara implemented activities to help children connect skip-counting to multiplication and solve multiplication word problems. For their fourth lesson, the triad decided the children were ready to deal with arrays and their connections to multiplication sentences. Ava and Clara believed the best way to approach this would be to begin by showing children an array of rectangles with four rows and three columns and writing the number sentence 4 X 3 = 12 alongside it. They would then give children 28 more arrays of objects on a worksheet and have them write a multiplication sentence for each array. The pair's faculty mentor and some of their peers worried that this approach might result in children simply mimicking the first example and not reveal or develop children's conceptual understanding of multiplication. Nonetheless, the pair strongly believed in the approach and implemented it during their lesson.

As children wrote multiplication number sentences for the 28 arrays they were given, they made few mathematical errors. At the conclusion of the lesson, Ava and Clara were satisfied that the children understood how arrays represent multiplication. Their faculty mentor, still concerned the children might not understand how the multiplication sentences represented the number of objects in each array on the worksheet, had Ava and Clara project a large rectangular array on a whiteboard and ask children to find a way to count all of the squares in it without counting one-by-one. As children came to the board, they proposed counting by 2s and 5s but did not suggest multiplying the dimensions of length and width. As they analysed lesson video with their mentor, Ava and Clara expressed surprise at this turn of events. It prompted them to pose conceptually oriented problems with arrays over the next two lessons rather than assuming the children had developed deep understanding from the original rote exercises.

In this case, making children's thinking more visible by using a different but related task allowed Clara and Ava to identify and resolve a SOI they initially did not recognise. Actions like these, which are not initially acknowledged to be sub-optimal practices but later are, have the potential to provide some of the best learning opportunities (Metcalfe, 2017). It is also important to note that the faculty mentor engaged the pair in recognising and resolving the SOI rather than just fixing the problem for them, because taking control of their teaching in such a manner likely would have resulted in a resolved, but unrecognised, SOI.

Jade and Eliza: Progressing from Recognised, Unresolved SOIs to Recognised, Resolved SOIs

Recognition of SOIs does not always lead to immediate resolution. Whereas Ava and Clara were able to progress promptly toward SOI resolution, Jade and Eliza lingered in recognised, unresolved SOIs for a considerable amount of time before moving toward recognised, resolved SOIs. During pre-interviews, children in their group often applied key-word approaches, using incorrect operations in solving problems, such as adding two numbers because the phrase "how many" appeared in the problem, even though addition did not yield a reasonable answer in the given situation. The interview data helped illustrate that key-word approaches usually result in surface-level understanding of the mathematical structures of situations (Karp et al., 2015). This provided a starting point for Jade and Eliza's mentor to work with them on alternatives to key-word approaches for teaching the group of children.

Jade and Eliza's faculty mentor recommended having children use manipulatives to act out word problems to help them engage more directly with problem context rather than attending only to surface-level features. The two pre-service teachers agreed with this advice and sought to implement it. The mentor also recommended using focusing rather than funneling classroom discourse patterns to prompt students' sense-making about word problems. Herbel-Eisenmann and Breyfogle (2005) discussed both patterns, explaining,

> Funneling occurs when the teacher asks a series of questions that guide students through a procedure or to a desired end. In this situation, the teacher is engaged in cognitive activity and the student is merely answering the questions to arrive at an answer, often without seeing the connections among the questions. (p. 485)

Focusing, in contrast, "requires the teacher to listen to students' responses and guide them based on what the students are thinking rather than how the teacher would solve the problem" (p. 486). Focusing is usually a more meaningful form of discourse in the sense that it elicits and extends students' thinking rather than funneling them exclusively in the direction of the teacher's thinking. The National Council of Teachers of Mathematics (2014) used the distinction between funneling and focusing to illustrate how teachers can discourage or encourage students to explain and reflect on their thinking during classroom discourse. Jade and Eliza actively sought opportunities for focusing discourse while teaching, though they were not immediately successful in implementing it.

As Jade and Eliza analysed video of lessons they had taught, they found it was difficult to diagnose and code children's reasoning because it was mostly the teachers, rather than the children, who were acting out problems. With their mentor, they discussed how they were funneling students toward solutions to the word problems (Herbel-Eisenmann & Breyfogle, 2005), which revealed the teachers' thinking about the problems but not the children's. In subsequent lessons, Jade and Eliza incorporated puppets because of their potential to engage children in critiquing mathematical reasoning (Keogh & Naylor, 2009), but video analysis once again revealed little of the children's reasoning because the teachers maintained control of the puppets. Moreover, some children were visibly disengaged from solving the problems posed. Jade and Eliza joked that this was because of the "puppet funneling" they had done. This insight prompted them to hand the puppets to students during their final lessons. They prompted children to use the puppets to act out word problems during these lessons, which they readily did. This made children's thinking more visible during video analysis and drew disengaged children back into mathematical activity. During post-interviews, children who used key-word approaches for some problems during pre-interviews explained their reasoning in relation to problem context instead.

Oliver and Molly: Functioning Mainly with Unrecognised, Unresolved SOIs and Recognised, Unresolved SOIs

Even though Ava and Clara and Jade and Eliza took different pathways in learning from SOIs, in both cases evidence existed that they recognised and resolved SOIs at some point. Evidence of SOI resolution was not apparent for the third pair of pre-service teachers, Oliver and Molly. They moved quickly, during the first 10 minutes of their first lesson, to ask children to reason about the compound probability situation of flipping two coins, even though most were not able to solve compound probability problems during pre-interviews. In subsequent lessons, they struggled to scaffold children's learning in ways that produced understanding of compound probability. As with Jade and Eliza, it was difficult to diagnose students' reasoning and code transcripts of Oliver and Molly's lessons because they also tended to use a funneling pattern of classroom discourse. Oliver and Molly were also advised to aim for a focusing pattern of discourse, but unlike Jade and Eliza, they did not commit to setting funneling aside. Post-interviews contained little evidence that children in their group understood compound probability situations. The case of Oliver and Molly indicates that being included in a positive error culture does not guarantee that all individuals will learn from their SOIs.

Discussion

Figure 3 uses a four-quadrant diagram to summarise the pre-service teacher outcomes we observed. Quadrant I contains the desired outcome of recognising and resolving SOIs. The remaining quadrants contain descriptions of other possible outcomes that may occur in a setting that encourages learning from SOIs.

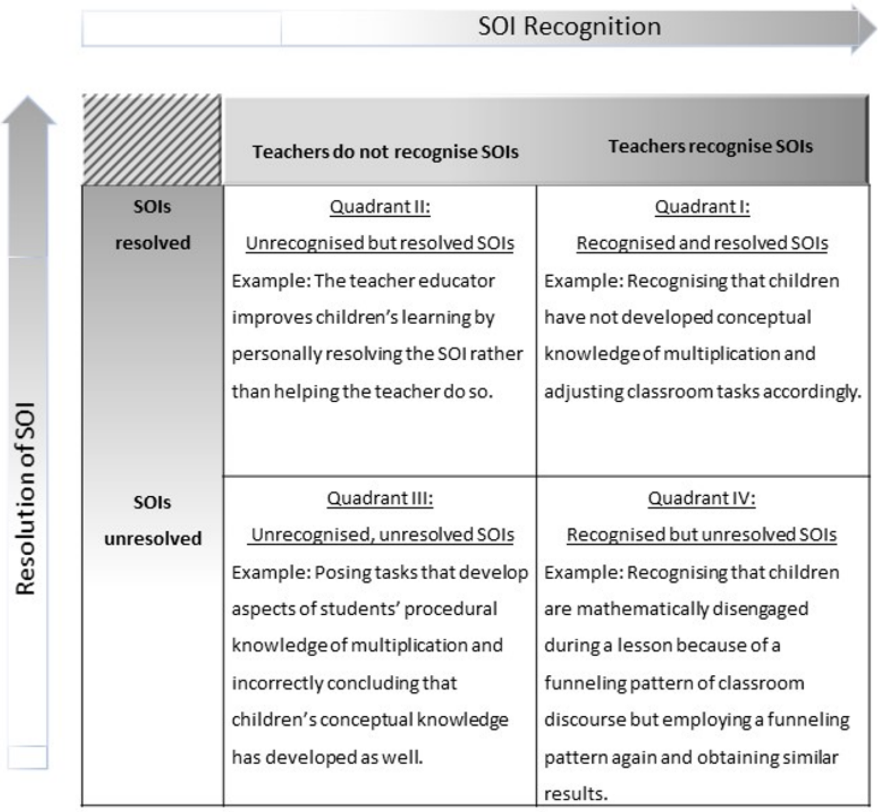

Figure 3. The four quadrants individuals may traverse within an environment that promotes learning from SOIs.

Our participants' movement (and stasis) among the quadrants in Figure 3 is summarised in Figure 4. We believe that the approach to arrays implemented by Ava and Clara led to an unrecognised, unresolved SOI (Figure 3, Quadrant III) that later became a recognised, resolved SOI (Figure 3, Quadrant I) as their faculty mentor helped them discover the limitations of their teaching approach. Jade and Eliza recognised SOIs related to their facilitation of classroom discourse for a period of time (Figure 3, Quadrant IV) and ultimately persevered to resolve them (Figure 3, Quadrant I). Oliver and Molly lingered in Quadrants III and IV, as they struggled to resolve their recognised and unrecognised SOIs related to facilitating classroom discourse and scaffolding children's mathematical learning.

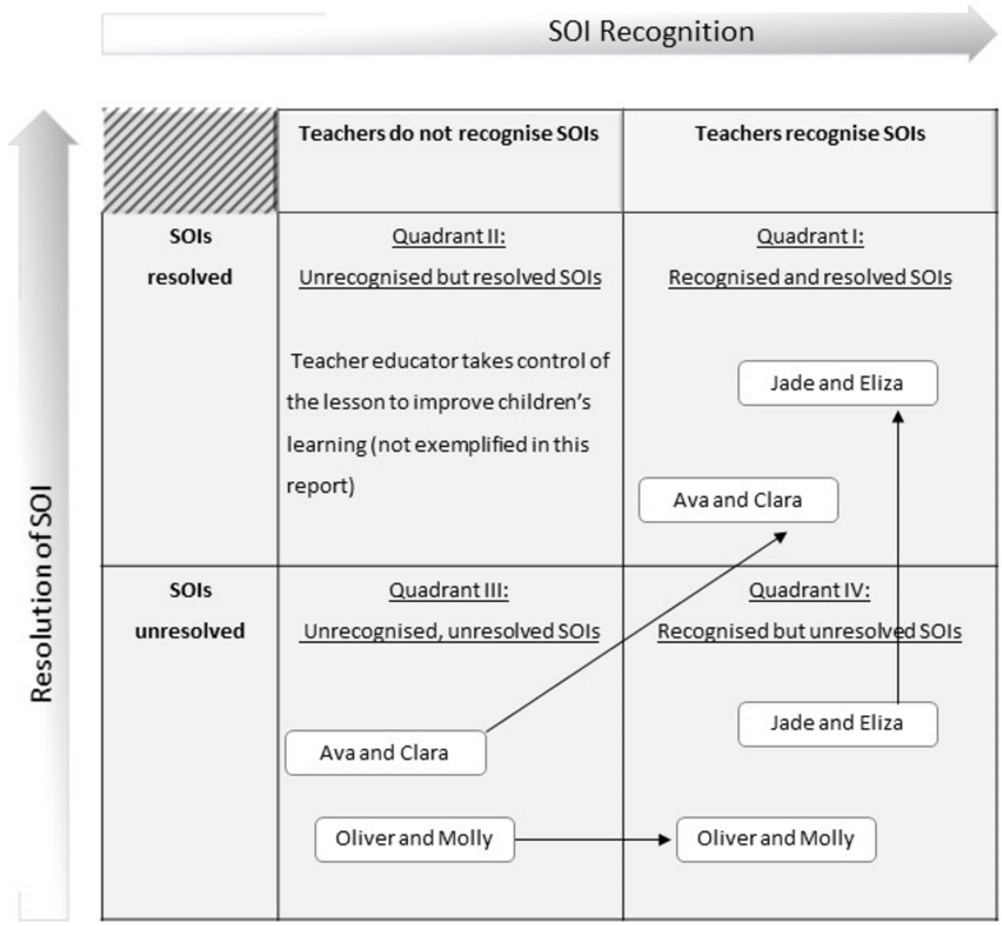

Figure 4. Patterns of teacher learning from SOIs illustrated by accounts in the present essay.

We hypothesise that sustained movement toward Quadrant I (Figure 4) is driven by the cyclic relationship among positive error culture, SOIs, and children's mathematical learning depicted in Figure 5. Ideally, participation in a positive error culture (node 1 in Figure 5) leads to identifying and learning from SOIs (node 2 in Figure 5). As the SOIs are recognised and resolved (i.e., Quadrant I in Figure 3), there is a positive impact on children's mathematical learning (node 3 in Figure 5), as in the cases of Ava, Clara, Jade, and Eliza. Seeing these improvements in learning can affirm the value of participating in the positive error culture. Hence, the cycle can repeat itself indefinitely, ultimately contributing to the long-term sustainability of professional communities like the one described in this report.

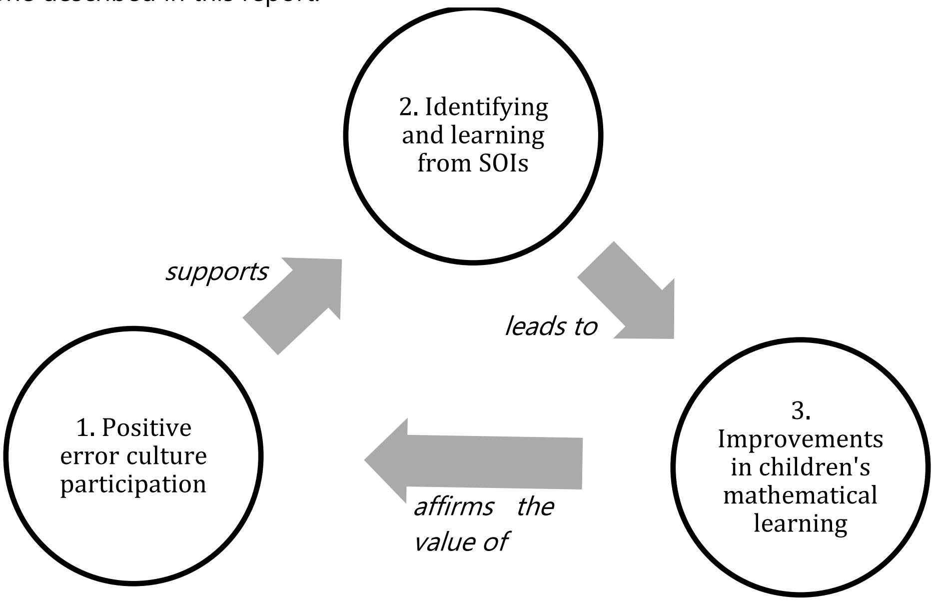

Figure 5. Hypothesised cyclic relationship among positive error culture participation, SOIs, and children's mathematical learning to sustain the process of learning from SOIs.

In all of our examples, various interventions by mathematics teacher educators were necessary to help pre-service teachers move through the Figure 5 cycle. Ava and Clara benefitted from working with a mentor who disagreed with their teaching approach but allowed them to carry it out nonetheless. Carrying out their approach to arrays provided an opportunity for the teacher educator to reveal resultant weaknesses in students' understanding that Ava and Clara otherwise would have overlooked (helping the pair make the transition represented by the link between nodes 1 and 2 in Figure 5). Recognising and resolving this SOI led to improvements in children's conceptual understanding (as represented by the link between nodes 2 and 3 in Figure 5). Seeing the improvements in children's understanding illustrated the value of their continued participation in the positive error culture (represented by the link between nodes 3 and 1 in Figure 5). Notably, these positive learning outcomes for Ava and Clara and their students occurred because they were allowed to make errors during teaching and then received guidance to identify and address them. Likewise, as Ava and Clara gradually attained the goal of focusing classroom discourse that was introduced by their mentor, they ultimately observed improvements in student thinking that validated their efforts to learn from the process of revising sub-optimal practices.

The case of Oliver and Molly, in contrast, emphasises the important task of finding additional strategies to ensure that participation in a positive error culture produces its desired results. Like their peers, Oliver and Molly were permitted to make errors while teaching, and they were advised to avoid funneling patterns of discourse. Unlike Jade and Eliza, Oliver and Molly did not acknowledge the limitations of the funneling patterns of discourse that pervaded their lessons. Persisting with funneling severely limited opportunities to identify SOIs because video data from their lessons revealed mostly what the teachers were thinking rather than patterns of children's mathematical thinking. Our informal day-to-day interactions with Oliver and Molly suggested that they held a strong transmission-oriented view of mathematics teaching (Barkatsas & Malone, 2005; Perry et al., 1999), thinking that students learn most effectively by being told how to do mathematics (Fennema et al., 1996). Such beliefs severely limit what can be learned in a positive error culture; even if teachers with these beliefs are open to learning from their errors, their attention will likely be limited to considering how accurately they have presented mathematics during lessons. Systematically assessing teachers' beliefs about teaching and learning mathematics before, during, and after their participation in experiences intended to foster learning from SOIs could provide insight on how best to support participants in accord with the beliefs they espouse. Such research could lead to identification of additional cognitive and affective factors (Figure 1) associated with translating SOIs to teacher learning.

Conclusion

The act of teaching mathematics inevitably leads to student learning difficulties in need of resolution. Eliminating all such difficulties is not an option. Fortunately, however, such difficulties need not be viewed solely in a negative light. Carefully drawing teachers' attention to them can enhance teacher education by creating opportunities to learn from SOIs. A positive error culture provides a setting compatible with such activities. Additional research can help further clarify and define productive roles for mathematics teacher educators to assume when facilitating teachers' learning within such environments. Our empirical illustrations and accompanying framework for characterising pre-service teachers' learning from SOIs provide starting points and infrastructure for such endeavours.

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