Mathematics Teacher Education and Development 2021, Vol 23.3, 166-181

Differentiation from an Advanced Standpoint: Outcomes of Mathematics Teachers' Action Research Studies Aimed at Raising Attainment

Alf Coles
University of Bristol, UK

Laurinda Brown
University of Bristol, UK

Received: 2nd February, 2021/ Accepted: 18th March, 2021

In this article we propose the notion of differentiation from an advanced standpoint as a teaching strategy, particularly valuable for working with students with low prior attainment. The notion arose from an enactivist analysis of the work of three teachers, engaged in action research in their own classrooms. All three teachers chose to teach their students (who were aged 15-16) topics that are usually only offered to those with relatively high prior attainment in mathematics. No intermediate or bridging topics were offered, instead, these teachers found ways to differentiate work for their classes, from this advanced standpoint. There is tentative evidence of students experiencing their relationship to mathematics in new ways, recognising they were doing "A-grade" work, and of gains in their attainment. There is also evidence of the teachers' own surprise at what their students could achieve.

Keywords · differentiation from an advanced standpoint · mathematics-teacher research · low prior attainment · action research · enactivism·

Introduction

We work with prospective and in-service mathematics teachers in a community of secondary schools serving students from 11 to 18 years old in England. In England, almost all secondary schools set students for mathematics teaching and learning, although there are current moves to shift this practice at primary and secondary levels. By setting, we mean grouping students by their prior attainment within a subject. Streaming would be understood as creating groupings of students that operate across all subjects studied. The sets an individual student would be placed in for different subjects would vary, so, they might be in the highest set for mathematics, say, but the lowest set for English. There are consequent implications for access to the curriculum from this cultural given. National examinations at 16+ in mathematics have higher-tier content on papers that are taken by higher-achieving students, whereas foundation-tier papers are taken by all other students. Effectively, in England, we have differentiation of content through differentiation of tasks. Access to study mathematics beyond 16+, what in England are called A-levels, is only possible through success in higher-tier examinations.

Nationally, teachers of mathematics have little experience of working in heterogeneous groups beyond the early years of secondary (11-18 year old students) education (Taylor et al., 2017; Taylor et al., 2020). Only recently is evidence emerging, in England, of how lower- attaining student contributions may benefit the work of higher-attaining partners, just as much as the other way around (Barclay, 2021). However, within any grouping of students, even in a setted class, there is a mix of achievement. This article explores the work of a collaborative action research Master's course of in-service teachers of mathematics, where a challenge became how to give access, for the lowest sets of students, to the higher-tier of the terminal examinations. We document the strategies used by the mathematics teachers in the action research group, as a challenge spread from the experience of one teacher working with their lowest attaining students to teach what is called higher-level work (i.e., the work needed for the higher-tier examination). We have come to label the strategies used by the teachers as *differentiation from an advanced standpoint* and, in the next section, place this notion in the context of other thinking about differentiation. We then offer more details about the action research group and the methodology of this study, before getting to the data from the teachers.

A Brief History of Differentiation in England

The word differentiation first emerged in curriculum documents in England and Wales in the mid-1980s (see, DES, 1985, pp. 27-28 and Appendix 2), becoming an important topic for teacher development. The illustrative example, commonly now called Max Box, "what is the largest volume of a box that can be produced from a fixed piece of card?", discusses a problem that can be given to any students but where there is differentiation by outcome: young children could change the dimensions of the corners cut off a sheet of card to form a box, filling different examples with sand; A-level mathematicians might find a solution through partial differentiation. Max Box became a popular activity that most teachers used, however, there did not seem, at the time, to be the expertise to generate other examples to cover the curriculum. The most commonly used form of differentiation, by task, leads to students with the lowest prior attainment not having access to the same curriculum as their peers. One barrier to access to the curriculum for all, and thereafter possibilities of further education, is setting, either across a year group in different classes, or using differentiation by different tasks for different groups within a heterogeneous class, with consequent reduced expectations for students of low prior attainment.

The interest in differentiation in curriculum documents in 1985 had grown out of paragraph 243 in the influential government document *Mathematics Counts*, commonly known as *The Cockcroft Report* (Cockcroft, 1982):

> Mathematics teaching at all levels should include opportunities for
>
> - exposition by the teacher;
> - discussion between teacher and pupils and between pupils themselves;
> - appropriate practical work;
> - consolidation and practice of fundamental skills and routines;
> - problem solving, including the application of mathematics to everyday situations;
> - investigational work. (p. 71)

Although exposition, consolidation and practice could be found in most classrooms, investigational work, in particular, was rare. This paragraph was said to be an attempt to make mathematics lessons "about something" (Brown & Waddingham, 1992, Introduction, second page). As the Cockcroft report describes:

> Mathematics lessons in secondary schools are very often not about anything. You collect like terms, or learn the laws of indices, with no perception of why anyone needs to do such things. There is

excessive preoccupation with a sequence of skills and quite inadequate opportunity to see the skills emerging from the solution of problems. (para. 462, p. 141)

Although the word differentiation was not around in 1982, the Cockcroft report describes in some detail what was meant by investigations:

> Investigations need be neither lengthy, nor difficult. At the most fundamental level, and perhaps most frequently, they should start in response to pupils' questions, perhaps during exposition by the teacher or as a result of a piece of work which is in progress or has just been completed. The essential condition for work of this kind is that the teacher must be willing to pursue the matter when a pupil asks 'could we have done the same thing with three other numbers?' or 'what would happen if ...?' […] The essential requirement is that pupils should be encouraged to think in this way and that the teacher takes the opportunities which are presented by the members of the class. […]
>
> Many investigations lead to a result which will be the same for all pupils. On the other hand, there are many investigations which will produce a variety of results and pupils need to appreciate this … Even practical routine skills can sometimes, with benefit, be carried out in investigational form; for example, 'make up three subtraction sums which have 473 as their answer'. The successful completion of a task of this kind may well assist understanding of the fact that subtraction can be checked by means of addition. (p. 74)

We have included extracts from these paragraphs to describe what came to be known as differentiation by outcome. This is not giving students a project to work on, on their own, but is a way of working as a community in a mathematics classroom. At the time that the Cockcroft report was published, Laurinda (second author) was working closely with the mathematics editor at the Resources for Learning Development Unit (RLDU) in Bristol. What seemed to be needed to support teachers in implementing paragraph 243 was more ideas for investigations spanning the curriculum (the report had published a Foundation List for all pupils). The Foundation List was sent to all teachers local to the RLDU with an invitation to send in any ideas they used for mathematical discussions, problem solving and investigations. The resulting publication, An addendum to Cockcroft (Brown & Waddingham, 1992) is still available freely (details in references) and includes accessible starting points, such as comparing samenesses and differences in two images or challenges such as "find me a square of area ten", that could open out in class discussions through the class focussing on students' contributions or questions. For each area of the curriculum, such as number, two exemplar lessons were written up showing how the discussions could develop. Interestingly, which is a point that will be picked up later in the article, working on, say, matrices and transformations with younger children was not so students were able to answer examination questions on matrices and transformations but so that they could apply other aspects of the curriculum and use, say, area, coordinates, transformations and directed numbers within a context.

In England, the common forms of differentiation are by task, by support and by outcome. There are many resources available that discuss these under different labels, for instance, flexible-pace learning, collaborative learning, progressive tasks, verbal support and variable outcomes (https://resourced.prometheanworld.com/differentiation-classroom-7-methods-differentiation/). For Sullivan et al. (2020), in present day Australia, when "facilitating and encouraging innovative practices" (p. 32), low floor and high ceiling tasks can provide the similarly accessible starting points that open out over time. Such tasks are said to be challenging in that they "allow the possibility of sustained thinking, decision making and some risk taking by students" (p. 32). Similarly to the Cockcroft report's description of investigations, they are "undertaken when there is more than one correct answer and/or more than one solution pathway" (p. 33). However, differentiation is limited to actions taken by the teacher "to differentiate tasks for students who might require additional support and those who finish quickly" (p. 33). For us, the nature of developing a classroom culture to support work on challenging tasks could be termed differentiation by outcome, within which there is differentiation by task and by support as necessary.

In this article, we will be discussing a form of differentiation which we have not seen before in the literature in mathematics education, a form which we have labelled differentiation from an advanced standpoint. This could be seen as a form of differentiation by outcome but without the expectation that there is a low floor or starting point that builds on what the students already know. The basic idea of differentiation from an advanced standpoint, is to choose a starting point for tasks with a class that is several years "beyond" what they would be expected to know according to curriculum documentation. The strategy is related to Sullivan et al.'s (2020) notion of challenging tasks, but different in the sense of the topics being so far beyond what students show they can do independently. How this operates, and what impacts it has, will be elaborated through the action research projects of three teachers, which we describe below. However, before getting to the work of these teachers, we set out some detail about the context of our work with them.

Collaborative Action Research Group

The University of Bristol is committed to diversifying its student body by empowering and supporting prospective students from underrepresented backgrounds to access university through its Widening Participation (WP) agenda. One action related to this agenda was inviting bids from members of staff to undertake longitudinal WP research pertinent to its own context. Professor Rosamund Sutherland was successful in obtaining funding for a three-year project, *Overcoming the mathematical barriers to participation in higher education*[1]. Data from that project is the focus of this paper.

The project used the Master's module entitled, *Professional development through collaborative working on an issue in education*, led by Alf and Laurinda (first and second authors), who are experienced in running such groups; Laurinda having written up her experiences from early days after the design of the course (Brown & Dobson, 1996), and in later use, as part of a research project (Brown & Coles, 2011). The module has provided a way into Master's study for teacher members of research groups, developing their thinking within a collaborative group, itself a form of differentiation. The course text (Altrichter et al., 1993) has supported the engagement of teachers through the process of action research, with mechanisms, activities to try out to support stages of their work, such as, the finding of an issue, or keeping a research diary. As part of the Master's module, each teacher might have a different issue they are working on. Experience tells us that, even though the issues might start out distinctly, the strategies for running the group as a reflective team, questioning an individual who is given time to present, leads to a rich and mutually extending atmosphere.

Ten mathematics teachers joined the collaborative Master's group, all of whom worked in schools that were classified by the university as falling within the scope of its Widening Participation (WP) agenda. These were schools where key measures were in the lowest quintile nationally, e.g., participation in Higher Education (HE); attainment at age 16; attainment at A-level; and proportion of free school meals. The project provided funds to cover the cost of two groups of teachers' enrolment on the Master's course, in 2013-14 and in 2014-15. The group we report on here is from 2013-14, which is the one where differentiating from an advanced

[1] We would like to acknowledge our debt to Ros Sutherland, who passed away in 2019, for her leadership of this project and also our thanks to the other project members, Richard Budd and Jeremy Rickert, for their work.

standpoint became a common feature (although not named as such at the time) across several teacher's work.

Data and Researching as Enactivists

We bring an enactivist methodology (Maturana & Varela, 1987) both to our work in supporting the teachers' inquiries and in organising the structure of this article through to analysis of the data. Enactivism is a philosophical stance which views knowing and doing as synonymous (Maturana & Varela, 1987). The enactivist view of cognition is profoundly anti-representational; perception is interpreted as an action, not a recovery or representation of a pre-existing world. We participate in the world in coming to know, and the world participates in us.

The principles for running the module sessions derive from the enactivist world-view and are about establishing a collaborative group among participants. What this means in practice is that:

> (a) the group size should be less than or equal to 10,
>
> (b) meetings should be spread out over an extended period of time,
>
> (c) the teachers should come from a range of schools and be volunteers rather than conscripts,
>
> (d) the leader of the group sets up a loose structure for meetings and time is given to each participant to discuss their emerging thoughts about their issue,
>
> (e) the leader of the group gives individual readings in between meetings to support participants thinking about their issue, or there could be tutorials for participants between meetings linked to their Master's study,
>
> (f) the leader(s) of the group will make one or more visits to each teacher's school to further support thinking about the issue and/or data collection, and, in some cases, the teachers visit each other's classrooms. (Brown & Coles, 2011, p. 865)

All of these principles were enacted in the research reported in this article, save the last one since there was not the resource available for the course leaders to make school visits. The set text for the unit, Altrichter et al. (1993) frames the work of the teachers around the idea of action research. Early meetings were focused on supporting teachers to find and clarify a starting point for their own action research. The teachers involved in the Master's module were given a pre-meeting task, before the first meeting, which was to think about a starting point for the action research they would undertake. The overall framing of the project needed to be 'widening participation', which included increasing the mathematical attainment of students as well as increasing participation in A-level and ultimately higher education.

Later sessions supported the finding of relevant literature and the planning of the actions that teachers wanted to investigate. Typically, teachers might try out data-collection methods, bringing to group meetings early data they collected, to share and discuss ways of analysing results. There may be one cycle of planning, implementing and evaluating a set of actions, or several cycles where each one is informed by the previous one.

We audio recorded all group meetings and were able to analyse the final 4000 word assignment for the course. Teachers were encouraged to keep a research journal, or diary, during the course and we had access to some of this writing, when it was shared by a teacher at a meeting. Of the ten teachers recruited, eight turned up to the first session and participated until the end. Our own analysis was undertaken following enactivist methodological principles (described, for example in Brown, 2015 and Coles, 2015); in particular, we used the notion of "equifinality" (von Bertalanffy (revised edition), 2015); Coles, 2015, p. 243) based on the insight that, in open systems that reach a steady state, "the same final state can be reached from different initial conditions and in different ways" (von Bertalanffy, 2015, p. 96). To give an example, a particular teacher sets up the culture of the classroom in which they work and the classroom is recognisably one of that teacher's, however, given the different students and the interactions over time, the particular behaviours might differ. This leads us to look first to the final items of data (in this case the assignments of the teachers). Patterns in that data are then used to trace the emergence of those patterns in earlier data. For instance, the initial level of analysis was to identify one-sentence descriptions of the projects the teachers undertook, which immediately threw up a pattern of doing higher-level work than expected with a class. We therefore focused on the four projects fitting this pattern. Three of those projects focused specifically on work with year 11 (age 15-16) classes, i.e., groups about to take their national General Certificate of Secondary Education (GCSE) examination in mathematics. For that reason, these three are the projects we report on here. We looked back over the contributions of those three teachers to group meetings and in their writing, picking out anything linked to the idea of doing higher-level work than expected. We have then selected, from this data, descriptions of what the teachers did and any evidence of impact on student attainment.

In working with a small number of cases, we are deliberately adopting an approach of "particularization" (Krainer, 2011, p. 47). We will be drawing out common themes in order to illuminate possibilities for action. We are not aiming to uncover general principles. We present the research of the three teachers in the form of narratives, to give a sense of how their work developed over time. Following these three narratives we then look at similarities and differences in order to draw out implications for supporting in-service teachers to plan for, and implement, differentiation in the classroom, including evidence of change in teacher views about differentiating instruction. We use the principle of equifinality and the exploration of "patterns which connect" (Bateson, 2002, p. 10) in order to structure the presentation and analysis of the teachers' projects.

Three Narratives

In this section, pseudonyms are used. All three teachers taught in schools serving areas with indices pointing to high levels of social deprivation in and around the city of Bristol.

Sally's Story

In some initial writing about why she wanted to undertake an action research project, Sally commented that the:

> school scheme of work does not allow for true independent inquiry – teachers end up re-teaching year after year. (Research journal, January 2014)

A scheme of work is a document, typically created within each school in England, that sets out what teachers should teach (in terms of topics) and for how long. Most curriculum images at that time were spiral. For some students this meant they were re-taught the same content year after year whilst making little progress. Elaborating on her starting point in her final assignment, Sally wrote:

> I came to the conclusion that over time I had subconsciously developed an almost resignation to pupils engaging during a lesson, yet not being able to retain and recall the learning and knowledge for the subsequent testing. I had almost separated the two parts of the process (learning and recalling) and grown to accept that as the way it normally would be. If I can engage a group of pupils during a lesson and they can achieve success and confidence then it is a great lesson!

> However, the topics that we teach are repetitive and pupil ability to retain what we teach them is not strong.

Reflecting on a lesson she taught to a year 7 (age 11-12) class about converting between units of metric measurement, Sally analysed the following features, related to retention and recall of learning (again, from her final assignment):

1. The lesson was a topic that had been covered before yet had not been previously mastered.
2. The teacher perceives that the pupils need a different hook to develop their learning.
3. The pupils face a topic where they may have previously been successful but could not remember all of it. (And this leads on to the possibility that they are struggling to engage because of disappointment/frustration at not being able to remember).
4. The teacher is trying to find yet another way of teaching a topic that should have been covered before yet needs to be done in a different way so that pupils can remain focused and hopefully have something that will remain in their understanding.

Sally was therefore wanting to explore what she could do in her teaching to both motivate students to engage in lessons (avoiding disappointment and frustration) and to increase what students can recall. A few weeks into the MSc sessions, Sally coined the phrase of a circular curriculum as a description of what many students experience (rather than the hoped for spiral curriculum of developing mastery). The circular curriculum captures the realisation that, for students who are struggling in mathematics, they effectively get re-taught the same content year after year between ages 11 and 16.

Sally chose, for her action research project, to focus on her examination class of year 11 (age 15-16) students. As mentioned earlier, in England, the examination for 16-year-olds was split between higher-tier (grades A* to E could be awarded) and foundation-tier (grades C to G could be awarded). In September 2013, Sally's year 11 class had finished the syllabus for the foundation-tier (they would take that examination in May 2014) and, instead of simply revising this content as a way of preparing them:

> with the support of my Head of Department, from October to December the pupils received a grade B Higher Tier diet including Trigonometry, Cumulative Frequency and the Quadratic Formula. I decided that the topics would […] reinforce the concepts needed for parts of the GCSE Foundation course. Trigonometry, for example, would hopefully cause the pupils to recall facts of triangles and consolidate their work with fractions and calculations. The pace was slow compared to how I would teach these to a higher achieving group. It took double the usual time as usual to teach but there was the flexibility within the structure this year for this to happen. The pupils were encouraged considerably that this would be something to try out to see if it helps their grades (positive re-inforcement), they were not going to be pressured into doing this in an exam and they were being told regularly that they were doing aspiring grade A/B work.

Sally was able to compare student achievement on a mock foundation tier paper in October and again in December, after the time spent working on higher-tier content. Overall, students improved their performance by more than one grade on average. Of the three students whose grade did not improve over this period (none regressed), two of them had already achieved their "target" grade; five student's scores increased by one grade and three students by two grades.

In more qualitative evidence, Sally recorded an incident in her research journal/diary:

> The Special Education Needs Co-ordinator (SENCO) from the school came up to me and relayed a story of one of my students offering to help another in the Special Education Needs department. His comment was "that's alright; I'll help him because I'm doing grade A work at the moment!" For this pupil to make that comment (and for the SENCO finding me to tell me) spoke volumes of the impact this learning was having on the pupils and how much progress was being made.

One of the barriers Sally had identified herself was student experiences of frustration and disappointment at not knowing things they had been taught. In this incident, the statement, "I'll help him because I'm doing grade A work at the moment", speaks to a perspective on mathematics that there are things this student knows and can share and wants to share. Sally concluded her assignment by looking back over what she had learnt:

> The first observation that I have seen in my teaching with year 11 is that I have raised the level of subject knowledge taught to similar year 11 groups in the past and it has been successful. Pupils can attempt and complete higher level work and seem to enjoy the challenge of something new. With support, pupils can achieve a huge amount and complete tasks to a similar standard to higher achieving pupils.
>
> The second observation that I have made, of my year 11 class, is that they are believing they can get a grade C in Maths. The whole idea of never being able to reach a C was very real for them in year 10 as their grades were similar to those at the October mock examination. Yet for them to have been able to complete higher tier work has produced a change in attitude in them and they are more positive about the maths and realising that they can succeed.
>
> And, finally, the effect of this research on my own practice has been highly significant. I have been teaching pupils topics which, in relation to their levels, I have not done before. It has challenged me to think again as to how I deliver these topics so that the entry point for each learner is accessible and they can see the progress being made […] I have had to be aware that pupils have not always been able to see the links between topics and that I still need to revise and review topics with them. However, this would be done very differently now, giving them ownership of the problem to start with, instead of explaining more quickly how to do something without giving them space to think and respond. My expectation of what pupils can achieve has increased, yet I still need to develop my skills in helping pupils find their own support frame (figuratively speaking) to work from.

Lack of a C-grade in mathematics can be a barrier to participation in higher education in England, i.e., many university courses will require a C-grade in mathematics GCSE as part of their entry requirements. It is therefore highly significant in the context of the overall research project that Sally was finding ways of engaging students in working towards a C-grade and that the introduction of higher-tier topics seemed so successful in improving performance on foundation-tier examination questions. The importance of challenging tasks in motivating students, as long as there is differentiation by support; time to dwell in the ideas and think their own thoughts; an accessible entry point for each student; and the ownership of the problem to begin with, links with both Sullivan et al. (2020) and the Cockcroft report (1982). Yet this is not in a context of using investigations or problem solving, what is focused on is trigonometry, cumulative frequency and the quadratic formula.

Adam's Story

In the first group meeting, Adam commented that he was interested in "engagement" and was coming to ask himself: "do students want to understand?". He elaborated on what he meant by this by offering an anecdote from his classroom in which students had got confused because in teaching them how to draw a graph of a function, he had set out a table of values without specifying which x-values they should use in the table. The students complained that their previous teacher had always given them the x-values and one girl in particular seemed unable to get over this difference. Adam reflected in his final assignment: "She had procedural knowledge of how to answer the problem from a given starting point and provided she was given this starting point she could answer the question correctly". Outside of this context she seemingly had no strategies on which to draw. He commented about his students:

> They're just saying that they don't know but I know that they know because they've shown me, several times, that they know […] is it just that they're not confident? Is it that they don't want to think about it and it's wrong, then […] I don't know what their previous experience is. (Meeting 5)

Adam was concerned to get his students thinking, to provoke conceptual knowledge and understanding, but was experiencing students who purported to want to be told methods and steps for solving specific kinds of examination questions. He, like Sally, chose to focus his action research on his year 11 class. He had a class whose grades in a mock examination in December included a fail, a G, many Fs and Es and one D. The action research he undertook involved teaching this group higher-tier content for six weeks. To guide his teaching, Adam found it useful to distinguish procedural from conceptual knowledge (Rittle-Johnson et al., 2001, p.346) as follows:

> Procedural knowledge: the ability to execute action sequences to solve problems. This type of knowledge is tied to specific problem types and is therefore not widely generalizable.
>
> Conceptual knowledge: implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain.

He commented that:

> I did not want to create an environment "which causes pupils to base their mathematical thinking on whatever they think the teacher wants them to do." (Boaler, 2002, p.103). I want my students to examine the grounds on which they base their mathematical thinking and knowledge. I wanted to identify procedural and conceptual thinking in my students and look at how this contributes to their lessons. (Final assignment)

Adam taught the higher-tier content. Although what he offered, he reported, remained largely procedural, "[s]tudents felt encouraged, if not surprised, that we believed they could be successful. They seemed proud to be able to do a "grade-A question" (final assignment). He reported (Meeting 8) an incident involving one of the students in his project class:

> I was doing an intervention session after school with some of my Year 11 students and a couple of others from different classes and one of them said something really interesting […] one of the girls was doing cumulative frequency [she] wasn't from one of my classes, she [was in] the class above mine […] There was a question that said "what's the modal class" from the frequency table at the beginning. I've never taught modal class before to my class, and one of my students just said "how do you not know what the modal class is? It's obviously the one with the most in it". How did she know that? Obviously we've done mode, they'd hopefully know that mode is the most but usually when you get to modal class they're like "what's that? I don't understand."

In this extract we can observe Adam being surprised by the connections that one of his students has been able to make. In that same meeting he referred to the work he was doing with this class on higher-tier content: "[i]t does seem positive, they seem to like it […] as opposed to what they would have been doing and they do seem to be picking up random things. You assume stuff and they just seem to do it without [being] explicit". The modal- class anecdote is an example of a student "picking up random things". Although not necessarily evidence of conceptual thinking, it is in contrast to Adam's initial stories of his students seemingly not knowing what he knows they do know.

> At the end of the six weeks, Adam gave the class a second mock examination. Unlike Sally, who gave her class a foundation-tier mock having taught them higher-tier content, Adam gave his class a higher-tier mock and: "made them aware that there would be parts they would not be able to do and that they should look through for questions they felt comfortable answering" (Adam, final assignment). Adam explained the results:

> Our school would define a student who makes a whole grade of progress in a year as making accelerated progress […] results seemed very positive. The students made an average progress of one grade in the space of 6 weeks of teaching, with one student making 3 grades. Only one student showed regression (Final assignment).

In the context of Adam's school, the progress made by seven of his ten students (according to these results) would be judged as accelerated if it had occurred over the period of one year. These results were obtained after six weeks of teaching. Adam analysed the reasons he thought such striking improvements had been possible for the class:

For a student to be successful in the higher paper they will need to know how to coordinate fewer sequential processes to get the required marks to achieve their target grade. The higher paper seems therefore to be more beneficial to them, looking at the evidence I have collected so far.

Adam did not, in the assignment, elaborate on the idea that the higher-tier examination paper requires fewer sequential processes. We have looked at the higher- and foundation-tier examination from June 2014 (for the examination board AQA) and picked out two questions, around half-way through each paper, that perhaps illustrate the idea:

- Foundation Tier (Q10): Andrew is paid £850 a month. Each month he spends 60% of his pay and saves the rest. How many months will it take Andrew to save £1700? (4 marks)
- Higher Tier (Q9): A tank is in the shape of a cylinder of radius 15cm and height 50cm. Work out the volume of the tank. (3 marks)

(Adapted from the June 2014 Linear Examination Series, AQA. Available at: http://www.aqa.org.uk/subjects/mathematics/gcse/mathematics-linear-b-4365/past-papers-and-mark-schemes.)

The higher-tier question requires the application of one formula, i.e., a single process. The foundation-tier question might require the calculation of 60% of 850, subtraction of this value from the monthly pay, division of 1700 by the amount saved and then interpreting the result in the context of the question, perhaps three nested processes – and yet this question would be judged as a lower GCSE grade than the one about volume.

What seems to be apparent in Adam's story is that he is learning to trust that his students are able to think for themselves. In giving them space to think, they do not expect to be told what to think, nor wait for a teacher to tell them what modal class means, for instance, carrying over to them giving the higher-tier questions a go.

Eleanor's Story

The starting point for Eleanor was a recognition that her students were, "not motivated and so not resilient" (Eleanor, meeting 2). She chose to focus her action research on her year 10 or "year 11 bottom set who aren't going to get a GCSE" (Eleanor, meeting 2) and wondered at that time: "[s]hould we be doing something else? They're going to be entered [for GCSE], but there is no way that they'll get a C". At that stage, given this assumption, Eleanor was thinking about alternative qualifications she could offer the class. Over the course of the first few meetings, in which several teachers were talking about the idea of working on higher-tier content with students expected to be entered for foundation tier, Eleanor moved towards a similar focus. In meeting 7, she commented:

> I'm still looking at my bottom set Y10 and the Y11 class […] they're supposed to be the same sort of class […] I think the bottom Y10 can definitely get Ds, some of them […] so why not a C, possibly […] as they're in Y10 and they've got that extra year […] I might try teaching them the higher stuff.

She went ahead with the idea of teaching higher-level content, something that would not be expected or suggested by her school's scheme of work.

In her final assignment Eleanor reflected on what her students gained from engaging in these relatively high-level topics:

> The research diary was a key element for the research. It enabled me to collect little snippets of information, views and ideas from the students. For example, a student who's [target] grade is a G and mock exam grade was a U, was able to tell me that the square root of 49 is 7 two weeks after having studied Pythagoras' Theorem. I was really impressed by this. Even though the student probably hasn't remembered how to use Pythagoras' Theorem, they have taken away how to square root. This was one of the factors that led me to think I should try teaching these students more of the difficult topics. Not just for them to try and grasp these harder topics, but because they might take something else away from it. The next topic I tried was factorising quadratics. This topic worked really well for practising factors. Few of the students will remember how to factorise a quadratic but it was much more engaging than any 'boring', 'easy', 'babyish' factors activity. (Final assignment)

For Eleanor, the purpose of teaching higher-grade content than would be expected, given these students' prior attainment, was not so that they ended up being able to do Pythagoras' Theorem or factorise quadratics but rather so that they had an engaging context in which to practise skills that they would need on their examinations (finding square roots and factors).

Part of Eleanor's action research involved interviews with students following the teaching of some higher-tier topics:

> When asked in the interview how they felt after completing C and B grade work they all said they were pleased with themselves and it gave them confidence to try more. They did also say they find it difficult to remember how to complete these topics after the lesson has ended. (Final assignment)

Eleanor reports on the shift in student confidence through engaging in work that was not a repetition of topics they had studied in previous years. In fact, the school blocked the move to actually enter these students for the higher-tier examination, with the worry that they could end up with U grades if they did not score sufficient marks for a D or E. They were entered for the Foundation Tier where they could access a C grade, but also get an F or G. Eleanor commented:

> I will continue to try and give these low achieving students the opportunities to complete as many qualifications as possible in order to boost confidence as well as sitting their GCSE exams. I will also be trying the more difficult C and B grade topics needed to sit the higher paper in order to give as many students as possible the chance to sit the higher-tier paper. (Final assignment)

It seems she has been convinced of the power of engaging students in work beyond the grade level of the examination for which they would be entered.

How do we, as teachers, know how to present work to students that will be initially accessible to them? Does this depend on our expectations of them in the first place? We are struck by the child remembering how to do square roots after working on Pythagoras' Theorem and wonder if there is something general here? GCSE work becomes easy when doing the A-level syllabus? We have started calling this differentiation from an advanced standpoint. What each individual student takes away from advanced work could be different since some students might already have been able to take a square root, however, starting with the challenge of learning an advanced topic gives the space and time for necessary skills to be rehearsed in a context.

Similarities and Differences

There are some striking similarities across the work of these three teachers. Where there is some pre-test and post-test comparison of GCSE mock grades, students made striking gains over a three-month period (averaging more than 1 GCSE grade) through the switch from foundation-tier to higher-tier work, in terms of their results on mock tests. The results across the two different schools where Sally and Adam conducted their research are remarkably similar in terms of the pattern of students' progress. We were not able to collect the final GCSE results of these students and so cannot corroborate if these gains translated into their final grades.

There were two interpretations of doing higher-level content with students. For Sally and Eleanor, higher-tier work was offered as an alternative way of preparing students for the foundation-tier examination. For Adam, the higher-tier content was taught and students sat the higher-tier examination. However, what seems clear is that, for all three teachers, their approach to teaching did not change drastically and included elements of rote or procedural learning; nor were intermediate or bridging topics offered, before jumping into the higher- level work.

In all cases, there are reports of changes in students' self-perceptions in relation to mathematics. We reported these changes in the words of the teachers: "they are believing they can get a grade C in Maths" (Sally); "[s]tudents felt encouraged, if not surprised, that we believed they could be successful. They seemed proud to be able to do a grade-A question" (Adam); "[w]hen asked in the interview how they felt after completing C and B grade work they all said they were pleased with themselves and it gave them confidence to try more" (Eleanor). While we cannot know, of course, what these teachers mean by words such as "proud" or "confidence", it does seem as though students recognised that what they were being offered, via the higher-tier content, was new and a shift from Sally's "circular curriculum". None of the teachers reported negative student views, which we assume they would have done had such views been voiced, as part of the evaluation of their action research.

Given the commonality of teachers' initial reporting that a lack of "motivation", "engagement" and "belief" were key barriers to attainment in mathematics (again recognising that we do not have access to what they mean by these terms), the changes in student self-perceptions are potentially significant. If teaching styles did not change radically, but the effect of presenting higher-tier work was an increase in student self-belief, and those students made rapid progress in their grades, we are tempted to conjecture a significant role for self-perception in relation to student attainment. There is some evidence, therefore, that teachers may have been right to identify "motivation", "engagement" and "belief" as key factors in student attainment.

The differentiation we observe, across the three narratives, is in the seemingly quite different things that students learnt from working on higher-tier topics than they would usually. This is particularly apparent in Eleanor's case, with the student who, after being taught Pythagoras' Theorem, had learnt what it meant to take the square root of a number. We would now describe all three teachers as differentiating from an advanced standpoint. They all offered their classes tasks that were within topics they would never normally have taught them, as a result of the relatively low prior attainment of those classes. We do not have access to the precise strategies used to make the work accessible but Sally reported spending longer on teaching the topic than she would have with a higher set. The differentiation aspect is captured by Eleanor's comment that the purpose of offering higher-tier content was "[n]ot just for them to try and grasp these harder topics, but because they might take something else away from it". We sense the teachers being able to let go of the necessity (sometimes imposed by school leaders) of having to define and predict the precise learning that particular students would take from a task or lesson (as though that is ever possible). Rather, in the offer of some complex, novel (to the class) mathematics, there is a sense of a wider range of mathematical connections becoming available to students and the differentiation coming from those students being able to make the connections that they are ready to make – inevitably different for each individual.

Teacher Views

A first conclusion to draw from the work reported here is further evidence (if any was needed) of the power of action research as a mechanism to support teachers' continuing professional development and the importance of meetings of a collaborative group to support, challenge and enrich the work of each other. The way that, for example, Eleanor's project was influenced by others' work is common in our experience. Eleanor's work, in turn, of course, also influenced the group. Although the research project set the overall framework of widening participation, we see it as significant that teachers chose to work on issues of personal relevance to themselves and their classrooms.

Taking the work of the three teachers, in the context of the suggestion that students from low socio-economic backgrounds may be excluded from opportunities to succeed in mathematics (Sutherland, 2014), we believe project results have potentially far-reaching implications. Although based on a sample size of only three, there is a striking similarity in the results reported by teachers working in different schools around the Bristol area. What we have evidence for is that if students, who have been grouped together on the basis of low prior attainment, are offered activities and mathematical topics usually reserved for their higher- attaining peers, then several things can happen. Students can recognise their teachers now seem to believe they are capable of A-grade work. Students can report an increased belief that they can attain the key benchmark (in terms of access to higher education) of a C-grade at GCSE mathematics. Students' attainment can rise (as measured either by their success on higher-level content or on the content at which they had previously failed).

What is also striking is that the differences observed in student behaviour do not appear to be strongly linked to changes in the way they were taught, nor to shifts in the teachers' own mathematical knowledge or understanding. Although Sally reports giving students more "ownership" within lessons and Adam wanted to move to a more conceptual focus, these teachers also comment on how similar their approach has been: Sally taught in a similar way to how she would with higher-attaining groups, just more slowly; Adam continued to focus on offering the procedures that students seemed to want; and, Eleanor does not comment either way in relation to changes in how she teaches. What comes through as a more evident pattern is a shift in how the teachers perceive their students: Sally comments, "[m]y expectation of what pupils can achieve has increased"; Adam is surprised at what his students retain, after starting on the higher-tier work with them; Eleanor goes from thinking her year 11 class cannot possibly get a C-grade to thinking that her (equivalent) year 10 class could do. A focus on differentiation from an advanced standpoint appears to have provoked a change not only in how students see themselves, but in how their teachers see them.

If lack of self-belief creates a barrier for students to access Higher Education (Walkerdine, 2011) then there may be something significant in a shift in what students perceive that their teachers believe about them. Through being offered higher-tier mathematical topics and through knowing that they are able to do A-grade work, it appears that students, within a matter of weeks, can experience changes in their view of themselves in relation to mathematics. While we do not want to claim anything general from a sample of three classrooms, we see these narratives as existence proofs of what is possible. These studies can be viewed as paradigmatic cases (Freudentahl, 1981, p.135) of just how quickly it is possible for students' perceptions of their own capabilities to shift, even students who have presumably experienced repeated failure within mathematics for up to ten years.

We are pointing, here, to the significance of relationships: teacher-student relationships and students' relationships with the subject of mathematics and the importance of differentiation within those relationships. The fact that relationships have emerged as significant from our analysis is in keeping with our overall enactivist stance. Bateson (2000) suggests that when analysing any context, a focus on particular objects or components is only ever achieved at the cost of insight. Any behaviour exhibited by an individual exists in relation to other behaviour and, therefore, Bateson suggests, should not be examined in isolation (ibid). What a focus on relationship works against is a sense of linear causality. We cannot say (and nor would we want to) that we have uncovered causal links that a negative student self-perception causes low attainment, for example, or that differentiation from an advanced standpoint causes change. What we have uncovered are patterns and connections and avenues for further inquiry.

Discussion

There are moves in several countries in Europe and elsewhere, to look to East Asian approaches to mathematics teaching as ways of potentially raising achievement. In England, there is a strong steer from government towards an East Asian inspired mastery approach to mathematics. A key statement in the national curriculum (DfE, 2013) advises not going beyond age expectations in terms of teaching mathematics, with the expectation that "the majority of pupils will move through the programmes of study at broadly the same pace" (p. 3). While the aim behind these recommendations, of raising expectations of all students, is laudable, we feel the work reported in this project offers an important alternative lighting on the way mathematical topics and concepts inter-link. If there is an assumption that mathematical competence builds in a linear manner, with solid building blocks needed, before any more complex work can be attempted, we fear that students may end up still being offered the kind of circular curriculum that has been written about in this article.

The notion, coined by Sally, of a circular curriculum echoes Watson's (2008) contention that some adolescents in mathematics classrooms are subject to "cognitive bullying" (p. 22) when teaching "ignores or negates the way a student thinks, imposes mental behaviour which feels unnatural and uncomfortable, undermines students' thoughtful efforts to make sense, causes stress, and is repeated over time" (p. 22). Watson gives the example of being taught fractions in grades 6, 7, 8 and 9 (which would be not untypical in England) in a manner that means some students are forced to "revisit sites of earlier failure […] which is at best marginally effective and at worst emotionally damaging" (ibid). The three teacher narratives above offer striking alternatives, for students with relatively low prior attainment, to re-visiting the same content, in the same way, year after year.

An implication we draw from the work of the teachers on this project is a demonstration that each link in a supposed chain of mathematical skills needed to access a particular topic does not need to be securely in place in order to access that topic. Indeed, through working on more complex topics, we have evidence that subsidiary concepts may be retained in an efficient and even surprising manner (e.g., Eleanor's story of a student understanding square roots for the first time through working on Pythagoras' Theorem; or Adam's surprise and what his students seemed to know about the modal class).

We offer these thoughts, on the question of how learners come to function effectively within mathematics, as provocations inspired by the work of the teachers reported here. We see an urgent need, in the cause of social justice, to find ways of thinking differently about how to support those learners in our care who just seem not to get mathematics. Differentiation from an advanced standpoint, in our project, served as a mechanism for students and teachers to loosen expectations of what is possible and surprise themselves by how capable they really are.

Conclusion

We have reported on a research project aimed at widening participation in higher education through supporting mathematics teachers to engage in action research projects within a collaborative group, as part of a Master's module at the University of Bristol. We highlighted a theme of *differentiation from an advanced standpoint*, as a pattern across the action research of three teachers, expressed in narratives of their work.

What came across strongly from the teachers' action research was the change in expectations of those teachers, who came to believe that previously low-achieving students could work at higher-level content. There is something about doing work at a higher level that strengthens the concepts needed for that work. So, rather than lower achieving students going through the same basic ideas again and again, doing higher-tier topics supports the learning of those more basic ideas, hence differentiation from an advanced standpoint. Every group is a mixed attainment group. The strategy of differentiating from an advanced standpoint could apply just as well to heterogeneous groups. Working on concepts from well beyond where a class is at, in a national curriculum ordering, would entail setting up a classroom culture of discussion and working together, similar to within the collaborative action research group of teachers itself, with management of learning given to the students. Differentiated support would be offered to support learners move towards a same goal but with the outcomes and results inevitably different. Ownership is key. A child once said to Laurinda that they knew what the teacher tells them and what's written in books but that that is different from what they think inside themselves. What seems common across Cockcroft, Sullivan et al. (2020)'s work and differentiation from an advanced standpoint is the affordance, for learning mathematics, of a classroom culture that supports students' expression of their thinking, in the form of questions and comments, so that students know what they are thinking in mathematics lessons.

References

Altrichter, H., Posch, P., & Somekh, B. (1993). *Teachers investigate their work: An introduction to the methods of action research.* Routledge.

Barclay, N. (2021). Valid and valuable: Lower attaining pupils' contributions to mixed attainment mathematics in primary schools. *Research in Mathematics Education*. https://doi.org/10.1080/14794802.2021.1897035

Bateson, G. (2000). *Steps to an ecology of mind.* University of Chicago Press.

Bateson, G. (2002). *Mind and nature: A necessary unity.* Hampton Press.

Boaler, J. (2002) Open and closed mathematics approaches. In L. Haggarty (Ed.), *Teaching mathematics in secondary schools* (pp. 99–112). Routledge.

Brown, L. (2015). Researching as an enactivist mathematics education researcher. *ZDM Mathematics Education, 47*(2), 185–196. https://doi.org/10.1007/s11858-015-0686-3

Brown, L., & Coles, A. (2011). Developing expertise: How enactivism re-frames mathematics teacher development. *ZDM Mathematics Education 43*, 861–873. https://doi.org/10.1007/s11858-011-0343-4

Brown, L., & Dobson, A. (1996). Using dissonance: Finding the grit in the oyster. In T. Atkinson, G. Claxton, M. Osborn, & M. Wallace, (Eds), *Liberating the learner: Lessons for professional development in education* (pp. 212–227). Routledge.

Brown, L., & Waddingham, J. (1982). *An addendum to Cockcroft. Resources for Learning Development Unit* (RLDU). https://www.stem.org.uk/resources/elibrary/resource/31296/addendum-cockcroft

Cockcroft, W.H. (1982). *Mathematics counts: Report of the Committee of Inquiry into the Teaching of Mathematics in Schools under the Chairmanship of Dr W.H. Cockcroft.* HMSO, http://www.educationengland.org.uk/documents/cockcroft/cockcroft1982.html

Coles, A. (2015). On enactivism and language: Towards a methodology for studying talk in mathematics classrooms. *ZDM Mathematics Education 47*(2), 235–246. https://doi.org/ 10.1007/s11858-014-0630-y

Department for Education (2013). *Mathematics programmes of study: Key stages 1 and 2.* Crown Copyright, https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study
Department of Education and Science (1985). *Mathematics from 5 to 16: HMI Series: Curriculum Matters 3.* Her Majesty's Stationery Office, http://www.educationengland.org.uk/documents/cockcroft/cockcroft1982.html
Freudenthal, H. (1981). Major problems of mathematics education. *Educational Studies in Mathematics, 12*, 133-150.
Krainer, K. (2011). Teachers as stakeholders in mathematics education research. In B. Ubuz (Ed.), *Proceedings of the thirty-fifth conference of the International Group for the Psychology of Mathematics Education, Vol. 1* (pp. 47–62). PME 35.
Maturana, H., & Varela, F. (1987). *The tree of knowledge: The biological roots of human understanding*. Shambhala.
Rittle-Johnson, B., Siegler, S., & Alibali, M. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology 93*(2), 346–362.
Sullivan, P., Bobis, J., Downton, A., Livy, S., Hughes, S., McCormick, M., & Russo, J. (2020). Ways that relentless consistency and task variation contribute to teacher and student mathematics learning. *For the Learning of Mathematics Monograph 1*, 32–37.
Sutherland, R. (2014). *Education and social justice in a digital age.* Policy Press.
Taylor, B., Francis, B., Archer, L., Hodgen, J., Pepper, D., Tereshchenko, A., & Travers, M.C. (2017). Factors deterring schools from mixed attainment teaching practice. *Pedagogy, Culture & Society, 25*(3), 327–345. https://doi.org/10.1080/14681366. 2016.1256908
Taylor, B., Hodgen, J., Tereshchenko, A., & Gutierrez, G. (2020). Attainment grouping in English secondary schools: A national survey of current practices, *Research Papers in Education.* https://doi.org/10.1080/02671522.2020.1836517
von Bertalanffy, L. (2015). *General system theory: Foundations, development, applications*. George Braziller.
Walkerdine, V. (2011). Neoliberalism, working class subjects and higher education. *Contemporary Social Science 6*(2), 255–271.
Watson, A. (2008). Adolescent learning and secondary mathematics. In P. Liljedahl, S. Oesterle, & C. Bernoche (Eds.), *Proceedings of the 2008 Annual Meeting of the Canadian Mathematics Education Study Group* (pp. 21-29). Universite de Sherbrooke.

Authors

Alf Coles
University of Bristol
School of Education
35 Berkeley Square
Bristol BS8 1TH, United Kingdom
email: Alf.Coles@bristol.ac.uk

Laurinda Brown
University of Bristol
School of Education
35 Berkeley Square
Bristol BS8 1TH, United Kingdom