Task Design for Differentiated Instruction in Mixed-ability Mathematics Classrooms: Manifestations of Contradictions in a Professional Learning Community

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The aim of this study is to investigate task design for differentiated instruction in mathematics in professional learning communities. Based on the cultural-historical activity theory, we conceptualize a professional learning community as an activity system and use the analytical construct of contradictions to give an account of structures that bring forward the teachers' work. Eight Swedish upper secondary teachers, engaged in designing tasks for differentiated instruction in mixed-ability mathematics classrooms, are studied. The analysis outlines three contradictions, manifested as three dilemmas, and shows how the teachers, by noticing a dilemma and making it an explicit object of inquiry, came to address a diversity of issues related to differentiated instruction in mathematics. For example, the teachers addressed students' different learning needs, problem-solving activities for a mixed-ability classroom, and design of tasks that are challenging for all students. With input from our findings on the three dilemmas, we then discuss the design of a task analysis guide as a means for facilitating the development of a professional learning community that is inquiry-oriented with a strong content focus on designing tasks for differentiated instruction in mixed-ability mathematics classrooms.

Keywords · differentiated instruction · content focus · challenging problems · in-service teachers · professional learning community

Introduction

When a mathematics task is appropriately challenging for most students in a class, it is often too difficult for some and too easy for others (Jäder et al., 2019). Some students will not be able to engage with the task or need to rely heavily on prompts from the teacher, while others will not be challenged and not learn anything from working with the task. Ability grouping has been common practice in many countries for catering to diversity in the mathematics classroom (Anthony & Hunter, 2017; Hunter et al., 2019). However, research has shown a range of negative effects from this approach. For instance, in ability grouping, students get fixed and static identities, which leads to teachers attributing different expectations to different groups and thereby not giving students the same chance and opportunity to develop (Mark, 2014). There is call for alternatives to ability grouping and, particularly, for a more inclusive perspective on differentiated instruction in mathematics (Hunter et al., 2019). In this study, we aim to make a contribution to this call by investigating professional learning communities on task design for differentiated instruction in mixed-ability mathematics classrooms.

Professional learning communities (PLCs) have received increased attention as a means for supporting teacher learning and the development of teaching practice (Brodie, 2020; Dogan et al., 2016). In a PLC, teachers are challenged to unpack and inquire into their teaching, in terms of trying out, analysing, and evaluating different ways of teaching (Stoll et. al., 2006). However, despite the promises of teacher collaboration it does not seem to be an easy task to develop a PLC that is inquiry-oriented with a strong content focus (Jaworski, 2008; Vangrieken et al., 2015). In this article, we report on a project in which two subgroups of Swedish upper secondary school teachers are engaged in a PLC with a focus on designing tasks for differentiated instruction in mixed-ability mathematics classrooms. We approach the analysis from the perspective of the cultural-historical activity theory (CHAT), viewing collective and individual actions in a PLC as an activity system (Engeström, 1987). We particularly focus on contradictions, which are central to learning in an activity system (Engeström & Sannino, 2011). We address the following research question:

What manifestations of contradictions can appear in a PLC, focusing on task design for differentiated instruction in mixed-ability mathematics classrooms?

With input from our findings, we then discuss the design of a task analysis guide as a means for facilitating the development of a PLC into an inquiry-oriented activity system with a strong content focus.

Previous Research

Professional Learning Communities

PLCs are those in which teachers work and learn together for "increasing teacher knowledge and skills and improving their practice, and which hold promise for increasing student achievement" (Desimone, 2009, p. 183). In a PLC, teachers can be challenged to make visible and explore their teaching and to test, analyse, and reflect on different ways of teaching (Vangrieken et al., 2015). More generally, in PLCs teachers can be supported in developing an inquiring attitude whereby they wonder, ask questions, and seek to understand teaching and learning by collaborating with others in the attempt to provide answers to their questions (Jaworski, 2008).

However, despite the promises of teacher collaboration it does not seem to be an easy task to develop a PLC that is inquiry-oriented and makes a difference to teaching and learning (Jaworski, 2008). It seems to be particularly difficult for teachers to establish and sustain a strong content focus. Often, the PLC is reduced to a practice in which teachers mostly discuss practical issues, such as their teaching schedule, teaching materials, and the pace and content of the curriculum (Plauborg, 2009; Vangrieken et al., 2015). Developing a strong content focus is key to a PLC (Desimone, 2009). Teachers should negotiate based on developmental needs that are grounded in their own practice (Slavit et al., 2011) and focus their collaborative work on subject matter students should learn, how they learn it, their difficulties in learning, and the means to support their learning (Darling-Hammond et al., 2017). In the next section we elaborate on the content and content focus of a PLC from a differentiation perspective, along with previous research on differentiation that is relevant to the present study.

Differentiated Instruction

To develop knowledge, regardless of their readiness or learning profile, each student should be provided with opportunities to be successful (Le Fevre et al., 2016; Tomlinson et al., 2003). Differentiated instruction is a pedagogical strategy used to design teaching according to students' readiness, interest, and learning profiles in the mixed-ability classroom (e.g., Hunter et al., 2019; Tomlinson, 2016). Differentiating instruction involves being proactive in relation to students' needs. Teachers can differentiate instruction through content, process, product, and learning environment (Tomlinson, 2016). In this study, we focus on content.

Content refers to "what teachers want students to learn from a particular segment of study, or the materials or mechanisms through which students gain access to that important information" (Tomlinson, 2016, p. 18). On this account, content can be differentiated to make it more accessible for all students (Gaitas & Martins, 2017). On a task level this means that teachers can differentiate content, for example through developing tasks that have a "low floor, high ceiling" or tasks with enabling and extending prompts (Bobis et al., 2021; Sullivan et al., 2015). In the present study, we focus on task design as a means for differentiating content according to differences in students' readiness and learning profiles.

Mellroth (2018) noted that, in PLCs, where the focus is on the inclusion of highly able students in learning activities, many teachers are unfamiliar with the techniques of differentiated instruction. Shayshon et al. (2014) found that teachers are aware that teaching needs to be differentiated in mixed-ability classrooms and are often confident in their own capability to develop differentiated instruction. However, research (e.g., Leikin & Stanger, 2011; Moholo, 2017) shows that teachers find it difficult to respond to the variety of abilities in the mathematics classroom in a differentiated way.

Gaitas and Martins (2017) found that differentiation of content can be particularly hard for teachers to accomplish. In a study by Bobis et al. (2021), teachers were instructed in several strategies for differentiating content through challenging tasks. Three strategies were found to be especially successful for mixed-ability mathematics classrooms: (1) using tasks with enabling and extending prompts, (2) using tasks with a low floor/high ceiling structure, and (3) letting students play games, through which mathematics naturally becomes differentiated through their choice of strategies.

Although teachers are aware that teaching needs to be differentiated, they need support in proactively differentiating content. On this account, the present study will investigate and characterize contradictions teachers can encounter when collectively working with task design for differentiated instruction. Examples of how contradictions can be manifested in a PLC will then be useful for facilitating the development of new PLCs to be inquiry-oriented, with a strong focus on content according to differentiated instruction.

Challenging Struggles for All

To develop conceptual understanding, students need to struggle with important mathematical ideas (Hiebert & Grouws, 2007). More specifically, students learn mathematics better when they have to struggle and create their own solutions to tasks, compared to being taught how to apply ready-made methods (Jonsson et al., 2014; Russo et al., 2020; Sullivan et al., 2015). Challenging tasks can support student learning, as they let students struggle (Sullivan et al., 2016). We refer to the opposite – guiding students around difficulties, and hence depriving them of the challenges and struggles in finding their own solution method – as funnelling (Bauersfeld, 1998).

However, developing tasks in mathematics with appropriate challenging struggles for all students is not an easy task for teachers (Mellroth, 2018). They need support in developing tasks that make all students struggle in the mixed-ability classroom (Leikin & Stanger, 2011; Mellroth, 2018; Shayshon et al., 2014). To make a challenging task accessible in mixed-ability classrooms, the teacher needs support in planning for multiple entry levels and variations of the task (Bobis et al., 2021; Sheffield, 2003; Sullivan et al., 2015). In other words, to meet the demands of a diverse, mixed-ability classroom, teachers need opportunities to engage in an inquiry of task design for differentiated instruction (Suprayogi et al., 2017).

We refer to tasks that offer appropriate challenges to a wide range of students as rich mathematical tasks (cf. Taflin, 2007; Sheffield, 2003). Sheffield's (2003) criteria framework for such tasks has shown promise in guiding elementary and lower secondary teachers in analysing and designing mathematical tasks in PLCs (Mellroth, 2018, 2021; Mellroth, et al., 2020). The framework gives some general characteristics of rich mathematical tasks: the task has a low floor (i.e., an entry that all students can access) and intermediate steps; offers challenges for all; stimulates higher-order thinking; allows different solution strategies; encourages students to generalize; promotes mathematical self-confidence and offers possibilities for self-assessment; and is open-ended. Moreover, rich mathematical tasks can be used to provoke mathematical curiosity, struggle, conversation, and insight into important mathematical topics (Benölken, 2015; Hsu et al., 2007; Taflin, 2007).

In addition, Mellroth (2018) showed that, when anchoring a PLC in Sheffield's framework, teachers can come to develop their discussions on how to provide learning for all students. In the present study, we use the criteria of Sheffield's framework on rich mathematical tasks to frame the initial design of a task analysis guide for facilitating content focus in a PLC on differentiated instruction. We present how we used the framework in the method description.

Cultural-Historical Activity Theory

We approach the analysis of collective and individual actions of a PLC based on the cultural-historical activity theory (CHAT) (Engeström, 1987). This means that we conceptualize a PLC as an activity system, constituted of a network of sociocultural elements (Figure 1) (Trust, 2017).

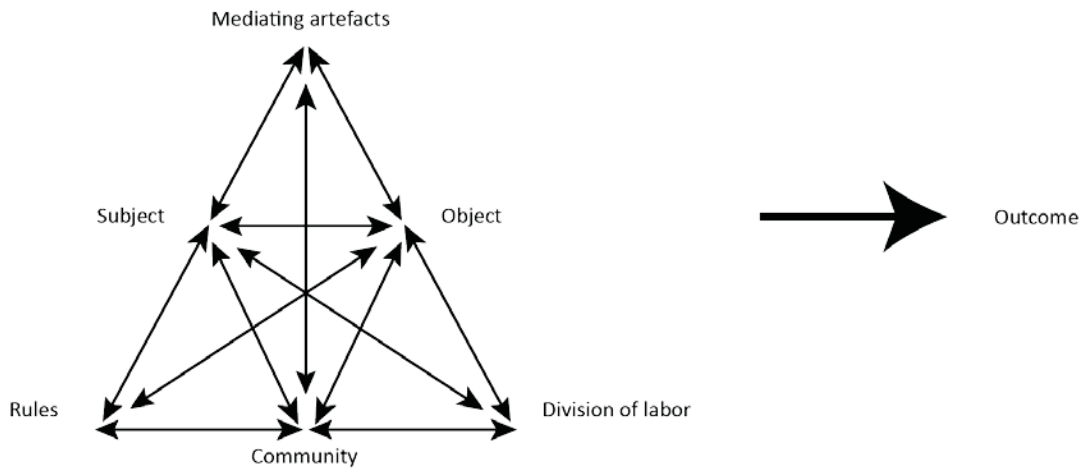

Figure 1. Engeström's (1999) model of activity system.

The *object* is the target of an activity system. What becomes the object of an activity system is an empirical question, generally constituted to meet the needs of the subject of the activity system

(Engeström, 1999). Answering to their object, the subjects of an activity system transform the object into outcomes, or desired results (Trust, 2017). In this study, we investigate PLCs in which upper secondary teachers (*subjects*) constitute an object that is focused on content in relation to differentiated instruction.

Vygotsky (1987) argued that human beings react to and act upon *mediating artefacts*, such as tools, signs, and instruments in their meaning-making processes. In an activity system, subjects react and act upon mediating artefacts in order to develop, understand, and accomplish their objectives.

Human activities are dictated by implicit and explicit *rules* (norms) that to varying degrees regulate the actions that take place. The rules are established and sustained in the social *community* to which the subjects of the activity belong. The *division of labour* refers to how different tasks and responsibilities are distributed among the subjects.

Activity systems develop on the basis of constant internal and external contradictions (Daniels, 2004). The meaning of contradictions is separated from mere problems on a subject-only level (Engeström, 2001). Their existence is a matter of structure. Contradictions are central to learning in an activity system (Engeström & Sannino, 2011). Through making contradictions visible and inquiring and resolving them, the activity system appropriates new knowledge and ways of learning. However, contradictions do not speak for themselves but are recognized in the talk and actions of the subjects. In the words of Engeström and Sannino (2011):

> In organizational life in general, and in change efforts and interventions in particular, contradictions are to an important extent manifested and constructed in patterns of talk and discursive action with the help of which actors try to make sense of, deal with and transform or resolve their contradictions (p. 371).

We use Engeström and Sannino's (2011) framework on manifestations of contradictions to account for teachers' collective inquiry in developing tasks for differentiated instruction (Table 1).

Table 1: Manifestations of contradictions (Engeström & Sannino, 2011).

Contradiction	Description
Dilemmas	are expressions of incompatible evaluations, either by one person or between people. They are commonly expressed in the form of hedges and hesitations, such as "on the one hand […], on the other hand" and "yes, but".
Conflicts	are situations in which participants oppose each other's opinions and actions, and are expressed through resistance, questioning, disagreement, argumentation, and criticism.
Critical conflicts	are situations in which a person encounters inner doubts regarding contradictory but mutually dependent motives and meanings that are unsolvable by the person herself. The situation paralyses her, and she often expresses feelings of violation and guilt.
Double binds	arise when requirements and conditions are incompatible, often expressed as helplessness or desperate rhetorical questions. People are forced to act in a certain way that they perceive they are not able to.

Method

Setting the Scene

This study is part of a joint project between the authors' university department and the local municipality. The project has followed eight mathematics teachers at a Swedish public upper secondary school (Grades 10–12) who participated in a PLC on developing tasks for differentiated instruction. In the terminology of CHAT, the PLC is an activity system with the teachers as its subjects. The participating teachers were the first to respond when the project was announced to public schools in the municipality, and were therefore chosen for the project. The project started in August 2019 and will end in December 2021.

Swedish upper secondary school is course-based, with different sets of mathematics courses for different programs. The pre-university programs that prepare students for higher studies in science and technology include five courses: Mathematics 1c, 2c, 3c, 4, and 5. The participating teachers form a team that teaches this set of courses to technology students. All of them have a teaching degree for upper secondary school, which means that they have at least two years' full-time mathematics studies at university level. Their teaching experiences vary from three to more than 20 years. The teachers are accustomed to collaborative work. For instance, most of them participated in the Swedish Boost for mathematics (e.g., Österholm et al., 2016), a national professional development program based on collaborative work that took place between 2012 and 2016.

This article is based on data from project meetings from the start of the project in August 2019 to March 2020, after which the project was paused due to the COVID-19 pandemic. During this period, there were seven 90-minute meetings and one full-day meeting. The first and second authors attended all meetings and acted as facilitators (coded F1 and F2 in the analysis). For an understanding of the PLC as such, the first three meetings are described in the next section. The description is based on the facilitators' fieldnotes. The remaining five meetings, coded M4–M8, were audio-recorded and provide data for in-depth analysis. All teachers were informed of the research aim and the data collection methods, and they all signed an informed consent form.

Meetings 1–3

Initially, the teachers needed meeting time to discuss what their overall object in the project would be. At the first meeting they decided to create a bank of challenging tasks (hereafter called the problem bank) that could be used as a complement to the textbook. They expressed that they lacked a source for such tasks that also linked to relevant parts of the Swedish national curriculum in mathematics. One ambition was therefore to design tasks that could simultaneously offer learning opportunities for the highly able students in mathematics and for all others. That is, the object was to create a problem bank with tasks that could be used to challenge all students in a mixed-ability mathematics classroom. Other options discussed involved developing strategies and solutions for highly able students to accelerate through the curriculum or giving them opportunities to meet and work with likeminded peers. However, these alternatives were rejected in favour of the problem bank, as such a bank would be beneficial to all students.

During the first months of the project, the meetings aimed to increase the participating teachers' knowledge of PLC and of the characteristics of mathematical tasks that can challenge all students (e.g., Mattsson, 2018; Sheffield, 2003; Szabo, 2017). In connection to the seminars on challenging tasks, task analysis guides from previous and similar projects (Mellroth, 2018, 2021) were collaboratively discussed. Based on the teachers' comments and suggestions, the facilitators designed a first version of a task analysis guide (TAG). The guide was meant to serve as a mediating artefact to support the teachers in their process of designing rich mathematical tasks to meet the needs of all students. The TAG included questions related to steering documents, teaching practice, and theory of rich mathematical tasks. A summary is shown in Table 2. The authors are willing to share the full guide on request.

Table 2: Summary of what the TAG asked the teachers to reflect on.

Theme	Description
Curriculum	Which of the five courses is the task suitable for, which mathematical content does it relate to, which mathematical abilities does it offer the students possibilities to use?
Purpose	Based on teacher experience, is the task an introduction task or an enrichment task, and how difficult is it for the intended group of students?
Criteria	Through rating: How well does the task fulfil the criteria of a rich mathematical task?

The TAG was one of several mediating artefacts in the activity system of this particular PLC. Other mediating artefacts were the organized meetings and seminars, recommended literature, and sources for tasks such as textbooks and websites, etc. The reason why we give the TAG special attention is that it was designed in close collaboration between the participants and the facilitators, and that it was made with more than one purpose in mind. It was a shared ambition within the project that one day the problem bank would be made available to all schools in the municipality, and include information collected through use of a digitalized version of the TAG.

Meetings 4–8

From the fourth meeting onward, the teachers were to bring candidate tasks for the problem bank to the project meetings. Meeting time was devoted to discussing and selecting tasks for further development and analysis, and to planning and evaluating classroom implementation of the developed tasks. The eight teachers were grouped into two subgroups: one for those teaching Courses 1c and 2c, and one for those teaching Courses 3c, 4c, and 5. The teachers in the first subgroup are coded T11, T12, T13, and T14, and those in the second subgroup are coded T21, T22, T23, and T24. Each meeting started and ended with a discussion among all eight teachers and the two facilitators. In-between, the subgroups worked separately on developing tasks for their specific courses, usually with one facilitator present in the room. In total, the meetings resulted in 15.5 hours of audio recordings.

Method of Analysis

The premise of our analysis is that collaborative learning is fuelled by contradictions within and between the nodes of an activity system. In the subjects' actions, in this case discussions, contradictions can be manifested as dilemmas, conflicts, critical conflicts, and double binds (see Table 1). To answer our research question, we have searched our data for manifestations of contradictions related to task design for differentiated instruction in mixed-ability mathematics classrooms. Episodes including utterances indicating such manifestations have been transcribed and thoroughly analysed.

The three authors discussed what sequences to search for in the audio recordings. To capture episodes related to task design for differentiated instruction we chose to distinguish manifestations of contradictions related to differentiation, in a broad sense, from those explicitly related to the use of the TAG. Here, differentiation includes dilemmas addressing the differing needs of students, the design of tasks that meet certain established criteria (including the evaluation of tasks related to such criteria), the implementation of tasks and problem-solving activities in a classroom context, etc. Contradictions related to the use of the TAG include uncertainties as to when and how to use the guide, difficulty answering its questions, drawbacks or limitations of the guide, etc. These types of contradictions are not attended to in this study.

One of the authors listened through all audio recordings and manually noted the time sequences of episodes related to the agreed-upon content. Thereafter, the same author listened through the selected recordings again, using NVivo and marking the episodes more thoroughly. Verbatim transcripts were produced for our in-depth analysis of manifestations of contradictions related to task design for differentiated instruction. The analysis of the transcripts was done in three steps:

First, transcripts from two meetings were analysed independently by the first and second authors, according to the four manifestations of contradictions.

Second, the two authors compared their coding. They agreed in most cases. In cases of disagreement, they negotiated a common interpretation and agreed on principles for the rest of the coding.

Third, the rest of the transcripts were coded by one author. Uncertainties in the coding were discussed between the authors until agreement was reached.

To increase the transparency and trustworthiness of the analysis, we discuss a typical example. The following episode was coded as a dilemma as it contained utterances including phrases such as 'but […] that's difficult', 'at the same time', and 'you don't want that either':

> M4:T22: To find a one-level problem, that's okay; but an open, rich problem with many depths, that's difficult.
>
> M4:T21: And at the same time, if you turn it into an a-b-c task, where Part a gives a clue to the whole solution, you don't want that either.

The teachers expressed no disagreement regarding the disadvantage of standard a-b-c tasks. If they had, the episode would have been coded as a conflict. If T21 had expressed an obligation to use a-b-c tasks against his conviction, it would have been a critical conflict.

When analysing the transcripts, we looked for recurring themes in the identified dilemmas, conflicts, critical conflicts, and double binds. Themes were formed during this process; that is, we had no a priori defined subcategories of the four kinds of manifestations. Double binds and critical conflicts were rare in our data, but we found it informative to distinguish between student-related, task-related, and classroom-related dilemmas. In the Results section we focus on our findings on task-related dilemmas and exemplify our analysis with paraphrased excerpts from our data.

Results

No critical conflicts were found, and double binds were rare. We will not go into any detail, beyond providing two examples.

The first concerns the contradiction between the time available for the PLC and the need for the thorough classroom testing of a task. The participating teachers have an idea of an ideal situation in which tasks can be inquired in a cyclic process over several iterations with implementations in different contexts. However, a lack of time forces them to go on; that is, they are forced to act in a way that is in opposition to the expectations of the inquiring cycle of the PLC:

M6:T22: And if you can split the class, you can use the task in one group, make improvements, and then use it again in the other. That feels very utopian here, though.

M6:T13: How much time do we have? [Laughter]

The second concerns the contradiction between effective collaborative work in analysing a task and the time needed to solve it individually. When one teacher presented a task (named The Beam), another teacher admitted that he could not see how to solve it. Ten minutes later, after having discussed another task (named Ferris Wheel), they had to make a decision:

M4:T21: Should we start with Ferris Wheel instead and solve The Beam later?

M4:T24: Yes, because I need to solve it first.

An hour later they spontaneously picked up the discussion of The Beam, but once again realized that to complete the analysis and revision they had to solve it first:

M4:T24: I feel like I have to solve this one.

M4:T22: I don't at all feel sure how to solve it.

Dilemmas and conflicts dominated the observed manifestations, with dilemmas being by far the most observed. Conflicts never took the form of agitated debate; rather, one participant's view was sometimes supplemented with another's, or with another participant sharing a different opinion. In our data, this means that the main difference between dilemmas and conflicts lies in whether the different points of view, or opposing perspectives, are asserted by the same participant or by different ones. Conflicts are merely interpersonal dilemmas. In what follows, we therefore choose not to distinguish conflicts from dilemmas.

Differentiation dilemmas are those that occur in relation to addressing the differing needs of students, designing tasks that meet certain established criteria to be 'challenging for all students', and implementing tasks and problem-solving activities in a mixed-ability classroom. These three examples can be described as student-related, task-related, and classroom-related, respectively. The list is neither exhaustive nor mutually exclusive. Other kinds of differentiation dilemmas will be discussed, but our focus will be on task-related differentiation dilemmas. Three types of such dilemmas are exemplified with paragraphed transcripts from our data, and can all be seen as contradictions within the object node of the PLC.

Guiding versus Funnelling Dilemma

The first task-related dilemma concerns the structure of the task formulation. On the one hand, the teachers look to find a suitable entry level and to bind the parts of a task together, so that all students can start with the task and not be overwhelmed by its generality or complexity. That is, they see a need to offer the student some guidance. On the other hand, they express that this must be done without giving too much away; that is, without funnelling. The task must still offer challenge and struggle to all students. When the teachers develop tasks for the problem bank, they often start with a problem that they soon realize is too difficult or lacks a sufficiently low entry level. They ask one another if the task can be adapted. To ensure an entry level for all students, they typically add simpler subtasks or split the problem into subtasks. In this way they offer enabling prompts. When they have achieved a lower entry level (low floor), they sometimes realize that the task lacks higher levels of mathematical thinking (high ceiling). When the task is split into subtasks, they also see a need to bind the parts of the task together. But already at the first meeting, before working with a specific problem, the teachers are clear that they want to avoid turning problems into standard tasks in which one part gives overly obvious hints and guidance for the next; i.e., they want to avoid funnelling:

M4:T22: To find a one-level problem, that's okay; but an open, rich problem with many depths, that's difficult.

M4:T21: And at the same time, if you turn it into an a-b-c task, where Part a gives a clue to the whole solution, you don't want that either.

Later in the same meeting, when one of the teacher subgroups had spent considerable meeting time on developing a task called Ferris Wheel into a series of subtasks, the same teacher realizes that what they have done contradicts their initial intention:

M4:T21: But once again we've split it!

At the next meeting, the teachers start working with a new task, The Beam, and the same topic is brought up again:

M5:T22: How do you formulate this? What kind of task structure should we build?

M5:T21: Yes, so it won't be the case that now we have a, b, c, so that it becomes like a standard task. But so that there's some kind of entry level so everyone can give it a try.

Obviously, this is a recurring dilemma for some of the teachers. On the one hand they want to guide the student through the chain of subtasks, while on the other they want to avoid funnelling.

Introduction Versus Enrichment Dilemma

At an early stage of the project, the teachers decided that the problem bank should contain two kinds of tasks: introductory and enrichment. On the one hand, they wanted inquiry problems that could be used to introduce new topics, concepts, or methods. On the other, the tasks should be suitable for enrichment and extra challenge for all students, including the highly able ones. The teachers also chose to include an item in the TAG in which they had to decide whether a task would be of one or the other of these two kinds. As far as we have understood the teachers' intentions, a single task must not fulfil both purposes, but both kinds should offer means for differentiated instruction and learning opportunities for all students. Still, during their work there were several occasions when this caused tension as certain criteria were difficult to meet for both kinds of tasks.

In the following episode, one teacher has recently presented a first version of Ferris Wheel for the problem bank:

M4:T21: This one, an introductory task, absolutely. But it's similar to a standard task from the book.

M4:T22: But we can include introductory tasks too, can't we?

M4:T22: It's just that the analysis becomes different, this isn't an open…

M4:T21: No, it's definitely not open.

M4:T22: We can't only have difficult, open, rich problems at the end for enrichment. This is also fun.

M4:T21: It introduces, it does, but it fails at a lot of the other criteria.

Obviously, they agree that the task has features that make it a good introductory task, and that such tasks can be included in their problem bank. On the other hand, it is pointed out that the task is not an open problem, which is a property they seem to connect to enrichment tasks.

At one meeting with the other teacher subgroup, they discuss the object of the PLC; that is, the problem bank they are creating and the importance of making its content accessible for teachers. They make clear that the bank has to be searchable with criteria for course, content, and whether you are looking for an introductory or enrichment task. At this stage, one of the teachers seeks confirmation of what she believes should characterize the tasks they are designing:

M6:T12: But isn't the point of them that they should be open and possible to use on different levels. Or am I wrong?

M6:T11: Especially if it's an introductory task. It has to be, like the Norwegians say, a LIST task, low entry level, high ceiling.[1] Anyone should be able to start, but few should be able to complete it on their own. Maybe no one, if it's a good task.

M6:T12: And then I feel like they differ from what you find in Kunskapsmatrisen.

M6:T13: Yes, they do, absolutely.

M6:T11: Yes, that's what I mean, but we're not competing with that, in my opinion. That's something different.

Note that teacher T11 connects openness especially to introductory tasks, and elaborates on the characteristics of a good introductory task. These characteristics are at odds with what the teachers are used to finding when using an extensive online material source called Kunskapsmatrisen. The dilemma is solved by pointing out that their problem bank is not intended to compete with Kunskapsmatrisen but to be a supplement.

There are also situations in which the teachers defend the use of a task even when it does not fulfill the criteria in the TAG. The contradictions need not always be between suitable properties for introductory tasks and enrichment tasks. There are other purposes that can be contradictory; for instance, a task may aim at assessing certain mathematics content instead of offering opportunities for conjecturing and making generalizations:

M5:T11: But it has to do with the fact that it's a pretty narrow task. It isn't intended to spread out in generalization and stuff, but to test whether they can solve a problem using trigonometry. That's why it scored low on a number of criteria. Because the aim wasn't great breadth.

Generality Versus Specificity Dilemma

The teachers share the view that general and open tasks are difficult for many students. Here, we give examples of dilemmas that occur when a general or abstract task is made more specific or concrete, even if the resulting dilemma is not related to generality issues.

To make a task more specific, an arbitrary situation can be replaced with a specific one. In the case of The Beam, the width of the corridors and the length of the beam can be specified:

M4:T22: It feels like a high entry level.

M4:F2: Are there arbitrary measures from the beginning?

M4:T22: Yes, but we thought you could give them specific numbers, and check if it works or not.

M4:T21: But you can't make a pattern from that.

In this situation, the dilemma is not that the specific measures remove opportunities for reasoning about general cases, but that the specific numbers will not aid students in seeing the general pattern and how to solve the general task. Another way to put it: specific numbers do not make for a generic case.

Another problem is that a more specified formulation gives a higher load of information, which is difficult for some. In the following episode, the teachers discuss two versions of The Beam:

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[1] In Norwegian, LIST is an abbreviation for low entrance level, high ceiling.

M5:T22: And already from the beginning, as it's formulated right now, it's very general.

The other teachers agree, and start sharing their experiences from earlier use of this task. Eventually, one of them presents another formulation:

M5:T21: Here it's formulated like this: An L-shaped corridor has two more than 10-meter long, perpendicular, and straight parts with widths 1 meter and 2 meters, respectively. What's the maximum length of a straight beam that can be carried horizontally through the corridor? So, you get a lot of information to handle. But there are specific numbers.

As in the previous example, the dilemma lies not in the loss of generality but in the fact that on the one hand the task is more concrete, which should mean a lower entry level, while on the other it contains a large amount of data that the student needs to handle, which may be problematic for a student with difficulties.

A different way to make a general task more concrete and to create an easy entry level is to supply manipulative materials of some kind. In the case of The Beam, one of the teachers suggests that they build a model of the corridor and supply sticks of varying lengths so that students can test what works. On the one hand this makes the situation very concrete and understandable, but on the other, some important aspects are lost. One is that the problem is to be solved in the ideal situation with a beam without thickness, a situation that cannot be represented with sticks in the physical world. Another problem with manipulatives is the differing roles such materials can play: to understand the problem, to test a specific case, to find a solution by trial and error, or to test a derived solution. The dilemma occurs if the teacher intends one role and the student uses the material for another, and concerns whether/how the student should be steered to a specific use of the manipulatives.

There is also an episode with another task in which the teachers express that possibilities to work with several different forms of representations are a positive feature of the task. However, they find that this is not enough to provide a sufficiently low entry level, and see little possibility to differentiate the task for students on different levels.

Other Task-related Dilemmas

In our data, we also saw that the teachers discussed where to put alternative formulations of tasks and supplementary questions meant for support or extra challenge; that is, tasks with enabling and extending prompts. They saw reasons to include them in the formulation of the task on the one hand, and on the other to place them in a separate teacher guide to the problem bank. Finally, at the beginning of the project there was a discussion about where to find suitable tasks for the problem bank. Tasks from Swedish mathematics competitions are (of course) very challenging. Nevertheless, the teachers' experience was that the starting tasks at competitions can be within reach for many students. These tasks are almost always related to elementary number theory and the contents of Courses 1 and 2. As the object is to produce a bank that includes tasks for all courses, this means that competition tasks might be less promising as a source for tasks suitable for Courses 3–5.

Discussion

This study was motivated by the need to find structures to support teachers inquiring into differentiated instruction in mixed-ability mathematics classrooms (Leikin & Stanger, 2011; Mellroth, 2018; Shayshon et al., 2014). To this end, we have investigated task design for differentiated instruction in a PLC, with PLC viewed as an activity system, constituted of a network of sociocultural elements (Engeström, 1987; Trust, 2017). Contradictions are central to the learning of an activity system. Through the inquiry of contradictions, an activity system appropriates new knowledge and ways of learning (Engeström & Sannino, 2011). This view of learning in PLCs implies the need to map out contradictions, which new PLCs in mathematics can use to support their collective learning regarding differentiated instruction. On this account, we addressed the research question "What manifestations of contradictions can appear in a PLC, focusing on task design for differentiated instruction in mixed-ability mathematics classrooms?" In the following discussion, differentiated instruction in mathematics always refers to the context of a mixed-ability classroom.

The analysis outlines three contradictions, manifested as three dilemmas: Guiding vs funnelling, Introduction vs enrichment and Generality vs specificity. Based on the analysis, we find the study's significance to be both theoretical and practical. Theoretically, it contributes insights into the process of content focus in PLCs in mathematics in general, and in PLCs on task design for differentiated instruction in mathematics in particular. For practical significance, we elaborate on the TAG as a means to support PLCs on task design for differentiated instruction in mathematics.

Theoretical Implications

Despite the promises of teacher collaboration (Brodie, 2020; Dogan et al., 2016), it does not seem to be an easy task to develop a PLC with a strong content focus (Vangrieken et al., 2015). However, what is meant by a strong content focus is not clear in the literature. What is, or can be, the content of a PLC is likely not an issue, but what does not appear to be discussed or problematized to any great degree in the literature is the meaning of focus or focusing and, particularly, the meaning of a strong content focus. For instance, in the present study, content (the object of the PLC) was task design for differentiated instruction in mathematics. This content can be focused on in many ways. One way could be that the teachers simply tell their colleagues what they usually do when they are trying to achieve differentiated instruction in mathematics. Such telling would be a case of a cumulative speech pattern (Wegerif & Mercer, 1997), whereby teachers do not evaluate each other's claims or ideas but uncritically accept them. Even if claims in cumulative speech pattern may be focused on content, it is unlikely that this increases the learning of the PLC in any substantial way (Plauborg, 2009).

A strong content focus in a PLC implies that the content is elaborated in processes of inquiry, through which teachers are encouraged and challenged to wonder, ask questions, and try out, analyse, and evaluate different ways of teaching in collaborations with other teachers (Jaworski, 2008; Stoll et al., 2006). We note that processes of inquiry are expressed in rather general terms, with no specific meaning. Looking at learning from CHAT, we can specify the meaning of inquiry in PLCs. Considering a PLC as an activity system, we see learning as a matter of approaching contradictions: Through making contradictions visible and inquiring and resolving them, the activity system appropriates new knowledge and ways of learning. Hence, when talking about a strong content focus in PLC from a CHAT perspective, we are talking about teachers' collective inquiry of contradictions; i.e., of being sensitive to, asking questions about, analysing, and dissolving contradictions.

Not only does this study contribute to the specification of inquiry for strong content focus in PLC in a general sense. Speaking directly to differentiated instruction in mathematics, it also contributes three empirically grounded characterizations of contradictions (Engeström & Sannino, 2011) that can appear in a PLC focusing on task design for differentiated instruction in mathematics. The contradictions manifest as the dilemmas Guiding vs funnelling, Introduction vs enrichment, and Generality vs specificity. In the analysis we saw how the dilemmas served the teachers in establishing a common focus, turning into details and approaching the task design from different angles. The teachers did not just contribute suggestions in a cumulative speech pattern (Wegerif & Mercer, 1997). Inquiring task design by means of dilemmas also encouraged them to give reasons for their suggestions or claims in general when developing tasks for differentiated instruction. By noticing a dilemma and making it an explicit object of inquiry, the teachers addressed a diversity of issues related to differentiated instruction in mathematics: students' different learning needs, problem-solving activities for a mixed-ability classroom, and design of tasks that are 'challenging for all students'. Bobis et al. (2021) showed that teachers experienced that tasks with a low floor and high ceiling, as well as tasks with enabling and extended prompts, were highly positive ways to differentiate instruction in the mixed-ability classroom. Interestingly, the teachers in the present study also seemed to strive to develop tasks that were differentiated through a low floor and high ceiling. However, at the same time they tried to avoid enabling and extending prompts within the task design. On this account, our findings differ from those of Bobis et al. (2012).

Hence, based on CHAT, and particularly on the construct of contradictions, we have provided a theoretical tool for conceptualizing inquiry with a strong content focus in PLCs. We can then use such an understanding to search for and develop feasible mediating artefacts to support PLCs in mathematics. Next, we will elaborate on such an artefact by discussing how the three dilemmas found in this study can contribute to the specification of the TAG for differentiated instruction in mathematics.

Implications for Practice – A Developed Task Analysis Guide

The TAG in this study

The role of the TAG was to stimulate teachers' discussions on developing tasks for differentiated instruction and, consequently, to provide us with rich data for identifying and characterizing manifestations of contradictions in the teachers' joint work. We have shown how attending to contradictions became central in how the teachers developed their understanding of task design for differentiated instruction in a PLC. There is therefore reason to reflect on how the TAG can be developed to increase the chance that future PLCs on differentiated instruction will develop by approaching similar dilemmas.

It should not be underestimated that the development of the TAG used in this study included seminars and discussions on the design of challenging tasks. In addition, the teachers whose practice was connected to the guide were invited to comment on and contribute to the design, which they did. The teachers also had support from their school leader and time dedicated for the project.

How the teachers used the TAG changed during the project. It was initially used before the teachers started their task revision, via being used parallel to the task revision, to being used after the task revision was finished. In the project's middle phase, during task revision, the time devoted to filling in the TAG increased; but on the other hand, when all the criteria were evaluated, the task was revised and ready for classroom implementation. In the final phase, the teachers revised their task candidates first, and then analysed the revised task with the TAG as a check to ensure that they had taken into consideration all the criteria. Thereby, the task was seen to have qualified as suitable for differentiated instruction and was added to the problem bank.

We have found some aspects in the TAG that are related to the manifestations of contradictions as well as aspects in the guide that did not facilitate contradictions. The teachers contributed most actively to the two TAG themes Curriculum and Purpose (Table 2). These parts of the TAG frequently caused contradictions, although they were quickly resolved. Of the criteria for a rich mathematical task, low floor, intermediate steps, and open end (Benölken, 2015; Taflin, 2007; Sheffield, 2003) led to dilemmas that made the teachers reflect on the task design. However, the criteria involving promoting mathematical self-confidence and stimulating higher-order thinking rarely caused contradictions.

Our contribution to teacher professional development in differentiating instruction in mathematics is the proposal of a developed task analysis guide that can facilitate teachers in their collaborative work of differentiating mathematical tasks for the mixed-ability classroom.

Developed task analysis guide

Table 3: A developed task analysis guide to use when teachers collaboratively develop mathematical tasks to differentiate instruction in mathematics for the mixed-ability classroom. Teachers are asked to adjust the task in the development process.

Theme	Description
Curriculum	When designing the task analysis guide, the participating teachers collaboratively add aspects that make them reflect on the steering documents that are of importance for their practice.
Purpose	As every teacher practice is unique, the teachers involved ought to, based on their teaching experience, discuss and add aspects of importance to them. Examples of what to reflect on are the complexity of the task in relation to the specific group of students, and whether the task is an introduction task or a task aimed to provide enrichment. To facilitate the dilemma *Introduction vs enrichment*, the following question can be discussed: • Is the task suitable to use to introduce or enrich a mathematics content, or does it fulfill both roles?
Criteria	For the dilemma *Guiding vs funneling*, reflect on how the task can offer sufficient guidance on the one hand, but leave room for discovery on the other; that is, avoid funneling. To facilitate this dilemma, the following question can be discussed: • How can student support be included through intermediate steps in the solution process and at the same time encourage mathematical creativity? *For example, this means that students should not be forced into a specific solution strategy.* For the dilemma *Generality vs specificity*, reflect on how the task can be adapted to be specific on the one hand, but offer possibilities for generalizations on the other. To facilitate this dilemma, the following questions can be discussed: • How can the task be designed so that the entry is accessible for all students, and still offer challenges for all students? *For example, this means that students may enter the task in different ways and that some may reach a general solution while others reach a special case solution.* • How can the task be designed to encourage students to find patterns and/or make generalizations, and at the same time offer a cliffhanger? *For example, does the task encourage students to ask follow-up questions such as What if? and Why?*

Based on experiences from our study, and also supported by Nordgren et al. (2019), it is important for the productivity of a PLC that teachers get time and support from their school leaders for PLCs. If the process of developing tasks is to be qualitative and time-effective, we recommend a preparation phase in which the teachers collaboratively develop theoretical knowledge of task design and differentiated instruction. For the preparation phase, we also recommend that the teachers discuss and tune in the task analysis guide to fit their curriculum. Therefore, the two first aspects in the task analysis guide, Rows 1 and 2 in Table 3, should be created by the teachers involved. Based on our results, the third row contains questions aiming to elicit manifestations of contradictions, related to the criteria of rich mathematical tasks (Benölken, 2015; Sheffield, 2003; Taflin, 2007).

Conclusion and Future Research

We have characterized manifestations of contradictions in PLCs, focusing on designing tasks for differentiated instruction in mixed-ability mathematics classrooms. We found three contradictions, manifested as three dilemmas: Guiding vs funneling, Introduction vs enrichment, and Generality vs specificity. The analysis shows that, by noticing and making the dilemmas explicit objects of inquiry, the teachers came to address issues of students' different learning needs, tasks that can meet certain established criteria of being 'challenging for all students', and tasks and problem-solving activities for a mixed-ability mathematics classroom. All are issues that are important for teachers to note in implementing differentiated instruction (Tomlinson, 2016). We have discussed how the findings of this study shed new light on the process of content focus in PLCs in mathematics in general, and in PLCs on task design for differentiated instruction in particular. With input from our analysis, we have elaborated the design of a task analysis guide, as a means for facilitating PLCs that are inquiry-oriented and have a strong focus on task design for differentiated instruction in mixed-ability mathematics classrooms.

We do not claim that the three dilemmas we have outlined constitute an exhaustive list of manifestations of contradictions in PLCs focusing on task design for differentiated instruction. There is reason to believe that there are not only more dilemmas to be obtained but also other kinds of manifestations of contradictions. One way to enrich the map of possible contradictions in PLCs on differentiated instruction would be to add questions to the task analysis guide that specifically address certain criteria of rich mathematical tasks. For example, no manifestations of contradiction arose in connection to the possible interplay between the criteria of stimulating higher-order thinking (analyze, evaluate, and create) (Bobis et al., 2021; Taflin, 2007; Sheffield, 2003) and mathematical self-confidence (Taflin, 2007; Sheffield, 2003). It can be assumed that a task aimed at stimulating higher thinking will be difficult for students, and that they may not succeed in solving it. The question is, then, what impact failures can have on their confidence in mathematics. To provoke teachers' joint work on such issues, one could ask them to consider: How can a task be designed to encourage higher-order thinking, without risking negative effects on students' self-confidence in a mixed-ability mathematics classroom?

Hence, to gain a more complete picture of inquiry processes in PLCs, with a strong content focus on differentiated instruction, we encourage future research to continue mapping and characterizing the manifestations of contradictions that can be present in teachers' joint work when designing tasks for differentiated instruction in mixed-ability mathematics classrooms.

There is a need to find structures for supporting teachers in designing tasks for differentiated instruction in mathematics (Leikin & Stanger, 2011; Mellroth, 2018; Shayshon et al., 2014). While the role of the TAG was not the object of our investigations, our analysis shows that the teachers' collective work grew focused and inquiry-oriented when they became aware of and struggled with the dilemmas. On this account we suggest that future research investigate how a task analysis guide that explicitly raises concerns about contradictions, in the form of questions, can support a PLC in developing into a community of inquiry (Jaworski, 2008) with a strong focus on content. To further develop the task analysis guide, and to understand its role in developing a PLC, there is a need for more qualitative research. Once we have a well-developed task analysis guide, and a clear picture of how it works, it will be time to examine the effect of such a guide in a randomized control trial.

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