# Differentiating Mathematics Instruction in Remote Learning Environments: Exploring Teachers' Challenges and Supports

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This report examines one teacher's attempts to differentiate instruction every day, in a dynamic teaching and learning environment. The report is part of a larger study that utilised a network of theories approach, coordinating the documentational approach to didactics (DAD) and Thompson and Harel's theory of meanings, to examine teachers' understandings as they engaged in and discussed their attempts to differentiate instruction in remote and hybrid learning environments. Analyses involved building models of teachers' understandings of differentiated instruction and the resources they utilised to support differentiation and exploring how these models persisted or changed throughout the study. The report's findings highlight the importance of teachers' meanings in their attempts to differentiate instruction and the role digital resources play in supporting or hindering such practices. Finally, this report adds to a growing body of research on teacher's work with and on digital resources as a means to support differentiated instruction, particularly as remote and hybrid teaching and learning continues throughout the U.S. and across the globe.

**Keywords** · differentiation · remote learning

## Introduction

Numerous international treaties and documents have affirmed the right of all children to education (e.g., Office of the United Nations High Commissioner for Human Rights, 1960; United Nations General Assembly, 1948, 1989, 2006). Preparing all students to participate in life within a diverse society is a major educational challenge. According to Suprayogi et al. (2017), "Meeting student differences is challenging since these differences can be related to a large variety of student characteristics, such as learner interests, learning styles, developmental level, . . . cultural background, language level, [and] attitudes" (p. 298). Differentiated instruction aims to deal with the inherent differences between students by providing them with the best possible opportunities to learn and thrive. According to Tomlinson (2017), differentiated instruction "provides avenues to acquiring content, to processing or making sense of ideas, and to developing products so that each student can learn effectively" (p. 1). Teachers can differentiate their classrooms around five key elements involved in learning (Tomlinson, 2014): content (the knowledge, understanding, and skills we want students to learn); process (how students come to understand or make sense of the content); product (how students demonstrate what they have come to know, understand, and are able to do after an extended period of learning); environment (classroom conditions that

set the tone and expectations for learning), and affect (how students' emotions and feelings impact their learning).

A long line of research supports the idea that differentiating instruction is a challenging and complex practice (e.g., Joseph et al., 2013; Lange, 2009; Moosa & Shareefa, 2019; Siam & Al-Natour, 2016; van Geel et al., 2019). Anthony et al. (2019), characterise differentiation as a "slippery concept . . . in terms of goals, teacher understanding, and practice within mathematics classrooms" (p. 117). Several reports detail the challenges to teachers' implementation of differentiated instruction, including a lack of teacher preparation time and resources (Rodriguez, 2012), the need for collaboration within and across schools (Smit & Humpert, 2012), and a disconnect between teachers' understandings and their implementations of differentiated instruction (Whipple, 2012). Furthermore, research has yielded mixed evidence of teachers' actual use of differentiated instruction, from teachers reporting they rarely or occasionally use differentiated instruction practices in their teaching (Moon et al., 2002; Pozas et al., 2020; Smit & Humpert, 2012) to moderate or high rates of such practices (Prast et al., 2015; Roy et al., 2013).

## *Differentiated Instruction during the Pandemic*

As a consequence of the COVID-19 pandemic, schools have had to adapt to fulfill their many functions, challenging teachers to rethink ways to support their teaching and their students' learning. According to the Organisation for Economic Co-operation and Development (OECD; 2020), an almost universal response to the pandemic has been the use of digital technologies to support teachers, students and their families. Digital technology allows for new solutions to "what people learn, how people learn, where people learn and when they learn. Technology can enable teachers and students to access specialised materials well beyond textbooks, in multiple formats and in ways that can bridge time and space" (OECD, 2020, para. 2). Unfortunately, not all students have the same access to digital devices and online resources, and access varies greatly across countries (OECD, 2020).

Although the pandemic has challenged all schools, students from low-income households and rural areas have faced especially significant barriers (Opalka et al., 2020). Globally, data accumulated by the United Nations Children's Fund (UNICEF; 2021) revealed "large gaps in access to electronic devices like computers or mobile phones, and to internet connections or other modalities of distance education, particularly in poor and rural areas" (p. 6). In Australia, low-income households and those in rural and remote areas are less likely to have reliable, fast broadband internet, and more likely to lack access to appropriate or individual computing devices (e.g., desktop computer, laptop, tablet) than those in metropolitan areas (Thomas et al., 2020). Furthermore, rural and remote schools are more likely to have teachers with low levels of expertise in delivering lessons using information and communications technology (Halsey, 2018).

The pandemic's restructuring of classrooms to remote or hybrid teaching and learning environments, along with teachers' motivations to find new and improved ways to support all of their students, makes examination of teachers' attempts to differentiate instruction in such environments an area prime for research. According to Hersh (2020), "Differentiated learning supported by technology has considerable potential, but is rarely used, largely due to lack of appropriate teacher education and other resources" (p. 2). Whereas prior research (e.g., Chamberlin & Powers, 2010; Dosch & Zidon, 2014; Morgan, 2014), strategy guides and texts (e.g., Preszler, 2006; Small, 2017; Small & Lin, 2015), and teachers' personal reflections have focused on differentiating a single or a sequence of lessons, a university-level course, or on differentiating instruction for individual students, the study reported here examines pre-tertiary teachers' attempts to differentiate for all students, every day. The study focuses on ideas of differentiation presented by Tomlinson (2014, 2017) and addresses the following research questions:

1. How do grades 6-12 mathematics teachers and math intervention specialists understand differentiated instruction?
2. How do grades 6-12 mathematics teachers and math intervention specialists understand the resources they utilise to support differentiated instruction?
3. How can knowledge of teachers' understandings support interventions that impact these understandings in productive ways (where "productivity" is defined in terms of student learning)?

Although the study focuses on addressing the three research questions for a group of 18 grades 6-12 mathematics teachers and math intervention specialists, the main focus of this report will be on the case of one mathematics teacher (Claudia). Such a focus allows for explication of Claudia's understandings of differentiated instruction and the resources she employs to support differentiated instruction, and how knowledge of these resources and understandings supported the design of interventions to promote refinements in and subtle modifications to Claudia's ways of operating.

## Theoretical Framework

The study utilised a networking of theories approach (Bikner-Ahsbahs & Prediger, 2010; Prediger et al., 2008) to connect frameworks and explore the research questions. Specifically, the study connected theoretical approaches using the networking strategies *synthesising and integrating (locally)*, which focus on the "development of theories by putting together a small number of theoretical approaches into a new framework" (Prediger et al., 2008, p. 173). The study "networks" the documentational approach to didactics (Gueudet & Trouche, 2009; Trouche, Gueudet et al., 2019) and Thompson and Harel's (Thompson, 2013, 2016; Thompson et al., 2014) theory of meanings.

### *The Documentational Approach to Didactics*

Developed from the French didactics tradition in mathematics, socio-cultural theory, the digitalisation of information and communication, and Rabardel's (1995/2002) instrumental approach, the documentational approach to didactics (DAD) analyses "teachers' work through the lens of 'resources' for and in teaching: what they prepare for supporting their classroom practices, and what is continuously renewed by/in these practices" (Trouche et al., 2018a, pp. 1-2). Although initiated with secondary school mathematics teachers, the documentational approach has been used from kindergarten to the university level, in both pre-service and in-service teacher education, and for a variety of subject matter (e.g., physics, chemistry).

In the documentational approach, resource is grounded in Adler's (2000) work, which defines a resource as anything likely to 're-source', or "to source again or differently" (p. 207), the teacher's work. That is, all the "resources that are developed and used by teachers and pupils in their interaction with mathematics in/for teaching and learning, inside and outside the classroom" (Pepin & Gueudet, 2020a, pp. 172-173). Such resources include text (e.g., textbooks, worksheets, tests) and other material resources (e.g., calculators); digital-/ICT-based resources (e.g., online textbooks, GeoGebra); discussions between teachers, orally or online; students' written work; teachers' discussions with mathematics teacher educators; and so forth (Pepin & Gueudet, 2020a). Finally, Gueudet and Trouche (2009) assert that a resource is never isolated; rather, a given resource belongs to a set of resources (p. 205). This conception of resource is particularly germane, since the study involved developing models of teachers' understandings of differentiated instruction in remote learning environments; that is, as they attempted to differentiate instruction using one or more digital resources.

The documentational approach focuses on the "interactions between teachers and resources, and on the consequences of these interactions" (Pepin & Gueudet, 2020b, p. 158). Central to the documentational approach is d*ocumentational* genesis, which comprises two interrelated processes (Pepin et al., 2013): 1) the process of *instrumentalisation*, where a teacher's knowledge guides the choices she makes among various resources and the way these resources are appropriated; and 2) the process of instrumentation, where the features of the resource or set of resources impact the teacher's knowledge she develops as a result of her interactions with the resource or set of resources. According to Gueudet et al. (2016), resources can "enable or constrain activity, which can be interpreted as the interplay between instrumentalisation and instrumentation" (p. 191). The process of documentational genesis results in the development of a document and can be represented by the equation: Document = Resource(s) + Utilisation Scheme.

According to Pepin and Gueudet (2020b), utilisation schemes include both procedural schemes (e.g., how to use particular resources) and cognitive schemes (e.g., knowledge about the means that the resource offers). The concept of scheme is central in the documentational approach. According to Vergnaud (2013), schemes are "stable forms for organising activity" (p. 57) and closely linked with the concept of "class of situations". Therefore, a scheme can be viewed as a structure organising a subject's activity with a resource or set of resources for a given goal.

## *Thompson and Harel's Theory of Meanings*

Thompson and Harel's (Thompson, 2013, 2016; Thompson et al., 2014) theory of meanings is based on Piaget's notion of assimilation to a scheme and focuses on teachers' (schemes of) meanings, where a scheme is defined as "an organisation of actions, operations, images, or schemes—which can have many entry points that trigger action—and anticipations of outcomes of the organisation's activity" (Thompson et al., 2014, p. 11). Such a focus enables researchers to identify and examine the assimilation of schemes, and the means for scheme transformation, via generalising assimilation, accommodation, and reflective abstraction.

According to Piaget, to understand is to assimilate to a scheme (Skemp, 1961, 1979; Thompson, 2013; Thompson & Saldanha, 2003). Therefore, attaching meaning (i.e., understanding) is constituted by assimilating to a scheme and the phrase "a person attached a meaning to a word, symbol, expression, statement, or action" means that the person assimilated the word, symbol, expression, statement, or action to a scheme (Yoon et al., 2015).

Thompson and Harel's system addresses issues of understanding, meaning, and ways of thinking. Such a system allows for discussions of and investigations into in-the-moment and stable understandings (see Table 1).

*Table 1: Definitions of understanding, meaning, and ways of thinking (Thompson et al., 2014).*

| Construct | Definition |
|---|---|
| Understanding (in-the-moment) | Cognitive state resulting from an assimilation |
| Meaning (in-the-moment) | The space of implications existing at the moment of understanding |
| Understanding (stable) | Cognitive state resulting from an assimilation to a scheme |
| Meaning (stable) | The space of implications that results from having assimilated to a scheme. The scheme is the meaning. |
| Way of Thinking | Habitual anticipation of specific meanings or ways of thinking in reasoning |

As characterised by Thompson and Harel (see Table 1), an understanding is a cognitive state of equilibrium, which may occur from assimilation to a scheme (i.e., stable understanding). According to Thompson et al. (2014), "A scheme, being stable, then constitutes the space of implications resulting from the person's assimilation of anything to it. The scheme is the meaning of the understanding that the person constructs in the moment" (p. 13). Alternatively, a cognitive state of equilibrium might be a state the "person has struggled to attain at that moment through functional accommodations to existing schemes . . . and is easily lost once the person's attention moves on" (i.e., in-the-moment understanding; Thompson et al., 2014, p. 13). Such understandings are specific to that moment in time and are "typical when a person is making sense of an idea for the first time" (Thompson et al., 2014, p. 13).

Aligned with Simon et al.'s (2018) work on students' understandings of multiplication, the study presented here attempted to design sequences of activities (i.e., interventions), such that "small changes in [teachers'] assimilatory structure could be provoked sequentially" (p. 155). That is, the study attempted to promote accommodation or generalising assimilation, which refers to "the process by which new, somewhat different objects or experiences are assimilated, causing at least a subtle change in the assimilatory structure… creat[ing] a more generalising or encompassing structure" (Simon et al., 2018, p. 155). Therefore, I examined teachers' interactions with digital resources through activities (e.g., individual interviews, group discussions) that were designed to support teachers' repeated (or repetitive) experiences—where what was intended to be repeated was "the constitution of situations in ways that are propitious for generalising assimilations, accommodation, and reflection" (Thompson, 1994, p. 227). As will be illustrated in the following sections, the local integration of the documental approach (with its focus on interactions between teachers' schemes and resources) and a theory of meanings (with its focus on teachers' in-the-moment and stable understandings) is particularly appropriate to address the study's questions.

# Methods

Since the study attempted to reveal teachers' understandings and explore how these conceptions supported or constrained their capacities to productively differentiate instruction (in terms of meeting the needs of all their students), it was necessary to make models of teachers' conceptions. The case study of Claudia described in this report express my second-order models (Steffe, 1995) of Claudia's understandings at various points throughout the study; that is, throughout the activities (i.e., interventions) designed to provoke sequential changes in Claudia's (schemes of) meanings.

## *Study Participants*

Study participants comprised grades 6-12 mathematics teachers ($n$ = 15) and math intervention specialists ($n$ = 3) from rural areas or small-town school districts. Participating teachers were self-selected (responded to email sent to district administrators) and met the following criteria: a) mathematics teacher or math intervention specialist in any of grades 6 to 12; b) teach in a rural or small-town school district in the U.S. state of Ohio; c) have an interest in investigating ways to differentiate instruction; d) have an interest in exploring how grade 6 to 12 mathematics teachers, math intervention specialists, and university mathematics education researchers can work collaboratively online to support mathematics teaching and learning; and e) have the availability and desire to spend approximately 10 hours of online collaboration over the course of 12 weeks.

*Table 2: Participating in-service content teachers and intervention specialists by grade level and years of experience.*

| Grades Taught / Years of Experience | 1 to 5 years | 6 to 10 years | 11 to 15 years | 16 to 20 years | Over 20 years |
|---|---|---|---|---|---|
| Grades 6-8 ($n$ = 8) | 1 content; 1 intervention | 2 content; 1 intervention | 1 content | 1 content | 1 content |
| Grades 9-12 ($n$ = 10) | 1 content | 1 content; 1 intervention | 6 content | 0 | 1 content |

Although the study comprised 18 teachers, this report will focus on the case study of one grades 9-12 mathematics teacher (Claudia) with between 11-15 years of experience.

## *Data Collection*

The study employed a modified version of the *reflective investigation methodology* (Trouche et al., 2018a) for data collection. According to Trouche et al. (2018a), the reflective investigative methodology is naturally associated with case studies and grounded by the following five main principles: (1) broad collection of resources; (2) long-term follow up; (3) in- and out-of-class follow-up; (4) reflective follow-up; and (5) confronting a teacher's views on her documentation work. Although data collection attempted to follow all five principles of the reflective investigation methodology, due to the length of the study (i.e., 12 weeks) and limits precluding in-person interactions, some principles could not be strictly adhered to, resulting in a modified version of the methodology. Specifically, regarding the principles of long-term follow-up and in-class follow-up, coronavirus restrictions precluded in-class observations of instruction (with students) and in-person observations of teachers' collaborations with colleagues and their planning of instruction (individually or with colleagues). Furthermore, several districts had policies prohibiting non-district personnel from observing online instruction, thus precluding any form of classroom observation.

According to Trouche et al. (2018a), the active involvement of the teacher is a practical necessity in reflective investigation methodology, as she "is the one having access to . . . her documentation work (beyond the direct observation of the researcher)" (p. 6). Furthermore, reflective investigation follows "teacher's interactions with resources, beyond the direct observation by the researcher, involving teachers closely in the data collection, and accessing their 'thinking' through reflective analysis and stimulated recall" (Trouche, Gitirana et al., 2019). Although reflective investigation asserts the need for long-term follow-up because "[g]eneses are ongoing processes and schemes develop over long periods of time" (Trouche et al., 2018a, p. 6), the study reported here was designed to examine teachers' initial (or early) engagements with a resource (or set of resources) or their engagements with a resource (or set of resources) in novel ways. As such, follow-ups comprised discussions, interviews, and examinations of teachers' lesson plans and materials, and their descriptions of instruction over the course of 12 weeks.

The study's data corpus consisted of video recordings and documents generated from individual interviews and online group meetings of teachers as they interacted with activities designed to promote sequential changes in teachers' meanings via accommodations or generalising assimilations. Each online group meeting was designed to be teacher-driven, with participants directing the subject and duration of issues and topics discussed with little interference from me. The rationale for providing teacher-directed online group meetings was to remove barriers (e.g., Trust, 2012), combat isolation (e.g., Slavit & Roth McDuffie, 2013), and promote collaboration and engagement in ongoing professional dialogue (Wagner, 2018).

Individual interviews, on the other hand, were designed to probe teachers' understandings in more detail and promote specific ways of thinking and reasoning.

Several individual and group activities involved provoking teachers to differentiate instruction utilising specific strategies and digital resources. Data consisted of video recordings and field notes from all online group meetings and individual teacher interviews; copies of and internet links to all materials teachers utilised during their lessons; teachers' daily or weekly reflections on their experiences; and teachers' responses to a pre-survey designed to make their meanings of differentiated instruction and digital resources to support such instruction explicit. Finally, the data corpus comprised two tools specific to reflective investigation methodology: reflective mappings of each teacher's resource system (RMRS) and inferred mappings of each teacher's resource system (IMRS). According to Wang (2018), an RMRS is a methodological tool created by a teacher where the teacher is asked to "draw a map to present her resources in a structured way based on her own reflection" (p. 197). Similarly, an IMRS is a methodological tool created by "the researcher based on the observations of and interviews with the teachers about their resource work" (Rezat et al., 2019, pp. 357-358).

As part of the pre-survey, teachers were asked to choose one element of differentiation to focus on (i.e., content, process, product, environment, affect). Teachers were then requested to utilise a digital resource (or resources) to support differentiation of their chosen element and implement strategies around this focus for three to five weeks. Teachers began implementing this initial focus element between Weeks 3 and 4 of the study, depending on their availability to meet for online group meetings and individual interviews. Teachers were also asked to describe their experiences through text (i.e., *Google Docs*), video (e.g., *Flipgrid*), or audio (e.g., *Audacity*) on a daily or weekly basis. At the end of three to five weeks, teachers were asked to evaluate the "success" of this implementation (in terms of their students' learning) and modify their chosen element, introduce a new element altogether, or add a second element to the first for another three to five weeks. By the end of the study (12 weeks), each teacher completed roughly two to three of these iterations. Finally, teachers participated in monthly online group sessions designed for teachers to share and reflect on their experiences. Teachers' sequential introduction of new, modified, or additional elements of differentiation, along with their daily, weekly, and monthly reflections, were intended to promote accommodation or generalising assimilation of their schemes.

## Results

Making sense of teachers' attempts to differentiate instruction in remote learning environments required me to develop models of teachers' ways of operating—models that represented my interpretations of teachers' interactions with study activities, digital resources, and their colleagues. Consistent with Thompson's (2008) characterisation of conceptual analysis, my analyses involved developing models of teachers' understandings in order to explain how these understandings persisted or changed throughout the study. Using data generated from study activities and reflective investigation, these models were tested, modified, and refined through ongoing and retrospective conceptual analyses (Thompson, 2008; von Glasersfeld, 1995) of the data corpus.

In the remainder of this report, I focus on a case study of one grade 9-12 mathematics teacher (Claudia), and explicate how being mindful of Claudia's understandings, not only supported the development and refinement of models of Claudia's understandings, but also interventions designed to provoke sequential changes in her (schemes of) meanings. Claudia taught Algebra I (three 50-minute sections or periods) and Geometry (two 50-minute sections or periods) each day to 9th and 10th grade students (ages 14-16 years). Claudia's teaching environment was fully

remote for the initial six weeks of school (which included Weeks 1-3 of the study), moved to a hybrid format for the next seven weeks (Weeks 4-10 of the study), and translated back to fully remote for the remainder of the semester (which included Weeks 11-12 of the study).

## *The Case of Claudia*

The initial model of Claudia's understanding of differentiated instruction was developed from her responses to the pre-survey, which was presented to participating teachers at the start of the study. In response to the pre-survey question "What does it mean to differentiate instruction?", Claudia stated that differentiation meant "trying to give harder or easier work to students who finish early or struggle." This statement was interpreted as indicating that Claudia partitions her students into two (possibly three) categories (i.e., students who complete assignments early and need more of a challenge, students who struggle, and implicitly the rest of her class—the group of "average" students). When asked on the pre-survey to rate how important she believed differentiated instruction was to her students' mathematics learning, between 0 (not important at all) to 100 (extremely important), Claudia indicated a "71". The average rating for the remaining 17 teachers was 90.8 (SD = 11.2), and Claudia tied with one other teacher for the lowest rating. Therefore, compared to almost all of her colleagues, Claudia did not believe that differentiated instruction was very important to her students' mathematics learning. Similarly, when asked to rate how important her students believed differentiated instruction was to their own (i.e., students) mathematics learning, Claudia indicated a "65". The average rating for the remaining 17 teachers was 80.3 (SD = 9.8), and Claudia tied with two other teachers for the second-to-lowest rating. Thus, compared to the majority of her colleagues, Claudia felt her students believed differentiation was not too important to their learning. According to Valiandes and Neophytou (2018), teachers' meanings for differentiation, "in terms of its feasibility and its effectiveness, shapes their attitudes and their actions" (p. 124); specifically, teachers' meanings for differentiation determine whether they will implement it or not, keep it or discard it after a brief trial, or persist and endure in using it even when faced with numerous challenges (Valiandes & Neophytou, 2018). Therefore, Claudia's pre-survey responses could have indicated an unwillingness to fully implement ideas and strategies for differentiating instruction or a willingness to discard implementation after a short trial. The potential for such views were identified as something to explore further and anticipate when designing interventions.

Claudia also indicated that she "very infrequently" provides her students with choices, regarding classroom activities, assignments, or assessments, but that she did "provide a more challenging extra credit problem on tests and some homework assignments." Finally, Claudia stated that she "never" discusses differentiated instruction with her students. From Claudia's pre-survey responses, her understanding for differentiated instruction was conceptualised as "to provide students with the types of problems they can solve in an allotted amount of time (which includes easier problems for students who struggle or more challenging problems for students who complete assignments and assessments early)." It was not clear at this stage what came to Claudia's mind when she thought about "easier" and "more difficult" problems (i.e., what these terms meant to Claudia), other than the amount of time it took students to complete assignments and assessments—perhaps as a proxy for level of complexity. Therefore, it did not appear that Claudia had a very robust understanding of differentiated instruction; that is, an understanding of differentiation that involves providing "different avenues to acquiring content, to processing or making sense of ideas, and to developing products so that each student can learn effectively" (Tomlinson, 2017, p.1). Claudia was one of six teachers (five content teachers and one intervention specialist) who exhibited such an understanding of differentiated instruction. These results are similar to those by Smit and Humpert (2012), who reported 80 of 162 primary and secondary school teachers studied in Switzerland exhibited a "limited view of [differentiated instruction] as

a reactive strategy to address advanced or slow learners" (p. 1157). Being mindful of Claudia's understandings anticipated the need for future discussions and activities that focused Claudia away from the time it took students to complete assignments and toward students' thinking and reasoning; away from teaching to the "average" student and toward providing all students with opportunities to engage in challenging mathematics; and providing students with variety and choices regarding activities and assignments.

During the study, Claudia engaged in several online activities designed to support reflection on and explication of her understandings related to differentiated instruction. Table 3 illustrates these activities. Due to a variety of reasons (e.g., internet problems, parent-teacher conferences, issues surrounding the pandemic), each online group meeting had teachers that were not able to attend. The only group meeting that had close to 100% attendance was the Week 9 meeting with Algebra 1 and Algebra 2 teachers, which involved four of five such teachers (80%).

*Table 3: Online activities Claudia engaged in by week.*

| Activity / Weeks | Individual Interview | Group Meetings involving Claudia | Daily / Weekly Reflections |
|---|---|---|---|
| Weeks 1-2 | Week 2 (50 minutes) | Week 2 - Grades 9-12 teachers (n = 8; 60 minutes) | No Google Doc entries |
| Weeks 3-4 | Week 3 (55 minutes) | Week 4 - Teachers who selected content as focus element (n = 7; 55 minutes) | Week 1 – One Google Doc entry<br>Week 2 – One Google Doc entry |
| Weeks 5-6 | Week 6 (50 minutes) | | Week 5 – One Google Doc entry<br>Week 6 – Two Google Doc entries |
| Weeks 7-8 | Week 7 (55 minutes) | Week 8 – Grades 9-12 teachers (n = 7; 50 minutes) | Week 7 – One Google Doc entry<br>Week 8 – One Google Doc entry |
| Weeks 9-10 | | Week 9 - Algebra 1 and Algebra 2 teachers (n = 4; 50 minutes) | Week 9 – No entries<br>Week 10 – One Google Doc entry |
| Weeks 11-12 | Week 12 (50 minutes) | Week 12 - Teachers who initially selected content as focus element (n = 6; 55 minutes) | Week 11 – No entries<br>Week 12 – one Google Doc entry |

**Weeks 1 and 2 of the study**

Claudia was one of eight grades 9-12 teachers to participate in an online group meeting during the study's second week. The lone grades 9-12 intervention specialist was not able to attend this meeting, so all participants were content teachers. Throughout the meeting, Claudia regularly agreed with her colleagues' assertions that differentiated instruction involved providing "most students with average problems, and the rest with problems that were a little easier or a little more challenging." Teachers did not explicate what they considered an "average" problem or what made a problem "a little easier" or "a little more challenging," or what student understandings or struggles would determine which type of problem they would be given. Since each online meeting was teacher-driven, I attempted to remain in the background as much as possible and simply observe the discourse.

During Claudia's first individual interview, which occurred three days after the online group meeting described above, Claudia stated that what determined whether a student received a problem that was "a little easier" or "a little more challenging" was each student's history at completing assignments later or earlier than most of their classmates. Therefore, it appeared that Claudia was focused on the time it took students to complete assignments, not necessarily what students understood or struggled with. Claudia also indicated she typically located "more

challenging" problems at the Illustrative Mathematics website and that this was her only use of the website. Therefore, Claudia's utilisation scheme (i.e., stable understanding) for the Illustrative Mathematics website was "where to locate challenging problems." Claudia stated that her use of Illustrative Mathematics was due in part to how professional developers had characterised the resource and her own view that the problems provided on the Illustrative Mathematics site were "much more involved" than the problems in her textbook. When pressed to articulate her meaning for "much more involved," Claudia stated the Illustrative Mathematics problems "took more time to solve" because they "required students to do more than follow the ways [she] model[ed] in class."

During the first and second weeks of the study, each participant was asked to construct and submit (via email) a reflective mapping of their resource system (RMRS); that is, a map illustrating, linking, and structuring the teacher's resources, using the teacher's own naming and categorisation of resources (Trouche, Gueudet et al., 2019). During Claudia's initial individual interview, we discussed her RMRS in order for me to get a sense for how she named, categorised, linked, and structured her resources. Directly following this interview, I created an inferred mapping of Claudia's resource system (IMRS), which was based on Claudia's RMRS, the pre-survey, Claudia's participation during the online group meeting, and her initial individual interview.

**Weeks 3 and 4 of the study**

Prior to her second individual interview, which occurred during Week 3, Claudia was asked to provide example problems, tasks, activities, in- and out-of-class assignments, and assessment problems that she intended to use for an upcoming sequence of lessons. Sample problems for the content standard "Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables" provided by Claudia are illustrated in Figure 1. Claudia identified each problem as being "average" or "challenging."

**Average Problems - for all students**
**(Big Ideas Learning, 2016)**

a) **Write a system of linear equations that has the ordered pair $(3, -5)$ as its solution.**

b) **Solve the following system of linear equations by graphing:**

$$y = \frac{1}{2}x + 3$$
$$y = -\frac{3}{2}x - 5$$

c) **Solve the following system of linear equations:**

$$y = -10x + 2$$
$$10x + y = 10$$

**Challenging Problems - for students who finish early or extra credit**
**(Illustrative Mathematics, 2021)**

d) **Without graphing, construct a system of two linear equations where (−2,3) is a solution to the first equation but not to the second equation, and where (5,−2) is a solution to your system. After you have created your system of equations, graph your system. Explain how your graph shows that your system satisfies the required conditions.**

e) **Jason and Arianna are working on solving the system of linear equations:**

$$6x + 17y = 100$$
$$5x + 9y = 86$$

**Rounding their answer to the nearest hundredth, Jason and Arianna find that $x \approx 4.04$ and $y \approx 7.31$.**

**Give a numerical explanation in terms of the slopes and y-intercepts of the graphs of the equations how you could tell that Jason and Arianna must have made a mistake.**

*Figure 1.* Example "average" and "challenging" problems.

During her second individual interview, Claudia indicated that "average" or "typical" problems—problems that she would assign to all students as in- or out-of-class assignments or on an assessment—were taken from her textbook and resembled the types of problems she used to model the lesson's important ideas. Claudia indicated that "challenging" problems were usually taken from the Illustrative Mathematics website and provided to students who finished

in-class work early or as extra credit problems in order to keep all students "busy and working." When asked for an example of an "easier" problem that she would provide to struggling students, Claudia stated, "I give them the same problems as everyone else, but just expect they will need more of my help or extra time to finish." Therefore, in selecting problems, Claudia did not focus on the complexity of the thinking and reasoning that students might engage in to make sense of the problem and work toward a solution, but the time it took students to complete the problems. Claudia's focus on the time it took her students to complete problems indicated a need to engage Claudia in discussions and activities that promoted a focus on students' thinking and reasoning and providing all students with opportunities to "stretch beyond their comfort zone" (Tomlinson, 2014, p. 46).

As part of the second interview, I confronted Claudia with her initial RMRS and the IMRS I created following her first interview. I use the term "confront" here in the sense of Brousseau (1997), and as employed in the documentational approach, to mean "a focused comparison; bringing together for careful comparison" (Interglot, 2021). This interview resulted in the refined IMRS shown in Figure 2, which illustrates the following resources: (1) the course textbook, (2) a document camera, (3) video conferencing for virtual instruction, and (4) course assessment resources.

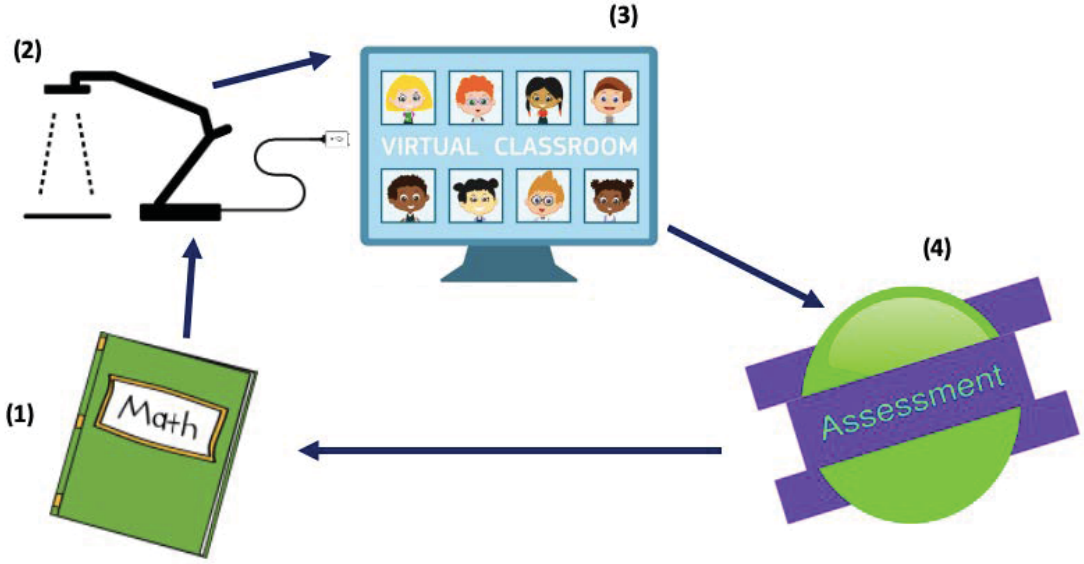


*Figure 2:* Initial Inferred Mapping of Claudia's Resource System (IMRS).

Claudia's initial RMRS did not include the document camera or the arrows and exhibited a non-connected set of resources. The refined IMRS (Figure 2) illustrates a cyclic process, where Claudia determines what to teach using her course textbook, delivers instruction in a virtual classroom (via Zoom) using her document camera, and then assesses her students using the educational assessment platform Edulastic (and sample state assessment problems in pdf form). Although Claudia models problem solving (of textbook problems) to her students using her document camera and assigns problems from the course textbook for students to solve that are similar to the sample problems, all of her assessments of student learning (e.g., quiz, unit test) are made using Edulastic. Claudia indicated that she had started using Edulastic during the pandemic because a colleague and her school district's mathematics coach had recommended the resource as a way to "assess students remotely without needing to transfer paper back and forth." Claudia

also stated that she, in general, did not like utilising digital resources in her classroom, because such resources required too much time to find and become familiar enough with (herself) to use "with few issues" in the classroom.

Although Claudia described the Illustrative Mathematics website as the resource she used to find "challenging" problems for students who finished work early or as extra credit problems, she neither included nor mentioned it when discussing her RMRS or my initial IMRS. When provoked to articulate the degree to which she actually used Illustrative Mathematics, Claudia stated, "Since classes have been remote since the start of the school year and not all students have fast internet or support at home, there hasn't been a need to use these problems." When asked to describe the situation in more detail Claudia asserted, "There are so few students completing assignments on a daily basis that each of my class periods are two to three weeks behind where we should be [on the district's pacing guide]."

Claudia's initial RMRS, her second individual interview, and the refined IMRS allowed me to develop models of Claudia's understandings for each resource indicated in Figure 2. Claudia's stable understanding of her course textbook was "where to find problems each day for classroom examples and assignments." Claudia's stable understanding of her document camera was as a proxy for her whiteboard; that is, as a resource "to display her notes, sample problems, and their solutions." Claudia's stable understanding of Zoom was a means for students "to observe instruction, and to ask and answer questions about instruction, synchronously." Finally, Claudia's stable understanding of Edulastic was as a means "to assess her students remotely and provide immediate feedback." When asked to describe the "immediate feedback" Edulastic provided to her students, Claudia stated, "it gives [students] an overall percent correct and the correct answers to each question."

Since Claudia was familiar with the Illustrative Mathematics resource, but had not recently used the resource due to many of her students' lack of participation, I requested that Claudia attempt to provide all of her students with opportunities to engage with problems from the website over the next two to three weeks (Weeks 4 and 5 of the study)—not just students who typically completed assignments early—by utilising the Illustrative Mathematics problems in pair or group activities during remote instruction. This action required Claudia to utilise the Illustrative Mathematics website in a novel way and Zoom in ways distinct from how she had been utilising the resource; specifically, to move away from using Zoom as a means for students to observe and ask questions about instruction in real time, and toward providing students with opportunities to become more involved in their own learning by using Zoom breakout rooms for "think, pair, share" or small group assignments. The intent of these actions was to subtly move Claudia toward thinking about providing all her students with opportunities to engage with challenging mathematics in collaborative environments. Furthermore, these actions were intended to not only explore how Claudia's meaning for differentiated instruction impacted her use of each resource (via instrumentalisation), but also how her use of Illustrative Mathematics and Zoom in novel ways impacted her understanding of differentiated instruction (via instrumentation).

Table 4 illustrates my models of Claudia's understandings during the first four weeks of the study. As highlighted in Table 4, my model for Claudia's understanding for differentiated instruction was modified and refined over the first four weeks as I interacted with Claudia, observed her online interactions with colleagues, and analysed data.

*Table 4: Models of Claudia's understandings during weeks 1-4.*

| Idea, Notion, or Resource / Model of Claudia's Understanding | Model 1 | Model 2 | Model 3 |
|---|---|---|---|
| Differentiated Instruction | To provide students with the types of problems they can solve in an allotted amount of time (including easier problems for struggling students and more challenging problems for students who finish early) | To provide three types of problems that allow all students to complete work in the same amount of time (struggling students, students who complete assignments early, and average students) | To provide two types of problems (average and more challenging problems); provide struggling students with extra support or extended time so that all students stay busy and complete work in the same amount of time |
| Illustrative Mathematics website | Where to locate challenging problems | | |
| Allowing students with choices | To provide a more challenging extra credit problem on tests and some homework assignments | | |
| Average Problem | Similar (in solving process) to problems shown as examples; majority of students can complete in set amount of time | | |
| Challenging Problem | Requires students to do more than follow ways example problems are solved; for students that typically complete problems early | | |
| Textbook | Where to find problems each day for classroom examples and assignments | | |
| Zoom video conferencing | For students to synchronously observe instruction, and to ask and answer students' questions about instruction | | |
| Document Camera | To display notes, sample problems, and their solutions | | |
| Edulastic assessment platform | To provide online assessments to students, grade assessments, and provide feedback | | |

Claudia was one of 10 teachers to select content as their differentiated instruction focus element. Seven of these teachers (including Claudia), all content teachers, discussed strategies and resources they found useful for differentiating content during a Week 4 online group meeting. One focus of this teacher-directed meeting was on using Webb's (1997) depth of knowledge to differentiate problems. Webb's (1997) depth of knowledge (DOK) aligns problems based on the level of cognitive expectation, or depth of knowledge, required for students to complete the problems. Although my intent was to mainly observe, I did ask teachers why they preferred Webb's (1997) depth of knowledge over the Revised Bloom's Taxonomy (Anderson et al., 2001), Hess' Cognitive Rigor Matrix (Hess et al., 2009), or Smith and Stein's (1989) Mathematical Task Analysis Guide—all of which most of these teachers had heard of or engaged with during prior professional development. Teachers unanimously indicated their preference for DOK because of its use to designate problems on the state end-of-course assessments.

The second focus of the online meeting was on the Edulastic assessment platform. Teachers stated the platform provided an "easy way" to provide students with problems similar to the state assessments and allowed for assessment in remote environments. Two teachers described their use of Edulastic to find problems of different "DOK-levels." Although each teacher indicated they were aware that Edulastic's item library could filter problems by content standard, the ability to also filter problems by depth of knowledge was new to five teachers, including Claudia. The group meeting concluded with all seven teachers indicating their intent to focus on differentiating content for the next three to five weeks and to utilise Webb's (1997) depth of knowledge and Edulastic to support their differentiation of content.

Although differentiating content in terms of the level of complexity of problems students engage with may be recommended for students based on their individualised education programs (IEP; Tomlinson & Imbeau, 2010), many professionals argue that differentiation "does not mean some students work at [DOK] level 1 and others at level 4" (Hunter, 2019). According to Tomlinson and Imbeau (2010), in order to differentiate content teachers should emphasise the "methods that students use to access key content (e.g., independent reading, partner reading, text on tape, text with images, listening comprehension, online research, communication with experts, group demonstrations, small group instruction) rather than change the content itself" (p. 15). Since the meeting was intended to be teacher-directed, and I expected that I could meet with each of these seven teachers within a week's time, I decided to engage with each teacher regarding what it might mean to "differentiate content" during the individual interviews.

Unfortunately, Claudia's next individual interview did not occur until Week 6. Therefore, I anticipated the need to remain attentive to subtle changes in her understandings of differentiated instruction and Edulastic to include "providing all students with problems of appropriate complexity, or DOK-level" throughout Week 5. Furthermore, I anticipated her Week 6 individual interview to focus on attempts to expand Claudia's understandings of DOK and Edulastic as a means "to provide all students with a variety of ways to access content."

Claudia completed two short Google Doc journal entries during Weeks 3 and 4 (one each week), which were two more entries than she had completed during Weeks 1 and 2. Claudia's Week 3 entry indicated that she "forgot" to post anything during Week 1 and had spent too much time thinking about her school's move to hybrid teaching and learning (which was to begin in two weeks) to write anything for Week 2. The remainder of Claudia's Week 3 entry and all of her Week 4 entry discussed her disappointment in the number of students who had done no work for most of the quarter and the challenges she was having contacting these students' parents. Claudia indicated that she had emailed and called home several times, left messages, and had her administrators call home and leave messages, but few parents had returned these messages or made any form of contact. These entries made it clear to me that Claudia's focus was not necessarily on ways to differentiate content, but on a much larger issue—how to get many of her students to complete any work. Therefore, I attempted to develop a future conversation with participants experiencing similar issues to discuss potential solutions and ways to mitigate the stress this situation was causing Claudia.

**Weeks 5 and 6 of the study**

Claudia's Week 5 journal entry highlighted her struggles with attempting to coordinate the use of Illustrative Mathematics problems with all her students and the Zoom breakout rooms (as requested during her Week 3 individual interview), particularly in a hybrid environment. Claudia stated that it was "not fair to pair students who had been doing their work with those who had completed nothing" in the Zoom breakout rooms and that "too many of [her] students were unable to understand the problems, because they were so far behind." Claudia stated that she "tried to use the [Illustrative Mathematics] problems with Zoom for three days but it was just too much, so [she] stopped." The fact that Claudia was not only trying to coordinate in-class and

remote students using Zoom, but also attempting to pair students in ways that supported all students' engagements with the Illustrative Mathematics problems makes it easy to understand why Claudia felt overwhelmed. As described in Courtney (2010), there are several forms of accommodation, not all of which involve the modification of schemes. One form of accommodation involves the avoidance of the situation. For example, Claudia, when confronted with an experience that caused perturbation (i.e., coordinating novel uses for Zoom and Illustrative Mathematics in a hybrid environment), may simply have decided to "give up" and continue to operate as she had been.

Unfortunately, examination of the instrumentalisation and instrumentation processes were not possible from these activities. Furthermore, although this initial attempt to promote generalising assimilation to produce changes in Claudia's understandings of differentiated instruction, Illustrative Mathematics, and Zoom was unsuccessful, it did highlight one particular struggle that Claudia (and several other teachers) experienced in their attempts to productively differentiate instruction throughout the study. Specifically, the reality that many students simply "disappeared" during the pandemic. Although these students were still enrolled in class (and school) they consistently failed to log into Zoom sessions or complete any assignment, and teachers' and administrators' attempts to reach out to these students and their families had resulted in little to no success. Such circumstances had negative consequences for teachers, who constantly worried about how to help these students and their families. Although this issue was not a focus of the study presented here, the extent of the problem across grade levels and schools suggests it should be examined and addressed in the near future.

During Claudia's Week 6 individual interview, I assured her that since her attempts to coordinate the use of Illustrative Mathematics problems with all her students during Zoom breakout sessions were constrained by so many of her students' lack of participation, that we would place this activity on hold and focus on differentiating content. To motivate Claudia to think differently about differentiating content, she was asked to select and utilise two or more resources that students could use to access content, and to provide students with opportunities to choose how content was accessed. Resources for accessing course content that Claudia was provided with and asked to select from included her course text; alternative texts (e.g., CK-12 free online textbooks); Khan Academy, Edulastic, existing pre-recorded videos (e.g., Teaching Channel) or videos that she would need to create herself (using free screencasting and video editing software such as Screencast-O-Matic); and small group video-conferencing sessions (using the Zoom video-communications app). Furthermore, Claudia was provided with resources to motivate her students to think about and reflect on relevant vocabulary through the construction of word walls, Frayer Models (Frayer et al., 1969), or math dictionaries or glossaries. Claudia was asked to explore these resources and select at least two to implement for three to five weeks. These interventions were intended to promote integration of the notion of "accessing content" into Claudia's understanding of what it means to differentiate content. Furthermore, over time, the intent was for such integration to support changes to Claudia's stable understandings of differentiated instruction (i.e., "to provide problems of two complexity levels so that all students stay busy and complete work in the same amount of time") to include more generalising or encompassing meanings (i.e., "ways for all students to access content").

Claudia made two Week 6 journal entries. These entries were short and focused on her concerns regarding how unacceptable her students' first quarter grades were going be, her hope that school administrators would provide her and her colleagues with some direction on how to deal with students not turning in assignments and the state end-of-year tests, and her desire to meet with other Algebra 1 teachers in the study—to get a sense for the issues they were dealing with and potential solutions.

**Weeks 7 and 8 of the study**

During Week 7, all grades 9-12 teachers were asked to view Rose's (2013) Sonoma County TEDx video on the "Myth of Average" and read Tomlinson and Jarvis' (2006) article "Teaching Beyond the Book." Teachers were asked to be prepared to discuss the video and article during a Week 8 online group meeting. Claudia made one Week 7 journal entry—an entry made before her Week 7 individual interview and before she had watched the Rose video and read the Tomlinson and Jarvis article—and focused on her selections for providing her students with ways to access content. Claudia indicated that she would provide the following resources for her students to access content: problems from the course textbook, topic-appropriate links to Khan Academy videos for students that might need extra help, and a classroom word wall to focus on important terminology.

During Claudia's Week 7 individual interview, I prompted her to articulate in more detail her selections for providing students with ways to access content, and how she envisioned her students (and herself) using these resources. Claudia stated that she would assign problems from the course text to all her students, since each student had a copy of the textbook and all of her examples came from the text; provide links to Khan Academy's pre-recorded course videos, because some of her students might benefit from "someone else's explanations"; and a word wall, since about 70% of her class was now in-person as part of her school's hybrid format. The broad focus of the discussion during Claudia's Week 7 individual interview was to reinforce the notion that learning goals should remain the same for almost all of Claudia's students, but that content could be changed for some students if required on their individualised education programs (Tomlinson & Imbeau, 2010). Furthermore, for students who were behind—due to lack of participation during remote instruction—some prerequisite content might need to be addressed. Claudia and I also discussed ways she could meet her students' needs while using the resources she had selected (i.e., course textbook, Khan Academy, word wall). Claudia stated that, since many (about 30%) of her students were still remote, an in-class word wall would be of no benefit to them. Therefore, I discussed and provided Claudia with a link to a free Google Slides interactive mathematics notebook template that she could use as an interactive glossary or vocabulary list with both her in-person and remote students.

Regarding Khan Academy, Claudia and I discussed four ways she could employ the resource. She could provide links to video of content she had already covered to those students who had been remote and fallen behind their classmates but were now in-person. These students would be assigned specific assignments that were connected to content standards they had failed to engage with, with the expectation they complete this work during class and at a pace that would allow them to be "level" with their classmates by the start of the third quarter (at the latest). For those students who were "on pace" with her instruction and in-person, Claudia could continue to utilise her textbook, but also provide links to Khan Academy videos and problems for students who needed or desired additional support. For those students who were "on pace" with her instruction and participating remotely using Zoom, Claudia could continue to utilise her textbook, but also provide links to Khan Academy videos and problems for students who needed or desired additional support or preferred using Khan Academy over their textbook. Finally, for those students who had fallen behind and remained remote, Claudia could assign Khan Academy videos and assignments that were connected to content standards they had failed to engage with. This final group comprised the students Claudia was most concerned about—those that had "disappeared" during the pandemic. Whether or not these students caught up to their classmates was somewhat out of Claudia's hands and required intervention by school administrators, counsellors, and social workers, if available. Although this four-fold strategy would require Claudia to attend to in-person students who were behind and those who were "current" with their assignments both in-person and remote, she felt this plan was more manageable than her

current way of operating and would provide more "support to each student and meet them where they are in the curriculum." This strategy also provided some students with choices for how they accessed the content, an idea that I intended for Claudia to reflect on over the requested three to five weeks of implementation.

During Week 8, Claudia participated in an online meeting with six other grades 9-12 teachers. Although the group-meeting was teacher-directed, I hoped that teachers would discuss and connect Tomlinson and Jarvis' (2006) assertion that "typical is not a synonym for all . . . [and] teaching to the 'typical' student doesn't work" (p. 17) with Rose's (2013) argument that there is no average learner, that individuals learn and develop in distinctive ways. The intent of this action was to provoke Claudia and several of her colleagues to move away from their attempts to meet the needs of an "average" or "typical" student, and toward providing opportunities to maximise each student's learning capacity (Tomlinson & Imbeau, 2010). Although teachers did not make an explicit connection between the article and video, even with subtle nudges from me, the majority of the discussion was productive and focused on Rose's (2013) assertion that you cannot apply a one-dimensional approach to complex "jagged" things like learning—that there is no average student. The video appeared to impact all seven teachers and provoked several (including Claudia) to assert their need to re-think "their instruction to really focus on students as individuals." This discussion anticipated a subsequent examination to ascertain whether any of these teachers had made generalising assimilations to their understanding of differentiated instruction to include "attending to individual learning differences and needs."

Claudia's Week 8 journal entry occurred the day after the online group meeting. Claudia indicated that the study's activities had been adding to her stress level over the past several weeks. Specifically, her attempts to utilise Illustrative Mathematics, Zoom, and Edulastic; the request that she watch the Rose video and read the Tomlinson and Jarvis article; and the request that she select and utilise at least two resources for students to access content was just too much to do with everything else that was going on in her practice. After reading Claudia's Week 8 journal, I realised that Claudia had agreed to differentiate content using depth of knowledge and Edulastic throughout Weeks 5-9, where her focus was on differentiating the complexity of problems she assigned to students; and using Khan Academy, her course textbook, and Google Slides interactive notebooks, where her focus was on providing her students with variety in how they access content throughout Weeks 7-11. Although Claudia did not mention the overlap of these two activities, I realised that I had inadvertently asked her to focus on differentiating content using two different strategies during Weeks 7-9. Unfortunately, this error in organisation on my part, caused Claudia to attempt to coordinate content complexity and access simultaneously—actions that contradicted my intent to provoke small changes in her assimilatory structure sequentially.

Claudia's Week 8 journal entry also indicated that her remote students were at least two to three weeks behind her in-person students, and that many students who had remained remote all year had basically done no school work. Claudia stated that although the Week 8 online group meeting had been interesting and made her want to focus on individual student learning needs, her current situation would not allow it. This entry made me inquire as to when I could meet individually with Claudia to see what support I could provide to her or if I could relieve some of the burdens that might be attributed to the study. Unfortunately, Claudia's next and final individual interview did not occur until Week 12.

**Weeks 9 and 10 of the study**

During Week 9 of the study, Claudia participated with two other Algebra 1 teachers and one Algebra 2 teacher in an online meeting to discuss issues and ideas involving these two courses. Three of these teachers, including Claudia, were teaching in hybrid environments and the sole Algebra 2 teacher was currently teaching fully in person. Part of this teacher-directed meeting

focused on resources that each teacher found productive for teaching Algebra 1 or Algebra 2. One Algebra 1 teacher, with about 40% of his students working remotely and the remaining 60% in-person, discussed his use of Desmos Classroom Activities. This online resource allowed him to assign activities to in-class and remote students simultaneously (individually or in pairs) and observe his students' work in real-time on the Desmos teaching dashboard. According to this teacher, the Desmos Classroom Activities allowed students to share ideas and stay connected, regardless of their location. Although Claudia indicated that this resource seemed "interesting and useful," I made certain to assert that teachers should only consider implementing Desmos Classroom Activities in lieu of, not in addition to, one or more of the other resources they were currently utilising as part of the study. This assertion was made for fear of Claudia, and other teachers, viewing the Desmos Classroom Activities as yet another resource to manage in what could be an already overwhelming experience. Fortunately, the vast majority of the meeting was spent discussing issues teachers were experiencing, including the problem with students failing to do work and missing what were considered mandatory remote classes. I believe this discussion was worthwhile and somewhat "cathartic" for teachers. Teachers also discussed their thoughts on whether the state would require mandatory end-of-course tests this academic year or cancel them as they did the prior year due to the pandemic. All four teachers indicated problems similar to those characterised by Claudia. Therefore, this meeting allowed each teacher to see that what they were struggling with was a problem across schools and districts.

Claudia did not provide a journal entry during Week 9. Claudia's Week 10 entry, which occurred two days after the online meeting, focused on the relief she felt knowing that other teachers were dealing with the same or very similar problems. Claudia indicated that using Khan Academy had forced her to accept the fact that many of her students were at "different places" in the curriculum and needed different support. Furthermore, Claudia indicated that trying to force all of her students to work at the same pace, while also attempting to differentiate content by assigning different Edulastic depth of knowledge problems was "exhausting and did nothing to support [her] students." Claudia stated that she had stopped using Edulastic to differentiate problem complexity (i.e., DOK) during Week 9 and did not intend to start up again. Finally, although Claudia never really started using the Google Slides interactive math notebooks with her students, she described her current attempts to differentiate instruction as "providing ways for all of her students to receive the support they need."

**Weeks 11 and 12 of the study**

Due to the timing of the study, Week 11 fell during the Thanksgiving holiday break in the United States, where participating teachers had most or all of the week off from school. As such, there were no individual interviews or online group meetings, and Claudia did not post a Google Doc entry during Week 11. Furthermore, Claudia's school went back to a fully remote environment when classes began after the Thanksgiving break.

During Week 12, Claudia participated in an online group meeting with five other teachers, all who initially selected content as their focus element. Five of the six participants (including Claudia) had participated in a similar meeting during Week 4. Four of the six Week 12 participants focused on differentiating content during Weeks 5-7, whereas Claudia and one other Week 12 meeting participant decided to focus on differentiating content during Weeks 5-9. Furthermore, all six Week 12 participants continued this focus, each with some variations or modifications, for the remainder of the study. The main focus of the Week 12 meeting was on teachers' attempts to differentiate the complexity of problems they assigned to students using Edulastic. Claudia asserted that she had stopped this activity during Week 9 and had begun to differentiate how her students' access content, using her textbook and Khan Academy. Claudia was the only Week 12 meeting participant to have made such a major shift in how she differentiated content, and, unfortunately, she did not get much time to share her experiences.

Claudia participated in her last individual interview two days after the group meeting. Claudia stated that, with her classes now fully remote until at least the third quarter, she was providing all her students with the choice of working with Khan Academy, the course textbook and synchronous Zoom instruction, or both. Claudia indicated that she would do what she could to get those students who had fallen behind earlier in the year "caught up" by the third quarter, using a combination of Khan Academy and periodic online Zoom instruction, but that she could "only do so much." Claudia asserted that, although her participation in the study had been extremely challenging at times, she believed her current focus on providing students with opportunities to access content in more than one way was beneficial to both her students and herself.

Claudia identified her course textbook, her document camera, the Zoom video conferencing app, and Khan Academy as her "go to" set of resources. When pressed to say more about each resource, Claudia stated that her textbook was "useful for presenting sample problems, since each student had a copy at home" but that she was using it less frequently because Khan Academy had "useful instructional videos" that students could watch and review any time and a "nice set of sample problems that are connected to video tutorials, hints, and practice problems." Therefore, Claudia was providing students with variation in how content was accessed and support that they could access independent of Claudia's availability. When asked about her document camera, Claudia indicated that she still utilised it "to present important content, so students could watch, in real time, how to solve sample textbook problems and ask questions" but that she was beginning to use Khan Academy in a similar but more student-driven way. Finally, Claudia asserted her intent to re-introduce Edulastic, with some focus on DOK, at some point prior to the end of the third quarter. Claudia stated that her main reason to utilise Edulastic and include some degree of DOK was to support her students in preparing for the state end-of-year tests should such testing occur.

# Discussion

In this section, I address the three research questions. Although the questions are asked broadly, in terms of the 18 participating teachers, the focus will be on addressing these questions in the case of Claudia.

## *Teachers' Understandings of Differentiated Instruction*

As illustrated in this report, Claudia's understanding of differentiated instruction was modified and refined as Claudia interacted with me, her colleagues, and the study's activities. As indicated in Table 4, Claudia's stable understanding of differentiated instruction (at Week 4) was "to provide two types of problems (average and more challenging problems) and provide struggling students with extra support or extended time so that all students stay busy and complete work in the same amount of time." Using Thompson and Harel's definitions (Table 2; Thompson et al., 2014) the space of inferences available to Claudia from this understanding were limited to providing her students with problems that focused on the time it took them to complete. Therefore, Claudia did not focus on providing all students with opportunities to engage with challenging mathematics but focused on the time it took students to complete problems. By the end of the study, Week 12, Claudia's understanding for differentiated instruction incorporated "providing opportunities for students to access content in more than one way." In addition, Claudia's Week 12 understanding for differentiated instruction had moved away from a focus on the "average" student and toward attending to individual learning differences and the needs of all her students (Rose et al., 2013; Tomlinson, 2014).

Although these models could have been more nuanced, had it been possible to observe Claudia interacting with her students, planning for her lessons, and interacting with her school colleagues, this was not possible due to coronavirus restrictions. In addition, I was unable to determine whether Claudia's Week 12 understanding was stable or the result of functional accommodations and thus specific to that moment in time and easily lost once Claudia's attention moves on (Moore & Thompson, 2015; Steffe, 1991; Thompson et al., 2014). Furthermore, even if Claudia had transformed her stable understanding for differentiated instruction, whether this understanding would "revert back" to a more limited focus when full, unrestricted in-person instruction resumes is a question that cannot be answered with the data corpus.

## *Teachers' Understanding of Resources Utilised to Support Differentiated Instruction*

At the beginning of the study, the main resource Claudia utilised to differentiate instruction was the Illustrative Mathematics website, although she indicated that she had not used the resource during the pandemic. Claudia's set of resources also included her course textbook, digital camera, Zoom web conferencing app, and Edulastic assessment platform. By the end of the study, Claudia had also utilised Webb's depth of knowledge, Khan Academy, and Google Slides interactive journals, although only Khan Academy had been integrated into her set of resources.

The study also attempted to modify Claudia's understandings of Edulastic, Illustrative Mathematics, and Zoom by provoking her to utilise each resource in novel ways (Gueudet et al., 2012; Trouche et al., 2018b; Trouche, Gueudet et al., 2019). For example, the study attempted to modify Claudia's understanding for the Zoom web conferencing app ("for students to synchronously observe instruction, and to ask and answer students' questions about instruction") to include "ways for students to collaborate" but this change in her assimilatory structure was not successful. In addition, attempts to modify Claudia's understanding for Edulastic ("to provide online assessments to students, grade assessments, and provide feedback") to include "providing all students with problems of appropriate DOK-level" was also not successful—although Claudia indicated her intent to re-introduce Edulastic, and DOK to a minimal degree, prior to the end of the third quarter. Conversely, Claudia's understanding of her textbook ("where to find problems each day for examples and assignments") was modified to "one of the places to find examples and problems for [her] students to engage with"; thus, allowing for student choice in accessing content (Tomlinson & Imbeau, 2010). Unfortunately, the data was such that determining whether attempts to modify these understandings were dependent on the process of instrumentalisation (Gueudet et al., 2012; Gueudet, Pepin, Courtney et al., 2021; Trouche et al., 2018b), or due to reasons unrelated to her understanding of differentiated instruction (e.g., student's "disappearing" during the pandemic) were not possible. Finally, as with Claudia's understanding for differentiated instruction, whether her understanding for the course textbook and inclusion of Khan Academy would "regress" to a more restricted focus when full, unrestricted in-person instruction resumes could not be answered with the existing data.

## *Designing Interventions through a Focus on Teachers' Understandings*

As demonstrated in this report, designing sequential interventions based on models of learners' (e.g., teachers', students') understandings to impact these understandings in productive ways is a complex endeavour (Frank, 2017; Hackenberg, 2010; Simon et al., 2018; Tillema & Gatza, 2017; Tunç-Pekkan, 2007). The study resulted in interventions that were successful at modifying Claudia's understandings, and those that were constrained to support changes to her meanings. The capacity for activities to promote sequential changes in teachers' meanings via accommodations or generalising assimilations was not only dependent on the activities

themselves, but also on the number and frequency of these activities. In addition, the potential for activities to promote sequential changes in teachers' meanings was dependent on the viability of the models of teachers' understandings that supported the design of activities, and the number and frequency of each type of activity (e.g., individual interviews, online group meetings) was dependent on teachers' availability and internet access. For example, there were at least three online group meetings that were postponed due to issues with several participants (including my own) internet. Finally, my error in keeping track of the sequence of all of Claudia's activities, an error that I did not recognise throughout Weeks 7-9, might have short-circuited the potential for changes in her assimilatory structure.

## Conclusion

As demonstrated here, synthesising and integrating (locally) the documentational approach and a theory of meanings not only supported the development of viable models of Claudia's understandings of differentiated instruction and the resources she uses to support differentiation, but also the design of interventions with the potential to provoke subtle changes in her assimilatory structures. Such an integrated framework shows potential in supporting the design of productive professional learning experiences for both pre-service and in-service teachers. Future research, particularly research free from pandemic restrictions, should attempt more nuanced models of teachers' understandings for differentiated instruction and the resources they utilise to support differentiation. In addition, future research should explore the impact co-teaching experiences (i.e., content and intervention specialists teaching concurrently) have of teachers' meanings. Although the focus of this study and the case of Claudia were on teachers' meanings for differentiated instruction and supporting resources, future research should utilise the framework to explore teachers' meanings for other cognitive and affective schemes.

The coronavirus pandemic forced teachers to try new things to provide engaging lessons that incorporated exciting technology and resources for their students. Attempts to incorporate new technologies pushed many teachers out of their comfort zones, making them learners alongside their students. Throughout the pandemic, attempts to manage dynamic teaching and learning environments and adapt instruction to meet the needs of all students has been a challenge for many teachers. The findings presented here highlight the importance of teachers' meanings in their attempts to differentiate instruction and the role digital resources play in supporting or hindering such practices. Study limitations included the small sample size, the lack of student interviews, and the inability to observe instruction (either in-person or online), observe teachers as they prepare for instruction, interview teachers face-to-face in their own environments, and observe in-person meetings with colleagues. Finally, this study adds to a growing body of research on teacher's work with and on digital resources as a means to support differentiation, particularly as remote and hybrid teaching and learning continues throughout the U.S. and across the globe.

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