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Editorial

What Knowledge is Needed for Effective Teaching of Mathematics?

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Each of the papers in this volume of *Mathematics Teacher Education and Development* addresses the question of what types of knowledge are needed for effective teaching of mathematics. There is an assumed relationship between teacher knowledge and student achievement – although this relationship is difficult to demonstrate, possibly because teacher knowledge is itself notoriously difficult to measure. However, teacher knowledge comes under the spotlight at times like the present when there is shortage of qualified teachers in subject areas such as mathematics. For example, the recent report of the Australian Council of Deans of Science on the preparation of mathematics teachers in Australia (Harris & Jensz, 2006) found that three in four schools had difficulty in recruiting suitably qualified teachers for mathematics classes, and many teachers had studied no mathematics teaching methods, including one third of those who taught only junior or middle school. The 2007 survey of beginning teachers conducted by the Australian Secondary Principals' Association found that significant proportions (up to 30%) of respondents were teaching outside their area of expertise, and mathematics was one of the subject areas in which this was most often the case. While mathematics teacher shortages may lead to calls for a greater emphasis on subject matter knowledge in pre-service programs as a compensatory measure, or to testing of pre-service teachers' subject matter knowledge before they are allowed to graduate, the knowledge base for teaching is much more complex and multi-faceted than this.

Over 20 years ago, Shulman (1987) suggested seven categories of knowledge that might make up the knowledge base for the teaching profession, including content knowledge, pedagogical knowledge, pedagogical content knowledge (PCK), curriculum knowledge, and knowledge of students, educational contexts, and the purposes of education. In mathematics education there has been much interest in the nature of pedagogical content knowledge (e.g., Chick, Baker, Pham, & Cheng, 2006) and the relationship between PCK and content or subject matter knowledge. For example, Deborah Ball and her colleagues have introduced the notion of Mathematical Knowledge for Teaching (MKT) to demonstrate this relationship (Hill, Ball, & Schilling 2008). They argue that subject matter knowledge comprises common content knowledge (knowledge used by teachers in ways in common with how it is used in other occupations that use mathematics), specialised content knowledge (mathematical knowledge that allows teachers to engage in particular teaching tasks), and knowledge at the mathematical horizon (knowledge that allows teachers to see how to link

concepts they are currently teaching to students' future mathematical learning). According to these researchers, pedagogical content knowledge is made up of knowledge of content and students, knowledge of content and teaching, and knowledge of curriculum.

Teacher professional associations have also been active in articulating the specialised knowledge base for teaching of particular subjects. The Australian Association of Mathematics Teachers' *Standards for Excellence in Teaching Mathematics in Australian Schools* (2006) identify three elements of professional knowledge required for best teaching practice: knowledge of students, knowledge of mathematics, and knowledge of students' learning of mathematics. Thus researchers and practitioners alike acknowledge the complexity of teacher knowledge and its significance for effective teaching.

The four papers in this volume report on research involving pre-service and practising teachers of mathematics in early childhood, primary, and secondary school contexts. Tunç-Pekkan and D'Ambrosio worked with primary pre-service teachers in their mathematics content course to investigate how they applied their mathematical knowledge in email communications with sixth grade children. In the emails the pre-service teachers and children exchanged ideas about mathematical problems they had been working on during class that week. The researchers were interested to discover how the pre-service teachers would construct the "voice of the children" (displaying knowledge of how children learned mathematics) and how flexibly they would use the "voice of the discipline" (mathematical knowledge) in these conversations. From analysis of the emails, it appeared that very few of the pre-service participants attempted to understand the children's mathematical thinking, with most simply asking procedural questions or commenting on the children's work in ways that revealed their own inadequate knowledge of mathematics. This analysis allowed the researchers to plot the position of each pre-service teacher on a two-dimensional set of axes representing the respective dispositions to use the child's voice and the voice of the discipline. Such a representation may be useful to teacher educators seeking to develop pre-service teachers' subject matter knowledge and knowledge of how students learn mathematics.

The focus of the research carried out by Afamasaga-Fuata'i was on developing secondary pre-service teachers' pedagogical content knowledge via vee diagrams – a way of representing the relationship between what is known and what needs to be known or understood, for example, in solving a mathematical problem. Participants in her mathematics methods course were instructed in the use of such representations. They then completed an assessment task using a vee diagram intended to assist them in designing teaching approaches for solving a given problem. Analysis of pre-service teachers' responses to the task revealed differences in the quality of their pedagogical content knowledge, in terms of their ability to identify mathematical concepts from the problem statement, prior knowledge that students needed to possess in order to solve the problem, and a range of solution methods. Vee diagrams proved to be a useful tool for investigating the extent to which pre-service teachers were able to "unpack" mathematical ideas for teaching.

Warren's research with three pairs of Year 1 teachers was concerned with simultaneously developing their mathematical and pedagogical knowledge for effectively teaching the Patterns and Algebra strand in the new Queensland mathematics syllabus. Her paper describes how she implemented a cyclic professional development model that guided the novice learners (teacher participants) towards becoming experts in teaching this new and unfamiliar material to young children. Key elements of the model were the sensitive scaffolding provided by the expert guide (the researcher) and the collaborative planning and sharing of feedback on lessons amongst the whole group as a community of practice. There was evidence that the positive changes in teacher knowledge and confidence achieved in the study were sustained long after the professional development intervention ended.

A teacher's relationship with mathematics shapes their cognitive and affective responses to the subject and also the way they design learning experiences. Namukasa, Gadanidis and Cordy argue in their paper that many teachers have formed negative and unproductive views about mathematics as a result of their own "cold" experiences of learning mathematics. They describe how they implemented a therapeutic intervention that engaged a large cohort (over 400) of primary pre-service teachers in "warm mathematics". This involved offering them problem-based tasks for exploration in pairs or small groups in a large lecture room, eliciting and sharing solution strategies with the whole group, and requiring written reflection on what they had learned. This approach helped the pre-service teachers to discover alternatives to traditional ways of doing and teaching mathematics while re-learning school mathematics subject matter.

Together these papers remind us of the breadth, depth and complexity of the knowledge required for effective teaching of mathematics, and they highlight the challenges for teacher educators in developing the necessary professional knowledge base in the pre-service and practising teachers with whom they work.

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