Teachers' Content-related Learning Processes: Teachers' Use of Professional Development Content on Teaching Approaches to Inclusive Mathematics Education

Jennifer Bertram
University of Duisburg-Essen

Katrin Rolka
Ruhr University Bochum

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The knowledge regarding teacher learning processes is fundamental to organising a professional development (PD) programme systematically and effectively. In the study reported in this article, we examined content-related teacher learning processes in the context of inclusive mathematics teaching. We first explain approaches to inclusive mathematics teaching (derived from the literature) and how these approaches can help teachers to differentiate with open-ended and challenging tasks. While focusing on the transfer of learning as one important part of teacher learning processes, the study investigated teachers' use of the PD content (i.e., the teaching approaches) after a PD session. For this, a case vignette as a method of inquiry was used. We analysed the written answers from 15 secondary teachers using qualitative content analysis. The teachers used many approaches to inclusive mathematics teaching and applied them in the context of the case vignette. Finally, we focus on implications for the design of PD programmes concerning teacher learning about inclusive mathematics teaching.

Keywords · teacher learning processes · professional development · inclusive mathematics teaching · open-ended and challenging tasks · transfer of learning

Introduction

Inclusion is one of the current challenges in the educational system and increases the need for subject-specific teaching development and corresponding arrangements for teacher professionalisation, for example, in the form of professional development (PD) programmes. To organise a PD programme systematically and effectively, the knowledge regarding individual learning processes of teachers participating in a PD programme is fundamental (Prediger et al., 2017). Especially, the focus should not be exclusively on the effectiveness of a programme but also on how it works to promote teachers' learning (Goldsmith et al., 2014). In addition to the request for more research on learning processes, Prediger, Rösken-Winter, and Leuders (2019) recommended investigating teacher learning for specific PD content because this elucidates which support teachers need for developing content-related knowledge. Therefore, we investigated teachers' content-related learning processes, i.e., teacher learning processes as they participate in a PD programme on inclusive mathematics teaching.

In this paper, we are interested in the teachers' use of the PD content on teaching approaches to inclusive mathematics education, more precisely their application of previously learned knowledge on these approaches in a new but similar situation. This so-called *transfer of learning* is one of three categories Schunk (2012) used to describe learning. As will be shown, the investigation of the transfer of learning allowed the acquisition of fruitful insights into teachers' content-related learning and thus allowed the derivation of implications for the organisation of a systematic and effective PD programme.

These considerations are based on the understanding of *inclusion* as a way of teaching and less as an ideology, whereupon teaching approaches for engaging all students in mathematics education are especially relevant (Roos, 2019). We focus on students who attend a regular school in inclusive settings, which means that teachers encounter "extremely heterogeneous groups in classrooms, so that a high degree of differentiation is needed" (Scherer, 2019a, p. 4680). Thereby, the goal of differentiation is "to improve learning for all students" (Russo et al., 2021, p. 1). In the context of inclusion, subject-specific and joint learning of all students are of particular importance (Jung & Schütte, 2017). The use of challenging tasks to differentiate is of particular interest because it allows the engagement of all students in mathematical learning (e.g., Bobis et al., 2021). Pulling these threads together, teaching approaches to inclusive mathematics education can be used to differentiate with open-ended and challenging tasks and should be part of PD programmes. After the PD, the teachers should be able to use the PD content to prepare tasks for differentiation, whereby this usage is understood as a transfer of learning; therefore, an important part of teachers' content-related learning processes. First in this paper, an overview of the topic content-related learning processes is provided, where upon the teaching approaches to inclusive mathematics education and aspects on transfer of learning within PD programmes are of special interest. Afterward, the methods for data collection and data analysis are presented, while concentrating on the task to which the teachers should apply their knowledge. The results contribute to understanding teacher learning within a PD programme on inclusive mathematics teaching. In the end, the possibilities for supporting teacher learning in a PD programme and the limitations of the study are discussed.

Literature Review

Following the recommendation to investigate teacher learning processes in a content-related way (Prediger, Rösken-Winter, & Leuders, 2019), we start with some general aspects on content-related learning processes in PD programmes. Instead of focusing on rather generic descriptions (less topic-specific) of what is important when conducting PD, a content-related focus concentrates on more content-specific approaches (Prediger, Rösken-Winter, & Leuders, 2019). Desimone (2009), for example, described a focus on content as one critical feature of PD. A connection between subject matter knowledge and how students can learn that knowledge can lead to "increases in teacher knowledge and skills, improvements in practice, and, to a more limited extent, increases in student achievement" (p. 184). Content-relatedness in this article refers to the topic of inclusive mathematics teaching and is described in the next section.

Content-related Teacher Learning Processes in PD Programmes

In the literature, different research approaches for investigating teacher learning processes in inclusive mathematics contexts exist. For example, one approach used knowledge (and other competence) domains, such as pedagogical content knowledge, to analyse the statements of teachers in the context of a PD programme on inclusive mathematics teaching (Bertram, 2022). Within this approach, the model of professional competence for teachers (Baumert & Kunter, 2013) was further developed for inclusive mathematics teaching (Bertram et al., 2020) to analyse teacher learning processes in a PD programme. By these means changes in the focus on beliefs or specific aspects of knowledge throughout the PD programme were identified. Another approach analysed the expertise of teachers by investigating the situational demands of inclusive mathematics teaching (Prediger, Kuhl et al., 2019). The following four situational demands were identified: the need to identify the demands of students, the need to set differentiated priorities for students, the need to support students adaptively, and the need to arrange joint learning situations. The existing work of these authors focuses especially on specifying what teachers should learn and which categories teachers use in response to the situational demands of inclusive mathematics teaching. According to Prediger, Kuhl et al. (2019), their future work is to focus even more on teacher learning processes.

Regarding a PD programme on language-responsive mathematics classrooms, Prediger (2019a) described another approach to investigate and promote the content-related learning processes of teachers by analysing one PD activity in detail. She exemplified her procedure in the context of a PD programme with the aim of "SUPPORTING students' language and IDENTIFYING the language demands in mathematically relevant activities" (Prediger, 2019a, p. 387, author emphasis). We pursued this idea of focusing on one PD activity because it offered the opportunity to investigate content-related learning processes in depth. Further, it was necessary to specify the PD content to analyse in detail what teachers need to know about the PD content (Prediger, 2019b). This specification is based upon the current state of research (Prediger, 2019b) and in this paper, it refers to the teaching approaches to inclusive mathematics education.

Teaching Approaches to Inclusive Mathematics Education

In general, a PD programme on inclusive mathematics teaching should help teachers to deal with heterogenous groups in inclusive settings. Possible central topics for PD programmes on inclusive mathematics teaching are, for example, diagnosis and support or differentiation (Scherer, 2019b). To specify the PD content as a necessary step for investigating content-related learning processes and based on the understanding of inclusive mathematics teaching explained earlier, we concentrated on teaching approaches that can be assigned to individual learning and joint learning (left of Figure 1). Considering the recent literature on inclusive mathematics teaching, we identified seven teaching approaches to inclusive mathematics education (right of Figure 1). These approaches are explained in detail because they are the basis of what teachers should learn within a PD programme on inclusive mathematics teaching.

Teaching approaches to individual learning in inclusive mathematics classrooms	• Access for all students • Differentiated learning opportunities • Action-oriented learning • Individual Support
Teaching approaches to joint learning in inclusive mathematics classrooms	• Joint participation on a common subject • Meaningful context for all students • Co-operative methods of instruction

Figure 1. Teaching approaches to inclusive mathematics education.

The first approach, *access for all students,* is about providing opportunities for access to a mathematical topic (e.g., using and combining multiple representations; cf. Knipping et al., 2017) so that all students are involved and can participate according to their abilities (e.g., using open problems; cf. Scherer 2017). The second approach, *differentiated learning opportunities,* includes different learning goals or different levels of difficulty, with the aim that each student works at his or her level of understanding using appropriate learning environments (cf. Schöttler & Häsel-Weide, 2017). Additionally, student demands are identified and differentiated priorities are set for students (cf. Prediger, Kuhl et al., 2019), consolidating basic competencies (cf. Häsel-Weide & Nührenbörger, 2017). Differentiation in the sense of offering more time, more or fewer tasks, worksheets at different levels of difficulty, etc. (cf. Scherer, 2017) are features of this teaching approach. The third approach, *action-oriented learning,* focuses on the idea that learning is based on practical experiences (e.g., with manipulatives) with the inherent possibility to gain a deeper mathematical understanding (more than just working practically; cf. Knipping et al., 2017) and that multiple sensory channels are used (cf. Häsel-Weide & Nührenbörger, 2017). The fourth approach, *individual support,* takes into account that students receive individual support appropriate to their individual needs (support students adaptively; cf. Prediger, Kuhl et al., 2019) and the support of students is guided by diagnosis of learning needs (cf. Häsel-Weide & Nührenbörger, 2017).

Further teaching approaches to inclusive mathematics education concretise the idea of joint learning. The fifth approach, *joint participation on a common subject,* is about enabling all students to participate and work on a common subject, where the common subject refers to a substantial mathematical idea (cf. Schöttler & Häsel-Weide, 2017). Teachers arrange joint learning situations (cf. Prediger, Kuhl et al., 2019) and the learning environments facilitate participation, collaboration, and mathematical discourse, while communication is essential for acquiring new mathematical knowledge (cf. Schöttler & Häsel-Weide, 2017). Co-operative settings also stimulate the exchange of mathematical findings (cf. Häsel-Weide & Nührenbörger, 2017). The sixth approach, *meaningful context for all students,* focuses on the use of tasks that are embedded in a realistic and motivational context (i.e., a story that offers a realistic situation; cf. Scherer, 2017). which entails the promotion of interest in the situation (cf. Bikner-Ahsbahs & Große Kamphake, 2016). The seventh and last approach, *co-operative methods of instruction,* takes into account methods of instruction that allow students to work together and individually. It connects to the idea of diversity in instruction methods and social-form changes (i.e., working alone or in a group; cf. Knipping et al., 2017).

This collection of teaching approaches is not exhaustive. Moreover, the identified teaching approaches to inclusive mathematics education may have overlaps and may especially have connections to each other. If, for example, an open problem with individual access options for all students is used that is embedded in a motivational context and the students work at their levels of understanding with different learning goals, the approaches *access for all students, differentiated learning opportunities* and *meaningful context for all students* are addressed.

Differentiation with Open-ended and Challenging Tasks in Inclusive Mathematics Classrooms

In this section, we explain how the use of open-ended and challenging tasks has a high potential for learning mathematics in inclusive settings and how they are connected to the teaching approaches described. Open-ended tasks allow students to solve a problem at their individual level and multiple strategies for solving the problem can be used. Open-ended tasks in inclusive mathematics teaching are considered a "powerful approach for addressing the diversity of inclusive mathematics classrooms due to their potential for natural differentiation" (Buró & Prediger, 2019, p. 4636). The underlying idea of natural differentiation concerning open-ended tasks can be explained in the following way:

> Natural differentiation means that the learning environment provided is substantial and complex and offers multiple ways of learning and multiple strategies for solving a given problem: the students can choose their level of working by *themselves*, work on several levels of the task and be successful at their level rather than being assessed against one that is predetermined. (Scherer et al., 2016, p. 641)

Open-ended tasks in inclusive mathematics teaching satisfy many of the teaching approaches mentioned to individual and joint learning in inclusive mathematics classrooms (Figure 1). For example, students can work on their level (*differentiated learning opportunities*) and the learning environment might unveil the opportunity to focus on a common subject (*joint participation on a common subject*). Differentiation with challenging tasks aims at maximising mathematics learning opportunities in heterogenous classrooms. Sullivan et al. (2006) describe three teacher actions for the use of challenging tasks: the use of open-ended tasks, preparing prompts, and posing extension tasks to quick students. The use of open-ended tasks and posing extension tasks can be seen as part of the above explicated teaching approach *differentiated learning opportunities.* Students with learning difficulties sometimes need even more support to solve an open-ended task (Rolka & Albersmann, 2019); they may need step-by-step prompts to solve these tasks successively. The prompts may refer to external structures or methodological frameworks that must still offer the opportunity for cognitive activation (i.e., the task especially remains cognitively challenging; Rolka & Albersmann, 2019). Therefore, the tasks should not be broken down into the smallest parts because a view of the whole must still be possible (Scherer, 1995). Additionally, it is important that the prompts allow the students to access the task without being restricted to a single way for the task solution (cf. Sullivan et al., 2006; part of teaching approaches *access for all students* and *individual support*).

The idea of combining individual and joint learning with differentiation was summarised by Buró and Prediger (2019) as follows: "Inclusive mathematics classrooms call for differentiated instruction with joint whole-class experiences and specific support for students with special needs" (p. 4636). The authors focused on the categories and practices that teachers activated in working with open-ended tasks in inclusive settings. Based on the literature for differentiating in inclusive mathematics classrooms, Buró and Prediger (2019) identified three demands placed on teachers when using open-ended tasks: analysing the task with a focus on the students' possible learning pathways, unfolding the differentiating potential of the task, and planning specific support for students with special needs. One of their results indicated that teachers often unfolded the differentiating potential of an open-ended task, but the support for students with special needs is not always focused on the intended goal of the task (Buró & Prediger, 2019). Currently, the literature emphasises that teachers need to know the content, how to differentiate with challenging tasks, how these tasks can be designed to be open-ended and how students can be supported by prompts (e.g., Bobis et al., 2021; Mellroth et al., 2021).

Transfer of Learning within PD Programmes

In our study, we focused on how the participants of a PD programme *transfer* the teaching approaches to a new task after they have completed a PD activity related to these teaching approaches. This section presents some general aspects concerning the transfer of learning as one part of the learning processes and specifies it in relation to the case vignette in our context.

Transfer of learning in general means that knowledge is "applied in new ways, in new situations, or in familiar situations with different content" (Schunk, 2012, p. 317)—the application of content beyond the original context in which it was learned (Goldstone & Day, 2012). The transfer of learning is a "core feature" of all learning because learning without transfer tends to be inefficient (Goldstone & Day, 2012) and "situationally specific" (Schunk, 2012, p. 24). The concept of transfer "also explains the effect of prior learning on new learning" (Schunk, 2012, p. 24). In this study the transfer of learning is discussed in the specific context of PD programmes, which can be connected to the effectiveness of a PD programme (e.g., Agyei & Voogt, 2014; Göb, 2018; McDonald, 2011).

For Schunk (2012), the transfer of learning includes "cueing retrieval" and "generalizability." Cueing retrieval refers to a learner's reception of "cues signaling that previous knowledge is applicable in that situation" (Schunk, 2012, p. 222). Generalizability is "enhanced by providing learners the opportunity to practice skills with different content and under different circumstances" (Schunk, 2012, p. 222). Goldstone and Day (2012) have pointed out the importance of investigating how learners transfer their knowledge and how educators can improve this transfer. A PD programme can be considered an experience that promotes teacher changes (Arbaugh & Brown, 2005) and initiates learning processes which are connected to the transfer of learning. Thus, the knowledge on teachers' content-related learning processes and particularly on transfer of learning can be used as a meaningful starting point for designing and implementing PD programmes that support teacher learning.

In the context of transfer, questions such as *What is transferred?* and *To where is it transferred?* arise (e.g., Barnett & Ceci, 2002). In this paper, the question *What is transferred?* refers to the PD content (i.e., the teaching approaches to inclusive mathematics education discussed in connection with the so-called Zoo task, which is an open-ended and challenging task to compare and calculate areas). The question, *To where is it transferred?* refers to the task in which teachers applied the knowledge learned in the PD programme (i.e., a similar task to the one focused on in the PD programme, dealing with calculating volume—the so-called House task), which can be considered a case vignette.

Research Question

In this article, we are interested in teachers' content-related learning processes and especially in the application of the learned PD content in a different context. The analysis of teachers' use of the PD content offers the opportunity to derive implications how teacher learning about inclusive mathematics teaching can be promoted. The PD content was about teaching approaches to inclusive mathematics education, considering heterogenous groups in classrooms through focusing on approaches to individual and joint learning for all students. Because open-ended and challenging tasks satisfy many of the approaches to individual and joint learning in inclusive mathematics classrooms, our analysis focused on answering the following research question:

Which teaching approaches to inclusive mathematics education (PD content) do the teachers use when elaborating an open-ended and challenging task in order to meet all students' needs?

In the analysis, we not only examined which teaching approaches were used, but also examined how the approaches were used in a way that aligned with the aims of the PD programme concerning differentiating with open-ended and challenging tasks.

Methods

In this section, we present some descriptive background information on the PD programme in general and the specific PD activity that focused on the open-ended and challenging task and which was used to concretise the teaching approaches to inclusive mathematics education. Next, the methods for data collection especially the task for investigating the transfer of learning, and data analysis are described.

The PD Programme

The PD programme "Mathematics & Inclusion" was a project lasting two years (2017-2019), addressing mathematics teachers and special education teachers in secondary schools in Germany. Various aspects that are of special interest in inclusive mathematics classrooms were considered in five modules, for example, inclusion in school and lesson development, teaching design and learning, diagnosis of learning difficulties and individual support, training and exercising, and implementation at the schools. The PD programme was conducted by 12 facilitators. Half of the facilitators were educated for supporting inclusive education and school development processes. The others were especially educated for mathematics teaching and lesson development. About 25 teachers from 10 different schools participated in the PD programme and each team of teachers from a school was accompanied by two facilitators. At the beginning of the project, the teachers were either working in inclusive mathematics classes in Grade five (approximately 10-year-old students) or in Grade seven (approximately 12-year-old students).

One main topic of the PD programme were the teaching approaches to inclusive mathematics education. The facilitators described these along three guiding questions (Figure 2). These guiding questions served as a structural framework throughout the whole PD programme and were illustrated and discussed in detail at different points over the two years. The guiding questions connect to the mentioned teaching approaches in various ways, which is marked with arrows (Figure 2). These connections are not the only ones possible, but we focused on naming those that were especially considered in the PD programme regarding the single questions. These connections also show how the guiding questions had been understood and used in the PD programme.

❖ Has planning considered the reduction of barriers to learning and participation for all? →	teaching approaches to individual learning: *access for all students, differentiated learning opportunities*
❖ Does the teaching unit include partner/group and individual work as well as work with the whole class? →	teaching approaches to individual learning: *individual support;* teaching approaches to joint learning: *joint participation on a common subject, co-operative methods of instruction;*
❖ Can every learner work in a cognitively challenging way on a common subject? →	teaching approaches to individual learning: *access for all students, differentiated learning opportunities, individual support;* teaching approaches to joint learning: *joint participation on a common subject*

Figure 2. Guiding questions in the PD programme and how they connect to the teaching approaches.

The Specific PD Activity (Zoo Task)

For this article, we consider a small part of the PD programme where the facilitators worked with the teachers on an activity on the Zoo task. Overall, this task served as an exemplar for the facilitators to specify and illustrate the teaching approaches to inclusive mathematics education. Focusing on one PD activity allowed an in-depth study of the content-related teacher learning in the PD programme.

The Zoo task (cf. Holzäpfel et al., 2011) was intended to introduce the topic of area and perimeter and the difference between them. First, the teachers examined a zoo plan with various animal enclosures that had different forms and sizes, which led to a discussion on how much space the different animals had. The teachers then considered the following question: With regard to the guiding questions in Figure 2, what would be suitable assignments for the students based on this plan? After collecting, discussing, and reflecting on some of the ideas of the teachers, the facilitators presented and discussed possible assignments with the participants. How these assignments could be used for the conceptualisation of the term "area" in inclusive mathematics classrooms was considered. To begin with, the facilitators proposed that the students individually work on the following assignment: "Choose two enclosures and compare them. Think about how you will make this comparison. Write down your thoughts. You can also paint into the zoo plan." The students would then work in pairs with the instructions described in Figure 3.

a) Tell each other about your thoughts.
b) Take a large zoo plan from the teacher's desk and start comparing two enclosures that you have agreed on (scissors, glue, help cards, and other copies or materials can be found at the front of the desk).
c) Present your method on a poster so that your classmates can understand it without further explanation. Invent a name for the method.

Figure 3. Further assignments for students, discussed in the PD activity.

As an optional further task for the students, the facilitators suggested that the students should carry out this assignment: "When you have finished comparing two enclosures, add more enclosures. Write down your results and describe how you came to your solution." Table 1 describes how the PD content for the different teaching approaches to inclusive mathematics education (Figure 1) is specified and concretised based on the Zoo task. These teaching approaches do have connections and overlaps. For example, the simplification of the task for ensuring different levels of demands (*differentiated learning opportunities*) can also be treated as an option for making the task accessible to all students (*access for all students*).

Table 1

Analysis of the Zoo task according to the teaching approaches to inclusive mathematics education

Approaches	Analysis
Access for all students	Students develop strategies for comparing the enclosure sizes according to their previous knowledge and choose different approaches to compare areas (e.g., estimating, cutting up areas and re-positioning them, laying out the areas with platelets and counting them, supplementing or disassembling into known areas). Students can use different supporting elements: material support (e.g., larger zoo plan, scissors, and cut-out enclosure), step-by-step prompts (e.g., help cards to differentiate between the area and perimeter) or prompts especially enabling mathematical understanding (e.g., square platelets, slides with centimetre boxes). The comparison of different approaches and the request for a written recording of the solution can stimulate the usage and linkage of the representations.
Differentiated learning opportunities	The openness of the task follows the idea of natural differentiation as students can compare the areas in individual ways (the different shapes of the enclosures result in various levels of demands for the area comparison). Some students may need to deepen their understanding of the terms "length" and "width" before understanding the multiplicative structures in terms of area (differentiated learning goals). By structuring the task (e.g., finding a way to compare areas, adding more enclosures to the comparison, and finding a ranking of all enclosure sizes) different levels of demands can be considered.
Action-oriented learning	Regarding action-oriented learning in a subject-specific manner, the students can make the transition from laying out the surfaces with platelets (action-oriented approach) to understanding surface area ("length times width" with multiplicative structures).
Individual support	Individual support can be implemented in the following way: weaker students practice multiplicative structures, ensuring that the basis of understanding is secured regarding the calculation of surface area. Individual support can also mean that stronger students are encouraged to investigate challenging forms of enclosures.
Joint participation on a common subject	The common subject in this task is given by the subject-specific central idea of surface area. The students work on a common task, which involves activities of measuring and comparing areas. Through the exchange in partner work, different approaches and solution strategies are reflected (social participation). The students present their results to others, which supports participation and communication.
Meaningful context for all students	The task of comparing areas is embedded in a meaningful context that arouses interest through the "zoo" context. It is introduced via the question "How much space do the animals need?"
Co-operative methods of instruction	Learning together also means choosing appropriate social forms (e.g., partner or group work) that enable exchange. In the sense of methods for instruction, an individual work phase (finding a strategy) was followed by a partner work phase (comparing strategies). The created posters provide the basis for an exchange with the whole class.

Figure 4 illustrates and summarises the specific PD activity as well as the investigation of the transfer of learning. The specific PD activity on the Zoo task is found on the left side. The teachers then applied the teaching approaches to inclusive mathematics education to another open-ended and challenging task in a case vignette on the House task (right side), which is going to be explained next.

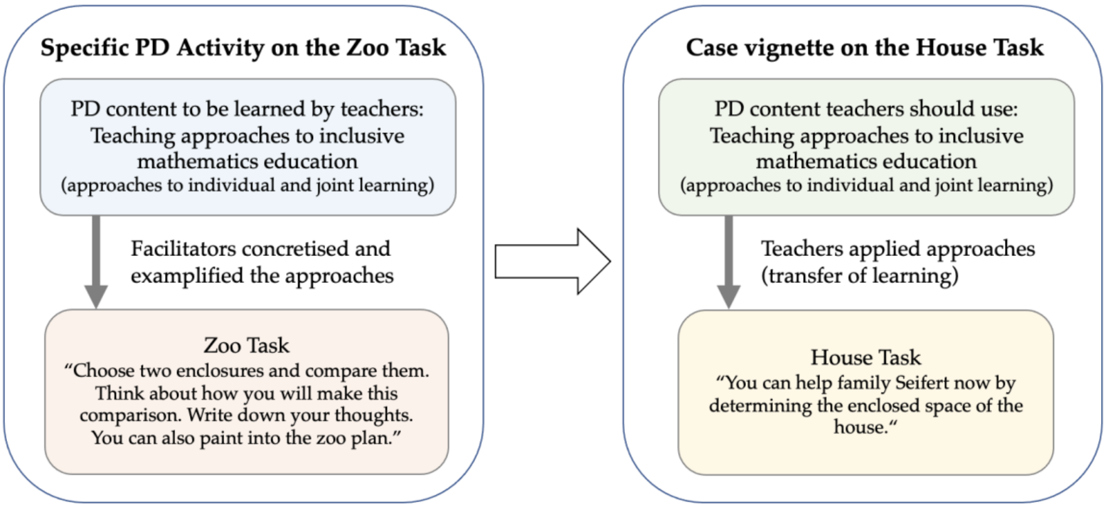

Figure 4. Overview of the specific PD activity (left side) and the investigation of the transfer of learning (right side).

Data Collection: The Task for Investigating the Transfer of Learning (House Task)

We used a case vignette as method for investigating the teachers' learning about inclusive mathematics teaching. Case vignettes are representations of self-contained cases that normally originate from everyday teaching or learning situations, combined with tasks that request the processing or analysis of the case vignette (von Aufschnaiter et al., 2017). In our case vignette, the teachers had to discuss the implementation of the House task (see Figure 6) in an inclusive mathematics class against the background of the content dealt with in the PD programme. The case vignette addressed a task on a situation of planning and aimed at supporting a fictitious teacher who wants to use the House task in an inclusive mathematics class. Employing this case vignette, the application of the PD content was evoked. Therefore, it could be considered as an opportunity for the transfer of learning of the PD content.

Figure 5 illustrates the first part of the developed case vignette, consisting of some background information on the planning situation. Figure 6 depicts the House task itself as the second part of the case vignette. Based on that, it was intended that the teachers would discuss the guiding questions distinctive for the whole PD programme for the House task: Has planning considered the reduction of barriers to learning and participation for all? Does the teaching unit include partner/group and individual work as well as work with the whole class? Can every learner work in a cognitively challenging way on a common subject?

In addition to the **teaching approaches to inclusive mathematics education**, in Module 2 you have become acquainted with an exemplary design of a series of lessons on the subject of "areas and surface area" and a corresponding introductory lesson, which follow precisely these approaches.

Mrs. Müller, a maths teacher of an inclusive Realschule Plus, has already had good experiences with the zoo task in grades 5 and 7. Now she discovered the task "enclosed space of a single family house". She sees a lot of potential in the task and is now working on designing a learning situation in such a way that the principles of inclusive teaching are taken into account. Please support Mrs. Müller and answer the corresponding guiding questions for the concretely given task.

Key data on Mrs. Müller's class:
- Learning group: 24 students in class 9a, three of them with special needs of learning
- Learning prerequisites: basic understanding of the volume of cuboids
- Subject of the learning unit: volume of bodies that are not cuboids

Figure 5. Case vignette part one.

Task "enclosed space of a single family house"
The family Seifert is looking for a new home. They are interested in the house on the photo[1]. Besides the living space and the room layout, they are interested in the energy costs. For the energy costs, the enclosed space of the house is important. You can help family Seifert now by determining the enclosed space of the house.

[1]The photo is a picture of a single family house. From a mathematical point of view, this house consists of prisms.

Figure 6. Case vignette part two—House task (cf. Hußmann et al., 2015, p. 234).

We used cues to signal that previous knowledge from the PD was applicable in that situation particularly in the first sentence (Figure 5). Within the case vignette, we considered some characteristics of the PD programme (a reference to Module 2) and of the participating teachers (in Germany, there exist different types of secondary schools, and we selected one type of school—a Realschule Plus—familiar to most of the participating teachers).

The case vignette also contained information about the classroom situation the teachers should think of, for example, the size of the class and the common subject are mentioned (see Figure 5). Regarding the topic "area" in the PD itself, we decided to choose the topic "volume" for the open-ended task, which was to be discussed in terms of the teaching approaches to inclusive mathematics education in the case vignette. The decision to use this topic was based on considerations about the parallels that can be drawn between the Zoo task and House task against the background of the PD content (Table 2). Consequently, aspects of the PD content, which could be assigned to the House task, are characterised in Table 2. It is important to mention that the teaching approaches to inclusive mathematics education can be applied to each open-ended and challenging task. For this project, one task with thematic parallels to the original task used for analysing the transfer of learning in a new but similar situation was chosen. These parallels demonstrate that the teaching approaches to inclusive mathematics education can be implemented in the transfer task in a way that is similar to the task in the PD programme. This offered us the opportunity to analyse the teachers' application of the learned PD content. We also asked the teachers for their self-perceived connections between the House task solution and PD content to assist us to interpret the applied PD content in the sense of the transfer of learning.

Table 2
Parallels between the Zoo and House tasks against the background of the PD content

Zoo task	House task
Strategies to compare the enclosure sizes (e.g., estimating, cutting up areas and re-positioning them, supplementing or disassembling into known areas)	Strategies to determine the volume (e.g., estimating, dividing the house, and putting it together again, filling the house with a unit of their choice, with sand/water or centimetre cubes, supplementing or disassembling into known fields)
Supporting materials for area comparison (e.g., larger zoo plan, scissors, cut-out enclosure areas, help cards, especially for differentiating between area and perimeter, square platelets)	Supporting materials for determining the volume (e.g., a tangible and detachable model of the house, centimetre cubes, sketches instead of a photo of the house, help cards, especially for differentiating between area and volume)
Practicing/securing multiplicative structures for a deeper understanding concerning the calculation of surface areas	Practicing/securing multiplicative structures for a deeper understanding concerning the determination of volume; reference back to the area content calculation
Simplification of tasks (e.g., from finding a way to compare areas to finding a ranking of enclosure sizes)	Simplification of tasks (e.g., transferring to the calculation of cuboids/triple prisms; finding a method to decompose the house into known fields)
"Zoo" context and introduction via the question, "How much space do the animals have?"	"Family Seifert and the house" context (introduction via a closer connection to the student environment is also possible)
Working alone (finding a strategy), working with a partner (comparing strategies), creating posters for presenting different strategies	Working alone (finding ways to divide/complete the house), working with a partner (comparing strategies), and presenting to others
Common subject: comparison of areas and surface areas; Common task: activities for measuring surface area and comparing areas	Common subject: comparison of rooms/fields and volume; Common task: activities for measuring volume and comparing fields
Reflect on different accesses and solutions	Reflect on different accesses and solutions

As generalizability means that different content is needed to demonstrate transfer, some further differences and similarities are explained. Similarities between the tasks were: both are tasks from the topic of geometry, they have connections to the PD content on open-ended and challenging tasks in inclusive mathematics classrooms, they have a similar degree of openness on the student level and similar adaptions for using the tasks in inclusive mathematics classrooms are possible. Differences between the tasks were: the Zoo task deals with areas, the House task with volume, the complexity of the task on student level is different (and different prior knowledge is needed) and the associated activities on the teacher level are different (the solution of the Zoo task in the PD programme is in oral form and in co-operation with other teachers and facilitators, whereby the House task is in written form and the teachers work on it alone after a PD session).

Data Analysis

Using qualitative content analysis (Mayring, 2014), we analysed the written answers to the House task of 15 participants (those who worked on the transfer task, the other participants of the PD programme did not respond to the case vignette). For the data analysis, we used the described teaching approaches to inclusive mathematics education as deductive categories (*access for all students, differentiated learning opportunities, action-oriented learning,* and *individual support* regarding individual learning and *joint participation on a common subject, meaningful context for all students,* and *co-operative methods of instruction* concerning joint learning). The above-mentioned explanations serve as definitions for these seven categories. For the coding, we used MAXQDA as software. Only a few words counted as a coding unit (i. e. smallest part that can be coded) because the teachers partly answered in notes (e.g., "specify lengths"). A context unit (i. e. information that can be used for interpretating a statement) consisted of the whole answer of one teacher to one of the guiding questions. The teachers' answers to the questions, which refer to the self-perceived connection between the solution of the House task and the PD content, were also analysed using the seven categories in the abovementioned manner. Additionally, we coded whether the teachers explicitly mentioned (the PD activity on) the Zoo task.

After viewing all the data material once, we started with the coding using the seven mentioned categories. We coded one teacher after another. In the next step, we reviewed the material again, category by category, and, using Table 2, examined which elements of the teachers' answers could be considered an application of the learned PD content. Aspects of the teachers' answers that could not be assigned to the PD content included further ideas for using the House task in inclusive mathematics classrooms. Because the teaching approaches have overlaps some of the teachers' answers were assigned to more than one category.

Results

To answer the research question "Which teaching approaches to inclusive mathematics education (PD content) do the teachers use when elaborating an open-ended and challenging task in order to meet all students' needs?" we illustrate some ideas formulated by the teachers. Based on the teachers' written answers, we present how they applied the teaching approaches to inclusive mathematics education to the House task, separately for each category. We focused on the transferred aspects and therefore, on the PD content which could be identified. The mentioned similarities and differences between the two tasks were also considered and we comment on the teachers' adaptations while referring to the central literature on differentiating with open-ended and challenging tasks.

Teaching Approaches for Individual Learning

Regarding the teaching approaches to individual learning, the teachers especially noted the following ideas on implementing the House task in an inclusive mathematics classroom.

Access for all students

The teachers described various concrete approaches to enable all students to access the task. For example, one teacher formulated with respect to the House task: "It can be rebuilt, drawn, estimated, and calculated. So, everyone should be able to participate in solving the task." Additionally, teachers indicated different approaches to determine the volume in the context of rebuilding the house. Some of these approaches could be assigned directly to ideas from the PD programme (e.g., working with centimetre cubes and, respectively, with centimetre platelets in the case of the Zoo task). Others were specific to the topic of volume (e.g., filling experiments).

Moreover, the teachers referred to students' necessary previous knowledge and suggested disassembling the house in such a way that known fields could be used to calculate the enclosed space.

The consideration of students' previous knowledge could also be regarded as a transfer. However, these considerations were more distinctive regarding the House task than the Zoo task, maybe because more previous knowledge was required to solve the House task.

The teachers also proposed the idea of providing supportive elements to the students. This could be considered a parallel between the House and Zoo tasks, although the concrete material differed because of the volume calculation instead of area measurement (e.g., sand and water for filling experiments). After a closer examination, the idea of using supportive material in terms of action-oriented learning (see also the results in the corresponding category) was considered a transfer. The particularities of volume calculation and the use of supportive material can also be discussed concerning the different access possibilities. For example, using centimetre cubes directly offers the possibility to derive formulas for the volume of fields, such as prisms, in contrast to using nonstandardised units for volume calculation. These possibilities reveal a variety of different ways to access this task and regardless of which particular way was used, one teacher stated: "everyone can go deep into the task according to his/her abilities".

To make the task accessible to all students, teachers also emphasised that new or unclear words must be clarified (e.g., what is meant by "enclosed space"). The idea of adding further information to the House task (e.g., measurements within the picture) to make it accessible to all students was not mentioned in terms of the Zoo task. The use of alternative (simpler) house forms and working with sketches or models was also suggested. While working with sketches seems to be of greater importance when calculating in three dimensions instead of two dimensions, the idea of using simpler forms has also been discussed in the PD activity concerning the concrete forms of animal enclosures (i. e., comparing different squares may be easier than comparing circles and parallelograms). In summary, the teachers used various approaches to make the House task accessible to all students. Some approaches had an explicit connection to the Zoo task and were interpreted as a transfer, whereas others had no explicit connection and focus on the particularities of the volume calculation.

Differentiated learning opportunities

The teachers indicated various aspects of the task that could be used for differentiation in the classroom. For example, regarding the learning content, the form of the house can be varied, and the volume can be determined in different ways. Some teachers distinguished between stronger and weaker students, which can be interpreted as a differentiation according to student performance, such as the following example provided by one teacher: "In the case of very weak students, it may be enough for them to be able to name the individual fields with confidence in different situations." In this context, it was found that the teachers focused on different learning goals with respect to different students. For example, one goal might be the estimation of the enclosed space, another the determination of the volume formula for prisms or even just being sure about how to use names of fields correctly. As the mentioned example reveals, some teachers also considered other learning goals while focusing on weaker students.

But not every learning goal the teachers focused on seemed to fit the intended learning pathway by students concerning volume calculation. For example, one teacher wrote: "A differentiation is possible by having stronger students build the house without a template and weaker students assemble the house from a construction sheet". Using a template or not might help students to rebuild the house, but it is not obvious in how far these different approaches for rebuilding the house can be interpreted as prompts that help students to determine the volume with respect to their individual learning pathway. The consideration of different levels of performance and various learning goals can be seen as parallels to the Zoo task. For example, the teachers also discussed (within the PD activity) whether the learning goal for students is solely the comparison of areas or the determination of formulas for different areas.

Action-oriented learning

Some teachers intensively dealt with the reconstruction of the house and viewed the action-oriented learning approach as the (only) possibility to determine the volume for some students. Other teachers started with an action-oriented learning approach and pursued this idea further until the volume and volume formulas were available. One teacher described the following procedure:

> The students can see that cuboids alone are not enough to build the house. If they have all the components together, they can try to combine them into a single cuboid or into several cuboids to trace back to what is known. By filling prisms with water and then determining their volume, they can develop ideas for the volume formula of prisms. By determining the two-dimensional area of the front surface, they can simplify the determination of the total volume computationally.

This aspect of action-oriented learning can be analysed further depending on the chosen action (e.g., reconstructing a house and filling it with water or disassembling a model of the house into individual known fields). From a subject-specific perspective, the teachers often considered the use of both non standardised and standardised units (see also further explanations above within the category *access for all students*). This was also an idea that was discussed in the PD activity on the Zoo task.

Individual support

With a special focus on lower-performing students, one teacher described that such students might experience the determination of the volume with the help of a filling experiment (see the example presented within the category *action-oriented learning*). However, other teachers described additional/other tasks for higher-performing students (e.g., discussing questions regarding the accuracy of the model, such as considering the thickness of the walls). Regarding the students' level of performance (higher or lower), the teachers used ideas out of the discussion in the PD activity on the Zoo task. Whereas a few teachers referred to the area content calculation (as a simpler component that is the basis for the volume calculation), they did not explicitly write about practicing multiplicative structures (which could have been another possibility for transferred PD content). Both the area content calculation and practicing multiplicative structures offer opportunities for the consideration of the previous knowledge of the students.

Teaching Approaches for Joint Learning

Regarding the teaching approaches to joint learning, the teachers indicated the following ideas on implementing the House task in an inclusive mathematics classroom.

Joint participation on a common subject

Some teachers explicitly chose the same task for all students (measuring volume and comparing fields) and then explained that different solutions on the common subject could be presented in class. Regarding the aim of having students learning together, one teacher wrote: "The important thing is that students get to know, compare, and evaluate different approaches." The reflection of different access and solutions within groups of students, as part of the discussion on the Zoo task in the PD activity, was understood as a transfer in this context.

Meaningful context for all students

Some teachers addressed the context (the enclosed space of a family's house) and added further motivational ideas, such as discussing the volume of the house in connection to the topic of saving energy. Therefore, the teachers used the idea that joint learning is embedded within an interesting context for the students (as it had been considered in the PD activity). Other teachers suggested a slight adaption of the task itself: Instead of directly asking for the enclosed space of the house, the students could collect ideas about which questions could be relevant while looking at the house, for example, how many rooms does the family need or how big are these rooms. The teachers noted that this might be an even more interesting context for motivating the calculation of volumes than the energy costs.

Co-operative methods of instruction

The teachers usually mentioned different social forms (e.g. individual, partner, or group work) and combined them with different phases within a lesson (e.g., exploring or exercising). Individual work was often indicated for working in an exercise phase or only for higher-performing students. In connection to the elaboration of the task, references were often made to partner or group work, with a subsequent presentation to the whole class. While the teachers used different social forms, as had been discussed in the PD activity, some differences in the concrete implementation could be identified. Individual work in the context of the Zoo task was discussed concerning individually different access opportunities for comparing areas, but concerning the House task, the teachers often described the abovementioned opportunities of social forms for phases of exercising and less often mentioned the idea of think-pair-share. However, partner and group work were implemented in a comparable way to the discussed options in the PD activity. Some teachers also referred to the different possibilities for grouping the students, either homogeneously or heterogeneously.

We used the teachers' answers to the question regarding their self-perceived connection between the case vignette and the PD content to generate support for our interpretation. These answers support the findings because half of the teachers explicitly indicated the PD activity on the Zoo task, which helped them to work on the House task. For example, the following comparison was mentioned: "Different area forms can now be different rooms of the house with the addition that the height of the rooms is considered. Instead of centimetre platelets, centimetre cubes can now be used for enactive work." The teachers also noted general ideas of inclusive mathematics teaching, which were discussed in the PD activity, for example: "The students work on the common subject at different levels or with other supportive materials."

Discussion

In this article, we investigated teachers' transfer of learning (i.e., their application of learned PD content to a case vignette). The PD content, which was classified as *what is transferred* (following Barnett & Ceci, 2002), consisted of teaching approaches to inclusive mathematics education, which were discussed in a PD activity using an open-ended and challenging task (Zoo task). The case vignette, which was classified as *to where is it transferred* (following Barnett & Ceci, 2002), comprised of another similar task (House task), where the teachers were given the opportunity to apply the learned teaching approaches.

In summary, the teachers used the teaching approaches to inclusive mathematics education (*access for all students*, *differentiated learning opportunities*, *action-oriented learning*, *individual support*, *joint participation on a common subject*, *meaningful context for all students*, and *co-operative methods of instruction*) to answer the guiding questions on the House task. In every category, elements of the PD content concerning the Zoo task were identified. With a focus on the category *access for all students*, additional aspects of volume calculation played a key role. While using the PD content and thereby the teaching approaches to inclusive mathematics education, the teachers met the demand of analysing the task with a focus on students' potential learning pathways and especially the demand of unfolding the task's differentiating potential, in terms of Buró and Prediger (2019). Most teachers did not explicitly address the demand of planning specific support for students with the help of prompts. One possible reason is that the guiding questions and therefore the case vignette did not ask the teachers to consider this point in more detail. It may have been helpful to address the teaching approach *individual support* even more in the PD itself and on the Zoo task as well as in the case vignette for the application of the previously learned PD content.

In the next section, we first discuss the implications of the results concerning the design of PD programmes and the support of teacher learning about inclusive mathematics teaching. Afterward, we concentrate on the limitations of the study and the issues that could be investigated in future work.

Implications of the Results

The teachers used the PD content on teaching approaches to inclusive mathematics education within the case vignette in various ways; therefore, the transfer of learning was considered to be successful. This is of special importance because facilitators want participants to be able to apply the learned PD content not only in the exact way it was discussed in a PD activity but also concerning other tasks and topics to meet the needs of heterogeneous groups.

First, it seems to be helpful to discuss the teaching approaches to inclusive mathematics education regarding a concrete task within a PD programme because the teachers explicitly indicated that the PD activity on the Zoo task helped them work on the case vignette. Nonetheless, the teaching approaches to inclusive mathematics education must be deeply understood, not just their exemplification in the application of the Zoo task. If, for example, some teachers had considered the House task without paying attention to the particularities of the volume calculation, it would have been helpful to discuss another open-ended and challenging task in the PD programme to deepen the understanding of the teaching approaches that concern not just one task or topic.

The teachers' answers often focused on making the task *accessible for all students*. They mentioned the idea of using tasks that allow every student to learn on her or his level of understanding while still being a challenging task for every student. This is in line with the idea of low-floor-high-ceiling tasks (e.g. Bobis et al., 2021; Mellroth et al., 2021) and was part of the discussion in the PD activity. Next to the use of such open-ended and challenging tasks, the teachers described *differentiated learning opportunities*, for example, in reference to students' performance. Because it was not clear how far the teachers' focused learning goal for the students fitted to the intended learning pathway (which is a parallel to Buró and Prediger's (2019) result) concerning volume calculation, this hurdle should be a central part in conducting further PD programmes on inclusive mathematics education.

Regarding the category *action-oriented learning*, the teachers often focused on the reconstruction of the house. As the teachers described the use of supportive materials for working on the House task, parallels to the discussion of using supportive materials for working on the Zoo task in the PD activity could be made. In this context, the question "How sustainable are the chosen approaches concerning individual students' preconditions?" arose. It seemed necessary to deepen the subject-specific interpretation of action-oriented learning (cf. Häsel-Weide & Nührenbörger, 2017; cf. Knipping et al., 2017) in the PD activity, based on the idea that all students should be engaged in subject-specific learning as well as in cognitively challenging tasks to deepen their mathematical understanding. Additionally, it was important that the teachers focused on a subject-specific learning goal. Working with manipulatives can motivate students, but we aimed to make the mathematical content accessible for all students with manipulatives and wanted to enable a deeper mathematical understanding. As Mellroth et al. (2021) point out, teachers must consider that no important aspects of the task are lost when adding manipulative materials for making the task accessible.

Concerning the possibilities of *individual support* explicitly addressed by the teachers, giving individual prompts (cf. Sullivan et al., 2006) to the students could have been displayed in more depth by the teachers. This could also be dealt with in PD programmes on inclusive mathematics education.

Mostly, the teachers had fruitful ideas about how all students could access the task, but regarding the *joint participation on a common subject* (cf. Häsel-Weide & Nührenbörger, 2017; Schöttler & Häsel-Weide, 2017), we suggest it could be helpful to discuss within a PD programme how an individual goal for the students could be reached while everybody works on a common task in more detail (cf. Prediger, Kuhl et al., 2019). Especially, this individual goal should be one with relevance for learning mathematics and not just a kind of superficial goal like "pupils are involved in rebuilding the house without focusing on volume calculation or naming fields." Within the category *co-operative methods of instruction*, some teachers referred to using group work either in homogenous or heterogenous groups. Maybe the teachers had reflected about different advantages or disadvantages of ability grouping (cf. Russo et al., 2021), but this had not become obvious within their answers. This is another point that should be addressed in PD programmes on inclusive mathematics education.

Limitations and Outlook

There are some limitations of the study, and we connect these limitations to ideas for future research. First, we only focused on one task for investigating teachers' use of the PD content in this article. Using another transfer task would have been helpful to better understand which teaching approaches to inclusive mathematics education could be applied, even if it was not linked to the initial task on so many points (e.g., using a geometry task that does not focus on the measurement of area or volume or an algebra task). Future research could consider different case vignettes on different topics of the PD programme, and it would be of special interest to investigate which similarities and differences could be found in the different tasks of transfer and the application of knowledge regarding the teaching approaches to inclusive mathematics education.

Second, for the interpretation of the aspects that the teachers use in a situation of planning (such as a case vignette) that are also of importance in their instructional practice, it would be interesting to focus even more on the teachers' actions in classrooms. Transfer of learning could then be investigated in terms of the question "Which elements of the PD content concerning the use of an open-ended and challenging task in inclusive mathematics classrooms can be identified in the instructional practice of the teachers?" so that classroom observation data could be added to the written answers to the case vignette.

Moreover, we cannot say how the teachers would have worked on the House task without knowing about the Zoo task, so this is another limitation of our study. Especially, if the PD programme is repeated, the teaching approaches to inclusive mathematics education could also be discussed without a concrete example, such as the Zoo task, to determine how the teachers use the teaching approaches to inclusive mathematics education within a transfer task.

In addition to these ideas, based on the literature on transfer of learning, it might also be interesting to compare the transfer of mathematics teachers and teachers for students with special needs because teachers in both professions were participants in the considered PD. Differences and similarities in their answers could be analysed, and the results could be used to derive implications for designing PD programmes that address both professions. The positive views on the PD content seem to be a predictor for the transfer of learning (Agyei & Voogt 2014); thus, it could be interesting to search for connections between teachers' views on inclusive mathematics teaching and their use of PD content when learning about inclusive mathematics teaching.

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Authors

Jennifer Bertram
University of Duisburg-Essen
Faculty of Mathematics, WSC O 2.44
Thea-Leymann-Straße 9
45127 Essen, Germany
email: jennifer.bertram@uni-due.de

Katrin Rolka
Ruhr University Bochum
Faculty of Mathematics, IB 1/131
Universitätsstraße 150
44801 Bochum, Germany
email: katrin.rolka@rub.de