

## Secondary Pre-service Teachers' Use of Vee Diagrams to Analyse Problems and Illustrate Multiple Solutions

Karoline Afamasaga-Fuata'i  
*University of New England*

This article presents data from a group of secondary pre-service teachers who used vee diagrams to illustrate their conceptual and methodological analyses of a problem, as part of a required mathematics methods course assignment. The individually constructed vee diagrams were analysed in terms of the extent to which they displayed principles and main concepts, adequately justified the mathematics applied in their multiple methods, and the appropriateness of the methods for early secondary level. Constructing vee diagrams prompted the pre-service teachers to repackage their own understanding of the mathematics embedded in the problem context, by organising the material in ways that were pedagogically meaningful, developmentally sound, and flexibly arranged to meet a range of ability levels. The vee diagrams' overall ratings indicate the degree to which pre-service teachers differentially achieved their intentions, conceptually and methodologically. Three exemplars provide further insights into the nature of responses that collectively differentiate between vee diagram types. Findings suggest vee diagrams provide a useful tool to illustrate the pre-service teachers' pedagogical intentions, understanding and interpretations of the mathematics contextualised in a problem.

*Standards for excellence in teaching mathematics in Australian schools* (Australian Association of Mathematics Teachers (AAMT), 2006) identified excellence in teaching mathematics as possessing: a rich knowledge of how students' conceptual understanding could be effectively developed; and the ability to plan for coherently organised learning experiences involving substantive mathematics (essential principles) to enable the development of students' new mathematical understandings, and effective application of these understandings to problem solving. Hence, assessment tasks for pre-service (PS) teachers should invite them to undertake tasks as "opportunities to unpack mathematical ideas or to make connections" (Ball, & Wilson 1990, cited in Mewborn 2001, p. 33) to assist in the development of their pedagogical content knowledge, skills and understanding. Therefore, PS teachers' assessment tasks should engage them in in-depth analyses of substantive mathematics, and in illustrating results in interconnected, meaningful organizations to purposefully develop future students' conceptual understanding.

This article presents data from a task used in a junior secondary mathematics methods course to assess the PS teachers' deep understanding of a mathematics problem. The problem was designed for use as a worked example within a lesson, with pedagogical organisation of the relevant mathematics illustrated by an individually constructed vee diagram (vdiagram). The focus questions guiding this report are:

- (1) In what ways do vee diagrams contribute to the development of pre-service teachers' pedagogical content knowledge?
- (2) What are the limitations of vee diagrams as communicative and problem solving tools?

Before presenting the data, the vee diagrams' theoretical underpinnings are described followed by the methodology and data analysis.

## Theoretical Framework

Gowin's epistemological vee, developed to assist in the understanding of meaningful relationships between events (phenomenon), or objects, is a tool that visually illustrates the interplay between what is known and what needs to be known or understood (Figure 1). With the point of the vee-structure situated in the event/object to be analysed, the two sides represent on the left, the thinking, conceptual aspects underpinning the methodological aspects displayed on the right hand side. All vee elements interact in the process of constructing new knowledge or value claims, or in seeking understanding of these for any set events and questions as promoted in Ausubel's meaningful learning theory (Ausubel, 2000) and the constructivist perspective. A completed vdiagram represents a record of an event or object that is investigated or analysed to answer particular focus questions (Novak & Gowin, 1984; Novak, 2002).

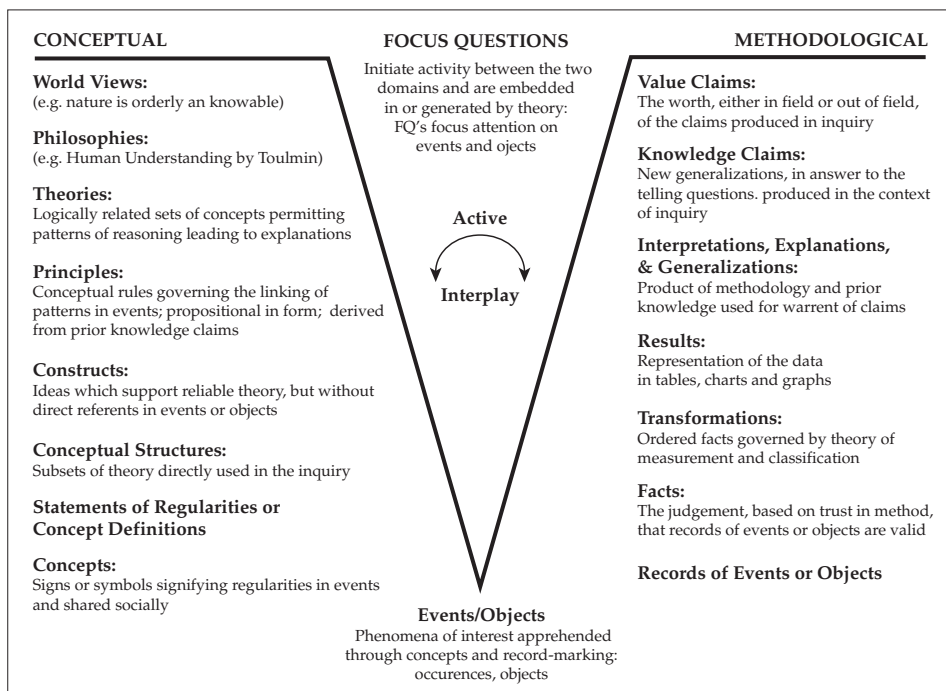


Figure 1. Gowin's epistemological vee (Novak & Gowin, 1984).

Shown in Figure 2 is an adaptation of Gowin's vee to solve mathematics problems (Afamasaga-Fuata'i, 2007) by analysing the problem statement for the 'given information' and 'focus questions', then using the information and the problem solver's (theoretical and methodological) experiences to set up a plan (or strategy), implementing the plan, and checking the reasonableness of the answers against the problem context, records and focus questions.

Having information about which to think analytically, creatively or practically is as important as the thinking process itself (Stermberg & Williams, 1998). For creative thinking to go beyond the given information requires that the problem solver has the knowledge of the given (records) and deep understanding of the substantive mathematics to interpret and transform the records; practical thinking to make use of knowledge of the situation (e.g., contextual knowledge); and adaptive expertise (Hatano, 2003) to effectively synthesise relevant knowledge and experiences. Furthermore there is a need for deep understanding (of principles, methods and procedures) to generate potential solutions and to establish the reasonableness of claims given the problem and its focus questions. The results may be strategically displayed on vdiagrams to effectively communicate publicly an individual's connected mathematical understanding and interpretations.

For the PS teachers in the secondary methods course, it was expected that their deep understanding of the pedagogical content knowledge would be

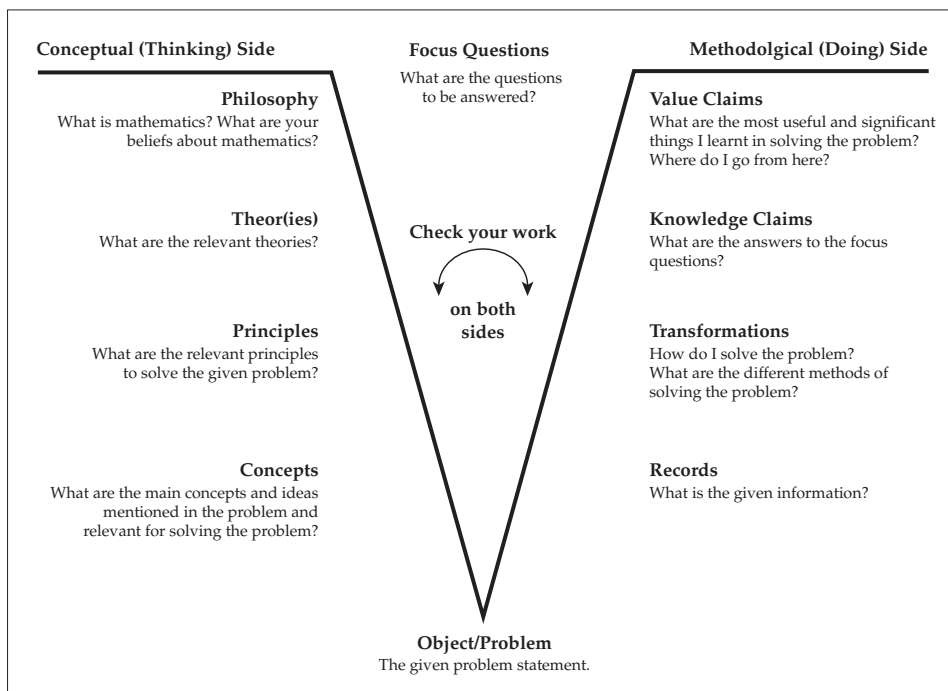


Figure 2. The mathematics problem solving vee diagram.

demonstrated by (a) their deliberate analysis of the given problem for the relevant principles and concepts, and (b) their use of creative designs of multiple methods or approaches, which use multiple representations, logically sequenced, and developmentally appropriate, to effectively scaffold the development of students' conceptual and interconnected understanding of the emerging mathematics.

Mewborn (2001) in her critique of research on the nature of teachers' mathematical knowledge identified three weaknesses two of which are relevant to this article. First, the need for teachers to "have a sense of what constitutes a valid mathematical justification" (Ball (1994), cited in Mewborn (2001, p. 33); second, the need to have rich data about the reasoning of teachers who do possess strong conceptual knowledge of mathematics and who are able to think through problems and provide suitable explanations (Mewborn, 2001, p. 33).

Findings from relevant studies investigating the usefulness of vdiagrams, as a tool to assess university students' integrated understanding of the mathematics underpinning solutions to problems, showed vdiagrams facilitated (a) the thinking and reasoning process; and (b) the public communication of students' integrated understanding (Afamasaga-Fuata'i, 2007). Teacher professional development workshops also showed vdiagrams have potential as teaching and assessment tools (Afamasaga-Fuata'i, 2005). These findings support those reported by Novak (2002) and Mintzes, Wandersee, and Novak (2000) in the sciences. This article reports a study conducted as part of a pre-service secondary mathematics methods course, to examine the usefulness of the meta-cognitive tool of vdiagrams to guide the reasoning process involved in solving mathematics problems and to make explicit visually PS teachers' pedagogical understanding of the substantive mathematics that is required to solve the problems in multiple ways.

## Method

Thirty-two secondary pre-service (PS) teachers, enrolled in the first mathematics methods course of their PS program in a regional Australian university, were introduced to vdiagrams during the four-day residential school for external students and weekly workshops for the internal students. During this one-year course, vdiagrams were used by the researcher, in presentations to the PS teachers as a means of displaying both the conceptual and methodological information underpinning a problem or activity, and as a pedagogical guide for the design of learning activities that challenge students' synthesis of conceptual and procedural knowledge and understanding.

The course's introduction began with class discussions of the PS teachers' philosophy, beliefs, and perceptions of what it means to 'do', 'learn', and 'teach' mathematics. They were encouraged to examine and reflect upon their own mathematical thinking and understanding and through small group activities, they collaboratively and cooperatively:

1. analysed investigation activities and application problems for the underlying principles and key concepts; and based on the results

2. identified the necessary appropriate prior knowledge to enable students to proceed with the activity/problem;
3. designed and developed multiple approaches that emphasize problem solving, reasoning, conjecturing, justifying using multiple representations; and
4. creatively extended activities/problems for subsequent learning.

Group presentations coupled with interactive, general discussions highlighted the importance of explicitly identifying, and appropriately sequencing, prior, new and future knowledge to develop students' conceptual understanding. Cross-referencing with the relevant Stage 4 and Stage 5 syllabus outcomes (NSW Board of Studies (NSWBOS), 2002) became an integral part of these group/class activities.

The PS teachers practised constructing vdiagrams either individually or in pairs. Vdiagrams were presented in-class with feedback provided by peers and researcher on how to improve. However, revised versions were not actually produced.

Two vdiagram tasks formed part of two of the four required course assignments as a different way of assessing PS teachers': (a) mathematical conceptions of what is important for students to learn, (b) mathematical flexibility in providing multiple methods or transformations to cater for a range of abilities, and (c) integrated pedagogical and mathematical sensitivity to student needs, curriculum imperatives and meaningful learning as demonstrated by their choice of methods and level of underpinning principles.

The assigned problem, *Determine the maximum area enclosed by a rectangular fence if 100 metres of fencing material is used*, was part of Assignment Two (2). PS teachers were to provide three different methods, which are appropriate for students in the junior secondary school who have not had much algebra and do not have a calculus background. As part of the assignment and course materials, PS teachers were provided with a vdiagram template, which included guiding questions for its completion (Figure 2) and an assessment rubric (Figure 3). Available for their use were the appropriate syllabus documentations (NSWBOS, 2002) and additional readings on vdiagrams.

### *Data Analysis*

This article focuses specifically on the vdiagram data. The analysis closely followed the marking criteria provided to the PS teachers (see Figure 3). All 13 criteria were weighted equally (7.7%) with each criterion rated on a scale of 1 to 4, namely, *not satisfactory* (1), *satisfactory* (2), *good* (3), and *very good* (4). A vdiagram's overall rating was based on further categorisation of the total score (out of 52) to identify types. For example, an overall rating of 1 is an *unsatisfactory* vdiagram for scores less than or equal to 25, 2 is *satisfactory* ( $26 \leq \text{scores} \leq 38$ ), 3 is *good* ( $39 \leq \text{scores} \leq 47$ ) and 4 is *very good* ( $\text{scores} \geq 48$ ).

| Vee Element          | Criteria   | % Weighting |
|----------------------|--|-------------|
| Philosophy           | A clear statement of your beliefs about mathematics learning.  | 7.7         |
| Theories             | Clearly stated main topic areas relevant to the Problem.   | 7.7         |
| Principles           | Comprehensive, clearly stated, and relevant principles to support and guide main steps of transformation.  | 7.7         |
| Concepts             | Comprehensive list of concepts relevant to the problem.  | 7.7         |
| Object/ Problem      | The given mathematics problem statement.   | 7.7         |
| Focus Question(s)    | Questions to be answered.  | 7.7         |
| Records/ Givens      | <ul style="list-style-type: none"> <li>• Given information in the problem</li> <li>• Methods of solutions to answer the focus question(s) generated by applying the listed principles to given information.</li> <li>• Multiple methods of solutions.</li> <li>• Justifications of main steps of each solution.</li> </ul> | 7.7         |
| Transformations      | The answer(s) to the focus question(s) generated by the transformations of given information using the listed principles.  | 7.7         |
| Knowledge Claims     | Significance and usefulness of knowledge claims and solving this problem to future learning.   | 7.7         |
| Value Claims         | ?  |             |
| Overall Organization | Overall relevance and correspondence between the information on the conceptual side and methodological side.   | 7.7         |

Figure 3. Marking criteria for the vee diagram.

## Findings

A summary of vdiagram scores and overall ratings are in Table 1. Of the 32 vdiagrams, six had the highest rating of 4, with eight rated 3 and the rest with a rating of 2; none had a rating of 1. An exemplar (marked \*) from each category is presented to illustrate similarities and differences between types.

Table 1  
Summary of vee diagram scores and ratings

| Map# | Score | Final Rating | Map# | Score | Rating |
|------|-------|--------------|------|-------|--------|
| 7    | 52    | 4            | 1    | 38    | 2      |
| 14*  | 51    | 4            | 28   | 36    | 2      |
| 32   | 51    | 4            | 4    | 36    | 2      |
| 5    | 49    | 4            | 24   | 33    | 2      |
| 23   | 48    | 4            | 11   | 32    | 2      |
| 3    | 48    | 4            | 2    | 32    | 2      |
| 9    | 47    | 3            | 30   | 31    | 2      |
| 31   | 44    | 3            | 29   | 31    | 2      |
| 15   | 43    | 3            | 26*  | 31    | 2      |
| 20   | 42    | 3            | 19   | 31    | 2      |
| 12   | 42    | 3            | 8    | 31    | 2      |
| 13   | 41    | 3            | 22   | 29    | 2      |
| 10*  | 41    | 3            | 16   | 29    | 2      |
| 6    | 39    | 3            | 21   | 28    | 2      |
| 27   | 38    | 2            | 18   | 27    | 2      |
| 17   | 38    | 2            | 25   | 27    | 2      |

\*Exemplars

### Example of a 'Very Good' Vee Diagram – Student 14

Figures 4a and 4b provide an overview of Student 14's (S14's) vdiagram. Using the given problem statement (What is the mathematics problem?) (Figure 4a), S14 crafted a focus question to highlight what needs to be obtained (What are the questions I need to answer?). The given information, extracted from the problem statement, is simply stated (What is the given information?) with S14's identified main concepts under What are the main ideas?. S14's suggested relevant principles (What do I know already?) are appropriately worded to align with Stage 4 and early Stage 5 Measurement Syllabus Outcomes (NSWBOS, 2002). The principles include simple definitions (P1, P2, P4, and P6) followed by formulas (P3 and P5) and P6 followed by formulas (P3 and P5)

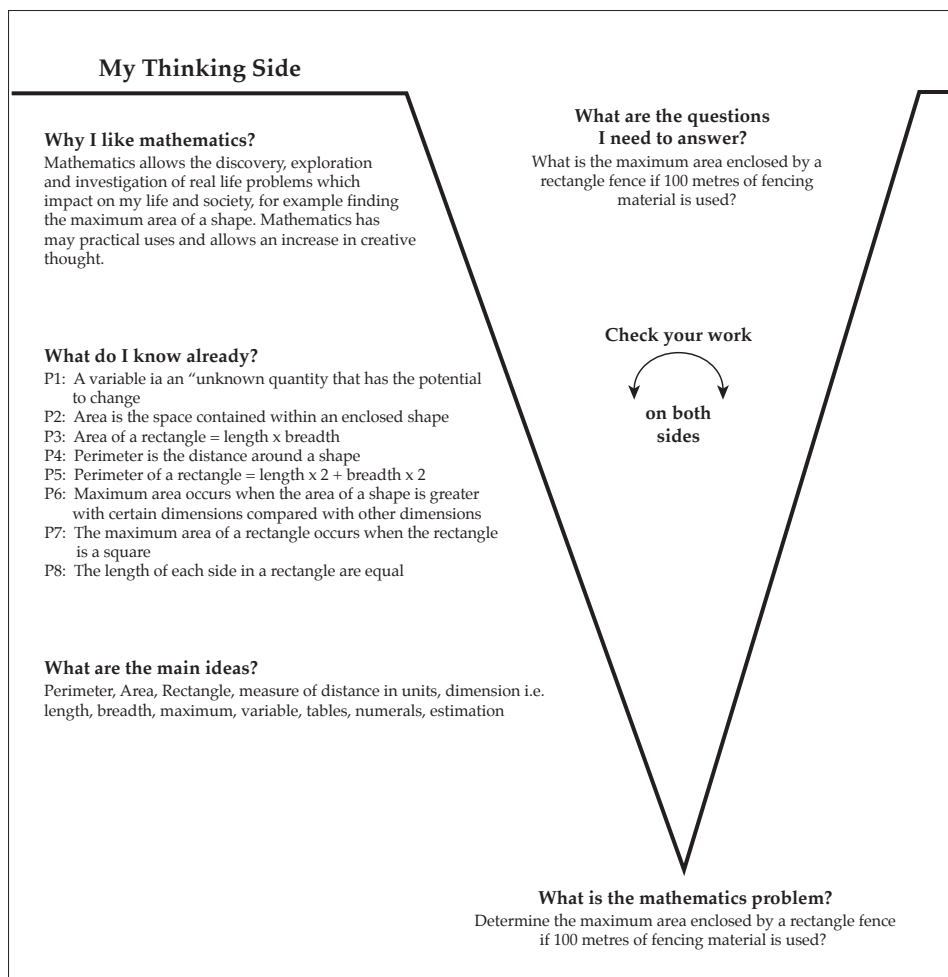


Figure 4a. A view of Student 14's vee diagram – Thinking side.



with specific prior knowledge (P7) and general property of rectangles (P8, correction is rectangle not triangle).

The displayed multiple methods (Figure 4b) include a trial and error method, utilising P7, and using a tabular approach. All three methods endeavour to encourage reasoning, investigating, conjecturing and confirming with very little algebraic manipulation but mainly numerically based reasoning and utilising a table to establish patterns. Detailed solutions and a list of facilitative questions were provided separately. While the *Knowledge Claim* answers the posed focus question (*What are my answers to the focus questions?*), the *Value Claims* expressed what students should learn and suggested potential extensions to other shapes (*What are the most useful things I have learnt?*). Overall, the vdiagram illustrates an overview of the conceptual and methodological information of the

### My Doing Side

**What are the most useful things I have learnt?**  
I have learnt that a square is a special rectangle which contains the maximum area. It would be interesting to investigate different shapes and the impact they have on the total area, for example with circles and triangles. I have learnt how to effectively problem solve to find a solution in regards to finding the total area of a rectangular shape.

**What are my answers to the questions?**  
The maximum area enclosed by a rectangular fence with a perimeter of 100 metres is 625 metres<sup>2</sup>

**How do I find my answers?**

|   |  |   |
|---|--|---|
| <p><b>METHOD 1 – Trial and error</b><br/>Choose 35<br/>From P5, find the breadth of the rectangle.<br/><math>35 \times 2 = 70</math><br/><math>100 - 30 = 30</math><br/><math>30 / 2 = 15</math><br/>length = 35 metres<br/>breadth = 15 metres</p> <p>From P3, find the area of the rectangle.<br/><math>A = 35 \times 15 = 525</math> metres squared</p> <p>Area for a 35 by 15 rectangle is 525 metres squared</p> <hr/> <p>Choose 20<br/>From P5, find the breadth of the rectangle.<br/><math>20 \times 2 = 40</math><br/><math>100 - 40 = 60</math><br/><math>60 / 2 = 30</math><br/>length = 20 metres<br/>breadth = 30 metres</p> | <p>From P3, find the area of the rectangle.<br/><math>A = 35 \times 15 = 525</math> metres squared</p> <p>From P6, the area increases as the length decreases. Therefore, by a number between 35 and 20.</p> <p>Choose 24<br/>From P5, find the breadth of the rectangle.<br/><math>24 \times 2 = 48</math><br/><math>100 - 48 = 52</math><br/><math>52 / 2 = 26</math><br/>length = 24 metres<br/>breadth = 28 metres</p> <p>From P3, find the area of the rectangle.<br/><math>A = 24 \times 26 = 624</math> metres squared</p> <p>From P6, the area has increased therefore choose a number between 24 and 26</p> | <p><b>METHOD 2</b><br/>From P7, I know that the perimeter must form a square. From P8, the four sides of a square are equal, therefore each side is 25 metres, for example<br/><math>100 / 4 = 25</math></p> <p>From P3, find the area of the square:<br/><math>A = 25 \times 25 = 625</math> metres squared.<br/>The maximum area enclosed by a rectangular fence with a perimeter of 100 metres is 625 metres<sup>2</sup></p> <hr/> <p><b>METHOD 3</b><br/>Since Perimeter = 100<br/>From P1, P4, P5 and P3, we can work out the following table:<br/>X 10 20 30 40<br/>Y 40 30 20 10<br/>A 400 600 800 400<br/>By P6, the maximum area enclosed must occur between 20 and 30, i.e. at 25 metres by 25 metres<br/>By P3, the area is<br/><math>A = 25 \times 25 = 625</math> metres<sup>2</sup></p> |
|---|--|---|

**What is the given information?**  
Perimeter = 100 metres  
the area enclosed within a rectangular shape

Figure 4b. A view of Student 14's vee diagram – Doing side.



problem, which S14 anticipated would be pedagogically appropriate for the targeted students.

*Example of a 'Good' Vee Diagram – Student 10*

Whilst the *Object/Problem* entry is the given problem statement (Figure 5), Student 10 (S10) creatively crafted 4 *Focus Questions* aimed to encourage students' reasoning (question 1); justifying (question 2); strategising (question 3); and reflecting (question 4). The given information under *Records* is simply stated with the listed main *Concepts* S10 identified or inferred from the problem statement. The prior knowledge students should possess (*Principles*) and relevant main topics (*Theories*) are listed, indicating most of the conceptual bases of the

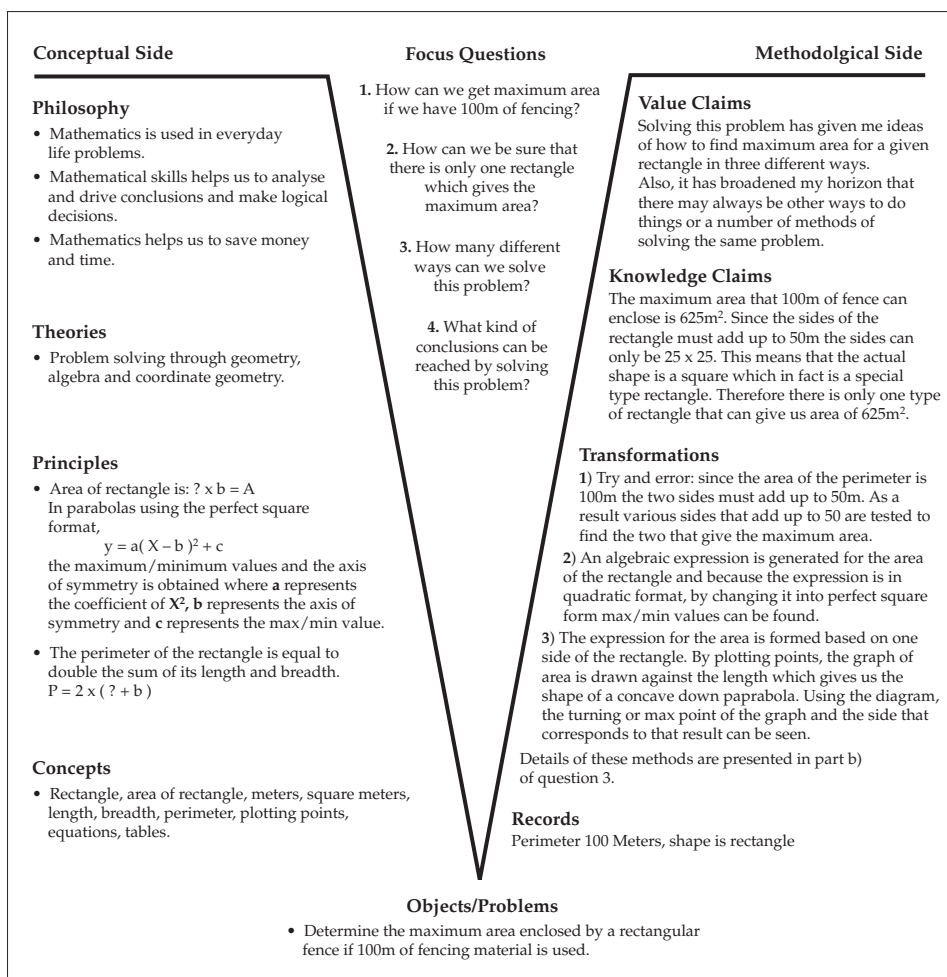


Figure 5. Student 10's vee diagram.

three methods (*Transformations*).

With detailed solutions provided separately, the first method included as well, a 3-column table (length, breadth and area) for recording the results. This formed the basis for identifying the maximum area. Both Methods 2 and 3 began with a derivation of the quadratic area formula, with Method 2 (M2) obtaining the maximum area algebraically using the perfect square format. In contrast Method 3 (M3) is by estimation from the parabolic graph, which, incidentally, lacked a supporting principle.

Juxtaposing the focus questions, listed principles, transformations, and detailed methods, it appears that there is a methodological leap from 'records' to the starting point of the first method (M1) (i.e., length and breadth add to 50 m) without a conceptual basis, either declared as a 'principle' (prior knowledge), or explicitly developed through investigation (new knowledge) as the first outcome of M1 and before moving to tabulate values. For example, S10 wrote "Driven by the fact that the perimeter of the rectangle is 100 m and opposite sides in rectangles are equal, it is known that the sum of the length and the breadth of the rectangle is 50m." This is a pedagogical concern; this 'fact' somehow appears 'known'. Ideally, it needs to be meaningfully mediated to ensure developmental understanding, either concretely through 'modelling', numerically through trial and error, or explicitly by reasoning, before generalising and systematically applied to compute table values. In contrast to M1, M2 and M3 demonstrably assumed a well-grounded algebraic and graphical understanding of quadratic functions and equations, which might be problematic for early secondary level.

The elaborated *Knowledge Claims* answer S10's four focus questions. *Value Claims* are his reflections that finding multiple methods "has broadened [his] horizon" and his mathematical views *Value Claims*. Given the dissonance between methods and conceptual bases above, the broadening of S10's horizon needs to be more developmentally based in accordance with the principles of meaningful learning and curricular imperatives. In the *NSW Years 7-10 Mathematics Syllabus*, M2 and M3 are more in the middle to upper Stage 5. Overall, the vdiagram presents a summative overview of the relevant conceptual and methodological information utilised to generate knowledge and value claims. Whilst the methods are predominantly algebraic, S10 needs to pedagogically re-conceptualise his approach in early M1, to meet the needs of all students.

### *Example of a 'Satisfactory' Vee Diagram – Student 26*

Student 26 (S26) had the correct entry for Problem as the given problem statement (Figure 6). However, instead of appropriately or creatively rephrasing the latter as a *Focus Question* to be answered (like S14 or S10), S26 reused the same entry. Shown alongside S26's entry for *Focus Question* is a minor change (i.e., What is) to convert the statement to a "question" instead of a statement. *What are the main ideas?* entries are phrases instead of succinct concept names as illustrated by example and discussed during practice sessions. For example, encapsulated by the phrase "maximising area for a rectangle" are two or three distinct concepts, namely, *maximising area* and *rectangle* or alternatively, *maximum*, *area*, and *rectangle* that

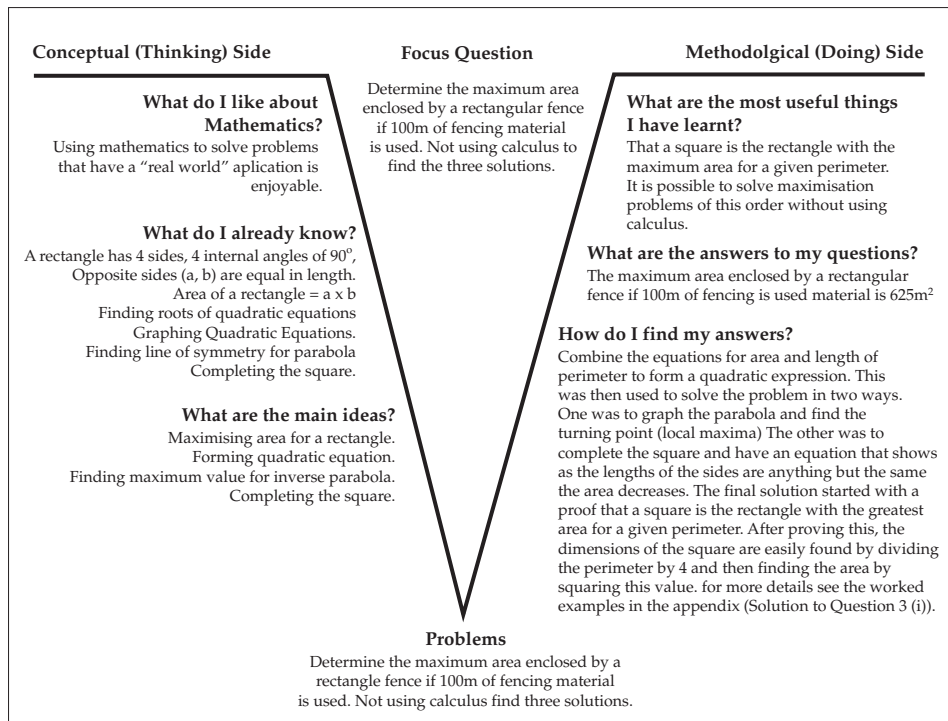


Figure 6. Student 26's vee diagram.

should be separately identified as "main concepts". This pattern of describing "main ideas" is also repeated for the next two entries instead of listing as distinct, main concepts as S14 and S10 did. Omitted from his vdiagram is an explicit identification of *What is the given information?* an explicit component representing an initial step of reasoning out appropriate information from the problem statement.

For *What do I already know?*, while the first three lines list the properties of rectangles and area formula, the rest are incomplete and do not represent complete principles or definitions. Instead, they appear to serve as pointers to the necessary principles. If the teacher is interested in assessing what students do know about *finding roots of quadratic equations*, a more elaborative statement should be provided; similarly for the rest of the listed phrases. Absent from the list is the essential principle for perimeter of a rectangle, one of two key principles. There is evidently confusion here with some of the entries and omissions, perhaps partially due to the newness of the vee tool. However, as a prospective teacher, S26 needs to theoretically clarify for himself the distinctions between principles and concepts and how to appropriately express each to minimise student confusion and to model clear articulation of conceptual understanding. Instead, the displayed entries suggest the need to re-examine his pedagogical understanding of the relevant mathematics.

On the "Doing Side" are descriptions of S26's strategies for solving the

problem (*How do I find my answers?*), using the principles on the “Thinking Side”, with reference to completed solutions provided separately. The first two methods are by graphing the parabola and completing the square with the third one based on proving that a square is the rectangle with the greatest area for a given perimeter. Inspection of detailed solutions confirmed the proof involves much algebraic manipulation, making it developmentally difficult for the targeted students. Instead, it should be re-conceptualised and anchored on concrete modelling or numerical investigations. All three methods assumed students would have a well-developed understanding of the relevant algebraic techniques. Under *What are my answers to my questions?* is his answer to the focus question with two statements describing S26’s expectations of what a student should learn (*What are the most useful things I have learnt?*). Directly opposite on the left hand side (*What do I like about Mathematics?*), are his perceptions of how a student might envisage the connection between the problem and real-life applications. Overall, S26’s vdiagram provided an overview of his interpretations of the conceptual and methodological information relevant to the problem. Revealed through the completed vdiagram are conceptual and methodological omissions, essential in the development of students’ conceptual and integrated understanding; incomplete expressions of principles; confusion between concepts and principles; and the need to bridge his advanced, formal mathematical thinking to early secondary to pedagogically mediate meaning effectively in the classroom.

In summary, the three examples illuminated areas where the PS teachers demonstrated thoughtful analyses of the problem and curriculum to identify appropriate mathematics principles/methods and those that were problematic. Hence, critical feedback can be used constructively by the PS teachers to confront their deficits while reflectively contemplating how to further enhance the completed vee and detailed solutions to effectively develop their conceptual and pedagogical understanding of the expansive content that can be covered by the multiplicity of solutions for this single problem.

## Discussion and Conclusions

The findings and exemplars showed there were qualitative differences between the 32 vee diagrams, namely, the cohesiveness of the displayed conceptual and methodological information, and the appropriateness of the multiple methods and principles for the specified secondary mathematics level, as evident by the scores and overall ratings. The variety of multiple methods displayed highlighted the diversity of approaches that PS teachers can potentially adopt depending on their philosophical and theoretical preferences in alignment with, or despite, clear guidelines as articulated in curriculum documentation and its constructivist underpinnings.

The *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics (NCTM), 2000) stipulate, like those by AAMT (2006), that teachers play an important role in developing students' problem-solving

dispositions. Hence, PS teachers need to create learning environments and activities that encourage students to explore, take risks, and work and communicate mathematically in order to develop students' confidence to explore problems and their ability to make adjustments in their strategies. To pedagogically mediate meaning effectively in these supportive environments, PS teachers need many "opportunities to unpack ideas or to make connections" (Ball & Wilson 1990, cited in Mewborn, 2001) during the course. For problem solving, developed in advance, flexible methods empower PS teachers to 'make intelligent decisions' promptly and competently about student concerns as they arise during the lesson (Ma, 1999). This was the main objective for the 'worked example' assessment task reported here, with the requirement that they use vdiagrams to communicate publicly this overall pedagogical understanding, for the purpose of evaluation. School students need to develop a range of strategies for solving problems. In the exemplars presented, there were methods that used diagrams, examined numerical patterns through trial and error or systematically in tabular form, in addition to variously presented graphical and algebraic methods. These strategies need to be explicitly taught and/or facilitated in the classroom if students are to learn them. In the study reported here, PS teachers were to provide evidence of their pedagogical intent through completed vdiagrams and detailed solutions. It does not, however, extend to the evaluation of these intentions in actual teaching practice.

As evident from the examples, PS teachers did apply and adopt a variety of appropriate strategies. Some of the multiple methods purposefully attempted to build new, or extend prior, mathematical knowledge as demonstrated by the transformations, knowledge and value claims of the three vdiagrams. The nature of the vee structure (i.e., its vee elements) explicitly invited the PS teachers to monitor and reflect on the process of solving the problem, as they critically analysed the problem statement, flexibly and creatively completed the conceptual bases of methods, in addition to providing multiple methods, philosophical views, and critical reflections as an overall visual summary. "Good problems give students the chance to solidify and extend their knowledge and to stimulate new learning .... Solving problems is not only a goal of learning mathematics but also a major means of doing so" (NCTM, 2000). The completed vdiagrams communicatively highlighted the diversity of potential pedagogical approaches, multiple methods and principles, value claims and philosophical views centred around a single problem, and made evident methodological and conceptual leaps and omissions that are non-trivial for ensuring meaningful learning, which need to be pedagogically mediated and developmentally bridged. By having PS teachers complete vdiagrams, they necessarily had to reflect upon their own knowledge and understanding of the relevant mathematics content, confront, and then make explicit their pedagogical interpretations in order to communicate visually and publicly their perceptions of appropriate and relevant mathematical principles and multiple solutions for early secondary.

Whilst there are clear benefits of using a vdiagram, especially in providing a big picture view, there are also limitations in terms of the level of detail they can

accommodate. While it may be a weakness, it is also a strength; it challenges the individual to provide a summative record or overview of the most pertinent information without getting lost in the details. The higher order skills of thinking, reasoning, analysing, synthesising, and reflecting necessarily permeate the process of completing vdiagrams. Another possible limitation is the need for the constructor to be familiar with the theoretical underpinnings of a vdiagram so that it continues to be used as a means of supporting the critical thinking and reasoning that is part of meaningful problem solving, but not used as a mechanistic structure with spaces to be completed haphazardly. A third limitation is the need for the critical feedback after public scrutiny (evaluation in this case as course assessment) to be constructively used for further pedagogical improvement before classroom implementation of the planned activity. Whilst this paper focussed only on PS teachers' pedagogical thinking and reasoning in the context of one mathematics problem, it does not include the evaluation of the implementation of these intentions in actual teaching practice.

Findings from this study contribute to the literature on PS secondary teacher education by suggesting one innovative means of facilitating the re-packaging of PS teachers' integrated and connected pedagogical content knowledge and understanding for public scrutiny and before teaching practice.

Ball and Wilson (1990, cited in Mewborn (2001)) recommended more opportunities in pre-service teacher education to "unpack mathematical ideas or to make connections". If PS teachers are to orchestrate classroom discourse to encourage students to share their emerging mathematical ideas, then they must have a sense of what constitutes mathematical justification (Mewborn, 2001) and how to evaluate connected knowledge and understanding. The findings reported here suggest that a completed vdiagram, with its conceptual and methodological sides including philosophical and values claims, provides one way of illustrating this complementary and integrated view of the mathematics necessarily involved in solving a problem. The research findings support the notion that following the preparatory work by the PS teachers, through to actual classroom implementation during teaching practice and subsequent evaluation of its impact on student learning over a period of time is a worthwhile area for further investigation.

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### *Author*

Karoline Afamasaga-Fuata'i, School of Education, University of New England, Armidale, NSW, 2351. Email: <kafamasa@une.edu.au>