# Influencing Teachers' Beliefs About Teaching Mathematics for Numeracy to Students with Mathematics Learning Difficulties

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This paper reports on the beliefs of a group of K-8 mathematics teachers about appropriate goals and methods of mathematics teaching for students with mathematics learning difficulties and for students generally. The teachers were involved in a brief professional learning program that aimed to provide them with effective strategies for mathematics teaching for numeracy, and to influence their relevant beliefs towards a more inclusive view of mathematics teaching. The questionnaire used in the study revealed differences between teachers' beliefs in relation to students generally and those with mathematics learning difficulties, and provided evidence that carefully designed professional learning may be able to reduce these differences.

Numeracy has its foundations in the discipline of mathematics. Definitions of numeracy typically emphasise the use of mathematics in everyday life and highlight the importance of affect (Australian Association of Mathematics Teachers, 1997). The Department of Education Tasmania (DoET) (2002) refers explicitly to the application of mathematics to everyday life and acknowledges affect in its use of the terms *disposition* and *confidence* in its definition of numeracy, which includes the following:

> Being numerate involves having those concepts and skills of mathematics that are required to meet the demands of everyday life. It includes having the capacity to select and use them appropriately in real settings. Being truly numerate requires the knowledge and disposition to think and act mathematically and the confidence and intuition to apply particular mathematical principles to everyday problems … it also involves the critical and life-related aspects of being able to interpret information thoughtfully and accurately when it is presented in numerical and graphic form. (p. 21)

Interpreting the Tasmanian definition necessitates some understanding of what the mathematical demands of everyday life might be and it seems self evident that such demands differ from individual to individual depending upon, among other things, occupation. Being numerate can thus be regarded as requiring different mathematical skills and concepts for different people. Nevertheless, the Australian Government remains committed to ensuring that "all students attain sound foundations in literacy and numeracy" (Department of Education, Training and Youth Affairs (DETYA), 2000, p. 6). DETYA (2000) also made clear that this goal was based upon a firm belief that all students can acquire the mathematical skills necessary for life in modern society and recognised that some students require, and should receive, additional support to this end. Similarly, in Tasmania, it has been recognised that recent and ongoing curriculum reform must include access to a broad, rich and challenging curriculum for students with special and/or additional needs (Atelier Learning Solutions, 2004). Atelier Learning Solutions (2004) noted that, for a variety of reasons, commitments to equity at a policy level are not necessarily translated into practice in classrooms. The study reported here was predicated on the hypothesis that at least in part the disjunction between policy and practice could be a consequence of teachers not sharing DETYA's (2000) belief in the capacity of all children, and of differing interpretations among teachers of what the everyday mathematical demands might be for various students, and hence of what it means for them to be numerate. The specific research questions addressed by the study were:

1. How and to what extent do teachers' beliefs about the goals and means of teaching mathematics for numeracy differ for students with mathematics learning difficulties and students generally?
2. How can professional learning influence teachers' beliefs?

# Theoretical Framework

## *Teachers' Beliefs*

The relationship between teachers' beliefs and their classroom practices is complex and subtle as well as powerful (Beswick, 2005a). Many studies of mathematics teachers' beliefs, including that of Beswick (2005a), have used aspects of Green's (1971) description of belief systems to understand ways in which an individual's many beliefs can interact. Green described how particular beliefs differentially influence behaviour according to their centrality or the number and strength of connections between a given belief and others. The more centrally held a belief the more dearly it is held and the more difficult it is to change. Green also acknowledged that the relative centrality of beliefs varies with context. This is consistent with Ajzen and Fishbein's (1980) assertion that beliefs are specific to all of context, which they defined as comprising place, action or behaviour, time, and subject. Illustrated in the context of the study reported here, this means that a teacher's belief about the capacity of a student to learn would depend upon: the particular physical classroom and school (place), the behaviours that the student would be expected to engage in both to learn and to demonstrate learning (actions), the date and duration of the lesson (time), the nature of the ideas/topic being considered, and characteristics of the student (subjects).

Green (1971) also described how beliefs can be held in clusters essentially isolated from other beliefs and hence possibly in contradiction of them. Such a situation can arise when beliefs develop in disparate contexts. A third aspect of Green's description concerns the primary or derivative nature of beliefs and recognises that some beliefs are held because they are logical consequences of other beliefs. It follows that changing a derivative belief may require restructuring of an individual's beliefs system such that the primary belief that is the ultimate reason for the derivative belief is altered or so that the logical links between beliefs are changed.

Importantly for those interested in promoting change in beliefs, beliefs may or may not be held on the basis of evidence. This is an important distinction, for beliefs in the latter category are held for reasons such as the authority of the source of the information, or because they support existing, centrally held beliefs. They are also more likely than evidentially held beliefs to be held in isolated clusters, and are, by definition, impervious to change even in the light of clearly contradictory evidence (Green 1971).

Many attempts have been made to define beliefs and to distinguish them from related constructs including attitudes and knowledge but there is little by way of consensus (Furinghetti & Pehkonen, 2002). In this study beliefs were regarded as anything that an individual regards as true which is essentially the definition of Ajzen and Fishbein (1980). It was considered that the 'facts' of a situation, however established, are less relevant to how a teacher acts in that situation than his/her beliefs about it.

## *Changing Teachers' Beliefs and Professional Learning*

Teachers' beliefs have been recognised as crucial determinants of what teachers do in their classrooms and hence a major area of research concerning teachers' beliefs has focussed on changing teachers' beliefs and practices (Wilson & Cooney, 2002) which is, of course, the objective of professional learning programs. Many studies have attempted to link teachers' practices with their beliefs and the interpretation of apparent contradictions between what teachers' profess to believe and the ways in which they act has attracted debate. Hoyles (1992) and Beswick (2003) argued that the contextual nature of beliefs renders distinctions between so-called espoused and enacted beliefs meaningless. There is, however, consensus that teachers' beliefs and practices are related in complex ways and develop and change together (Cobb, Wood, & Yackel, 1990) and that both need to be addressed by professional learning programs (Wilson & Cooney, 2002). This section, therefore, includes and attempts to make connections between findings concerning changing teachers' beliefs as well as ideas from the professional learning literature that does not specifically mention beliefs.

Hawley and Valli (1999) listed eight characteristics of effective professional learning programs, all of which have also been mentioned by other researchers (e.g., Ball, 1996; Britt, Irwin, & Ritchie, 2001; Farmer, Gerretson, & Lassak, 2003). According to Hawley and Valli (1999) professional learning should be: 1) driven by student performance, 2) driven by teachers determining what they have to learn, 3) school based and connected to the classroom, 4) based on collaborative work on shared concerns, 5) continuous and supported; and should provide: 6) access to and evaluation of multiple sources of information, 7) opportunities to develop relevant theoretical understandings, and 8) time to learn and to implement new practices.

Ball (1996) also acknowledged the relevance of the beliefs that teachers bring with them to professional learning experiences and studies reporting change in teachers' beliefs have identified characteristics of mathematics teachers whose beliefs and practice seem to change most profoundly as a result of their involvement in professional learning. These include a desire to improve the understanding of their students (Breyfogle & Van Zoest, 1998) and recognition of the need to change (Chapman, 1996). Teachers thus motivated would arguably be more likely than others to engage with data about students' performance, and would be likely to respond positively to opportunities to direct their own learning. In addition, Nespor (1987) explained that teachers are able to change their practice only if they have available to them an alternate paradigm that they believe is plausible in their context. Professional learning that provides alternate theoretical frameworks and is also school based and classroom focussed would be likely to meet this condition. Arvold and Albright (1995), in their study of 15 preservice secondary mathematics teachers, identified those most likely to adopt beliefs and practices advocated in their preservice education as relativistic rather than dualistic thinkers. That is, they were able to consider multiple possibilities: were willing to consider alternate ideas, and resisted prematurely judging the relative merits of alternatives. Professional learning based on collaboration, and that provided multiple sources of information would seem appropriate for such people. While not specifically mentioned by Hawley and Valli (1999), continuous and supported professional learning in collaborative contexts with input from multiple sources creates conditions conducive to effective reflection, which also has been identified as a driver of professional learning (Wilson & Cooney, 2002).

In fact, all of the factors mentioned as being relevant to whether and how much teachers change in response to professional learning experiences, including the informal experiences within their classrooms, are the out-workings of various aspects of their pre-existing belief systems. For example, a teacher's inclination to reflect can be regarded as a result of that teacher's beliefs about the value of reflection; a focus on student learning is likely to be related to beliefs about such things as the role of the teacher, and students' capacity to learn; openness to new ideas can be seen as a reflection of beliefs about the nature of knowledge, and the role of change and could also be related to professional self-esteem. Indeed there is evidence that teachers' beliefs about themselves, their performance and the perceptions thereof of significant others, may be among the most crucial determinants of the extent to which teachers can change (Beswick, 2004). Other relevant beliefs concern such things as the nature of teaching and learning, the role of the teacher and the importance of ongoing professional learning. An understanding of both the content and the structure of teachers' belief systems is therefore key to understanding and consequently increasing the effectiveness of efforts to change teachers' beliefs and practice.

## *Mathematics Learning Difficulties*

Research on mathematics learning difficulties (MLD) has been conducted principally from a psychological perspective with a view to identifying causes. Definitions of MLD and related terms tend to refer to sustained underachievement in mathematics relative to expectations based on IQ (e.g., Geary, 2004). Other characteristics of MLD include: the use of immature calculation strategies for longer than other children (Geary, 2004; Torbeyns, Verschaffel, & Ghesquiere, 2004), delays in learning mathematical procedures, and chronic difficulties retrieving basic facts (Micallef & Prior, 2004). These features were identified by the teachers in this study as characterising up to 10% of their students and formed the basis of the meaning of MLD that was negotiated with them. The IQ of students was not considered and so it is likely that some of the students that the teachers had in mind had low IQ.

Relatively little is known about effective mathematics teaching for students with MLD (Baker, Gersten, & Lee, 2002), however, that which is known largely accords with recommendations for effective mathematics teaching generally. One exception might relate to students exhibiting a particular subtype of MLD described by Geary (2004) as characterised by difficulty in using and interpreting spatial representations of mathematical material. Geary acknowledged that relatively little is known about this subtype and observed that students with other types of mathematics learning disabilities appear to have spatial abilities comparable to those of other children. Evidence from other sources (e.g., Bobis, 1996) suggests that visualisation has an important role to play in the development of children's number sense and fact retrieval. Other studies have shown that strategy instruction is more effective than drill and practice for all students. Tournaki (2003) and Keeler and Swanson (2001) suggested that strategy instruction in relation to remembering may also be helpful for struggling students. Such findings lend weight to the assertion of Aubrey (1993, cited in Robbins, 2000) that "the majority of children identified as having special needs require not specialist teaching but good, high quality and effective teaching" (p. 55). This is consistent with the trend noted by Treuen, van Kraayenoord and Gallaher (2001) toward providing the same pre-service education for regular and special educators.

Very little has been written about teachers' beliefs in relation to students who experience difficulties in learning mathematics but several findings about students with learning difficulties in general are relevant. For example, Treuen et al. (2001) reviewed evidence that suggested that while most teachers report positive views about inclusion a sizable minority reject the implementation of inclusive practices in their own classrooms. Teachers' attitudes to inclusion are dependent upon their perceptions of both the severity of students' disabilities and the extent of support available, and most believe that more professional learning in the area would be beneficial (Treuen et al., 2001). Interestingly, Treuen et al. noted the failure of efforts to change teachers' attitudes to inclusion simply by addressing their skills and pointed to evidence that suggests that challenging school cultures could be at least as important. In discussing the views of teachers, managers and policy makers in school systems of the preparation of beginning teachers, Skilbeck and Connell (2004) reported widespread concern with, among other things, their ability to cater for the diverse learning needs of students, and noted the consistency of this with beginning teachers' own concerns in relation to including students with disabilities.

There is also evidence that high teacher expectations of students in relation to academic tasks are associated with improved achievement (Schoen, Cebulla, Finn, & Fi, 2003). Such expectations are likely to be underpinned by positive beliefs about student capabilities. This study revealed that teachers' hold different beliefs in relation to students with MLD compared with other students and also provides encouragement that their unhelpful beliefs are susceptible to change in response to carefully designed professional learning experiences.

## The Study

The study was part of the evaluation of a brief professional learning program planned and delivered by the researcher and aimed at improving the teaching repertoires of K-8 teachers of mathematics in relation to students with MLD. The subjects of the study were the 22 teachers who participated voluntarily in the professional learning program. The teachers taught classes from K-8 and were from government, independent and Catholic schools. The numbers of primary and secondary teachers from each school sector are summarised in Table 1. Of the 13 primary teachers, five identified as early childhood teachers.

Table 1
*Primary and secondary participants by sector*

| | Independent | Catholic | Government | Totals |
|---|---|---|---|---|
| Primary | 3 | 2 | 8 | 13 |
| Secondary | 2 | 4 | 3 | 9 |
| Totals | 5 | 6 | 11 | 22 |

### *Instrument*

The questionnaire, *Numeracy for Students with Mathematics Learning Difficulties* (NSMLD), comprised three sections. Section 1 comprised nine belief statements requiring a true or false response. These are shown in Table 3 and were drawn principally from the 12 "maths myths" identified by Kogelman and Warren (1978) in their work with mathematics avoidant students and used in subsequent surveys of preservice teachers (Beswick, 2006; Beswick & Dole, 2001; Frank, 1990). They represent myths in the sense that they are held without consideration of evidence (Frank, 1990) and express beliefs that are unlikely to result in inclusive or student-centred mathematics teaching. Section 2 comprised eight statements to which participants responded on 5-point Likert scales ranging from Strongly Agree to Strongly Disagree. The items related to the respondents' own attitude to mathematics (six items) and the nature of mathematics (two items). The 22 items in the third section were reported on by Beswick (2005b). They concerned beliefs about mathematics teaching and learning and required responses on two 5-point Likert scales — one to indicate the extent of agreement with the item in relation to students with MLD, and the other with respect to students generally. All of the Likert scale items were scored such that five indicated the highest level of agreement. Participants were also asked to provide a code name to facilitate matching of their surveys at the beginning and end of the program. Completing the entire questionnaire took 10-15 minutes.

## *The Professional Learning Program*

The program comprised three spaced 3 hour workshops and was designed cognisant of the characteristics of effective professional learning described in the literature and mindful of the key role of teachers' beliefs in affecting lasting change. Specifically, the content of the second and third sessions was based on discussions about their perceived needs that occurred in the first session; participants were encouraged to try ideas introduced in each of the first and second sessions and to share the results at the subsequent workshop; much of the time in the workshops was devoted to discussion and sharing of ideas and experiences; additional resources in the form of readings and a website were provided; constructivism was explicitly mentioned as the theory of learning underpinning the ideas presented; and explicit discussions about relevant beliefs were facilitated. The researcher's own beliefs, which guided the design of the program were:

1. All students are entitled to a rich, broad and challenging mathematics curriculum (Atelier Learning Solutions, 2004).
2. All students are able to learn mathematics (Ollerton, 2001).
3. A belief that mathematics makes sense is an essential part of being numerate (Van de Walle, 2004).
4. All students should experience mathematics teaching aimed at the development of deep conceptual understanding.

The particular needs of students with whom the teachers were working were diverse and hence the emphasis of the professional learning program was on the provision of high quality teaching for all students rather than specialist teaching for students with particular needs (Aubrey, 1993, cited in Robbins, 2000). Nevertheless, teachers with particular shared concerns were able to discuss these and share experiences and resources.

## *Session 1*

The first session began with a discussion of the meaning of MLD and the proportion of students in the teachers' classes to whom they believed the term applied. The teachers were then asked to brainstorm the kinds of difficulties that their students experienced with mathematics and to list the issues and mathematics topics that were of most concern to them with respect to teaching students with MLD. The lists were collected and used as the basis for planning subsequent sessions such that topics and issues mentioned most frequently were prioritised. The collated issues and topics are shown in Table 2 and represent the content of the program. Of necessity, some items were covered very briefly indeed while others were recurrent themes in discussions. Asterisked items received more attention than others.

Table 2
*Topics and issues addressed by the program*

| Topics | Issues |
|---|---|
| Place value* | What survival life skills students need |
| Mental computation* | When to use which concrete materials Mathematical thinking and reasoning* |
| Visualisation* | Developing metacognition* |
| Time | Meaning of numeracy — application |
| Money | Affective responses of students |
| Fractions* | Time implications of teaching for understanding/with concrete materials* |
| Estimation | Retention of knowledge including strategies for fact reconstruction* |
| Decimals* | Repetition of content |
| Rational numbers | Engaging older students with concrete materials |
| | Promoting student autonomy |
| | Role of language in mathematics |
| | Encouraging students to verbalise their thinking* |
| | Moving from concrete to abstract reasoning |
| | The place of calculators/spreadsheets |

Having established a shared understanding of the meaning of MLD the questionnaire was administered. Following this teachers worked on a task based on the 1-100 board, discussed the mathematics inherent in the task, its suitability for students with MLD, possible adaptations of the task for these students, the extent to which the task could be considered open-ended, other open-ended tasks using the 1-100 board, and the role of open-ended tasks generally in catering for diverse student needs. The teachers were then asked to discuss in groups and then share with the whole group their thoughts on the following questions: What does numeracy mean for students with MLD? What are appropriate/realistic goals for these children? To what extent is conceptual understanding a realistic goal for these students? Finally a brief synopsis of literature related to inclusion was presented and discussed, and six readings distributed. The six readings were: Carpenter and Lehrer (1999), Charles, (1999), Grouws and Cebulla (2000), pages 125 to 129 of Luke et al. (2003), Ollerton (2001), and van Kraayenoord (1999). Teachers were encouraged to read the first three of these and to either trial one of the 1-100 board activities or some other open-ended task and to come prepared to share the results of this, in whatever form they chose, at the next session.

## *Session 2*

The second session began with a discussion of the three readings that the teachers had been asked to consider. These focussed on teaching mathematics for understanding. The teachers also shared relevant experiences since the previous session. Material was presented on the development of number sense, mental computation focussed on the basic facts of addition, subtraction, multiplication and division and with an emphasis on visualisation including the use of ten frames, the 1-100 board and empty number line. The difference between the approaches advocated and rote learning were discussed at some length. One teacher described and demonstrated her use of Linear Attribute Blocks (Stacey, Helme, Archer, & Condon, 2001) for teaching decimals in the middle grades. This led to a more general discussion of the development of place value including the use of calculators and number slides. Using number lines to teach negative numbers was also discussed and, in particular, the confusions that can arise between the number line and place value chart. The teachers were provided with a display folder for handouts, notes and readings, and a set of materials selected to remind them of some of the ideas presented during the program. These were a four function calculator, a laminated 1-100 board, and assorted dice. Teachers were encouraged to read the second three articles and to either trial one of the activities mentioned or some other related task and to come prepared to share the results of this at the next session.

## *Session 3*

As participants arrived they were asked to write a number (decimal, fraction, percent, or integer) on a card and to peg it on a line stretched across the room. The mathematical ideas that could be accessed through such an activity were discussed followed by a brief discussion of the readings. Participants then divided into groups of their choice to discuss their experiences since the last session related to the use of one of: 1-100 boards, mental computation, empty number lines, the use of calculators for developing place value, ten frames, or teaching multiplication facts. All teachers were engaged in a discussion and were asked to focus particularly on the effectiveness of the activities for students with MLD in terms of developing these students' mathematical understandings. Further ideas related to common misconceptions and developing understandings of decimals and fractions were introduced and related to the initial activity involving pegging numbers on a line. A significant discussion of the difficulties involved in getting older students to engage with concrete materials arose during this time. Approaches to teaching children to tell the time were also discussed as well as the role of metacognition and possible ways to assist students to develop their metacognitive abilities. The final 10-15 minutes was used to complete the NSMLD questionnaire for the second time.

Paired sample t-tests were used to compare differences between the teachers' mean responses to items in Sections 2 and 3 of the survey at the beginning and end of the program and, in the case of section 3, between their responses in relation to students with MLD and students generally. Effect sizes were calculated by dividing the mean by the standard deviation (Burns, 2000).

## Results and Discussion

The number of participants responding "True" to each Section 1 item on each occasion is shown in Table 3.

Table 3
*Responses to the nine "Maths Myths"*

| | Number responding "True" | |
|---|---|---|
| Beliefs about Mathematics | Pre-test (N=22) | Post test (N=22) |
| 1. Some people have a maths mind and some don't. | 18 | 13 |
| 2. Maths requires a good memory. | 11 | 1 |
| 3. There is a best way to do a maths problem. | 1 | 2 |
| 4. Men are better at maths than women. | 2 | 3 |
| 5. Maths is easy to teach because the answer is either right or wrong. | 0 | 2 |
| 6. Mathematicians do problems quickly, in their head. | 6 | 1 |
| 7. Children who have difficulty with maths naturally do not have a maths mind. | 5 | 3 |
| 8. Maths requires logic not intuition. | 11 | 3 |
| 9. Maths is a series of rules to be memorized and followed. | 4 | 0 |

Small numbers of participants agreed with Items 3, 4 and 5 on both occasions. Overall the changes that occurred were overwhelmingly in the desired direction and pleasing in view of the relative brevity of the program. The change in relation to Item 2 was pleasing given the concern expressed by several teachers in the first workshop about the inability of many students who experience difficulty with mathematics to remember basic facts. The teachers may have taken on board ideas about the importance of teaching strategies for fact reconstruction should they be forgotten.

Similarly the change for Item 6 is consistent with a focus on understanding and thinking rather than simply speed, and those for Items 8 and 9 are consistent with the program's emphasis on the importance of students viewing mathematics as a sense-making activity.

While there was a marked reduction in the number of participants indicating agreement with Item 1, the fact that more than half of the teachers still professed this belief is of concern. The use of this item with pre-service teachers indicates that this is one of the most widely held and difficult beliefs to influence (Beswick, 2006). Fortunately the participants in this program seemed reluctant to believe this about children (see Item 7) but nevertheless it suggests an underlying tendency of teachers to cite the cause of students' difficulties beyond the influence of teaching, as well as an acceptance of, or resignation to, the fact of any inadequacies with respect to mathematics that they perceive in themselves. Neither is helpful. A sustained program providing opportunities for teachers who lack confidence in mathematics to experience success (defined in terms of understanding) would be likely to help, but it is pleasing to note that even this brief program may have made some impact in a positive direction. The apparent contradiction between some teachers' responses to a statement that they may have related to themselves and a similar one that related to children is possibly an example of beliefs clustering as described by Green (1971).

The second section of the survey administered at both the beginning and end of the program concerned the participants' attitudes to mathematics and beliefs about mathematics. Unsurprisingly, given the deep seated nature of these constructs, the fact that they were not directly addressed, and the brevity of the program, there was a statistically significant change in relation to just one item, "Mathematics makes me feel uneasy and nervous". The mean response changed from 2.14 to 1.86. The change was significant at the 0.05-level ($p$=0.030) and the effect size, 0.50, was medium. It is possible that the teachers considered this item in terms of the anxiety they felt about teaching students with MLD and that the program had provided them with some useful ideas in this regard. It could also be related to changes noted in relation to several items in Section 1, particularly Items 1, 2, 8, and 9. To the extent that the teachers regarded themselves as struggling with mathematics as a result of innate lack of ability and/or an inadequate memory their anxiety concerning the discipline could have been alleviated by changes in their beliefs regarding the importance of these factors to learning mathematics.

As reported by Beswick (2005b) there were statistically significant differences for some items in Section 3 of the NSMLD survey. These included differences between beliefs in relation to students with MLD and students generally, both at the beginning and the end of the program, and differences between responses in relation to either group of students between the two administrations of the survey. Items for which there were statistically significant differences for the two groups of students are shown in Table 4. Items that elicited different mean responses on the second administration of the questionnaire are italicised.

Table 4
*Items eliciting significantly different responses for all students and students with MLD*

| | Mean (all students) N=22 | Mean (Students with MLD) N=22 | Mean diff. (All-MLD) | Std Dev. | Sig. (2-tailed) | Effect size |
|---|---|---|---|---|---|---|
| 3. Conceptual understanding is an appropriate goal of mathematics students. | 4.09 | 3.81 | 0.29 | 0.56 | 0.030* | 0.51 |
| *3. Conceptual understanding is an appropriate goal of mathematics students.* | 4.43 | 4.24 | 0.19 | 0.40 | 0.042* | 0.47 |
| 8. Students should not rely on concrete material rather than thinking for solving mathematics problems. | 2.05 | 1.64 | 0.41 | 0.67 | 0.009** | 0.61 |
| 11. Providing students with 'survival' mathematical skills is an appropriate goal of mathematics instruction. | 3.27 | 4.18 | -0.48 | 1.15 | 0.001** | 0.79 |

*p<0.05. ** p<0.01.

Higher mean scores indicate greater agreement with statement, and italics indicate differences that were obtained on the second administration of the survey. The effect sizes obtained were medium in the case of Item 3 at both administrations of the survey and medium and large for Items 8 and 11 respectively.

The participants began the program significantly less inclined than at the end to see conceptual understanding as an appropriate goal for students with MLD compared to students generally. Rather, they regarded survival skills as more appropriate for these students and were more inclined to see concrete materials as supporting answer getting, rather than the development of understanding for these students. This is consistent with their conversations in the first professional learning session about the problems such students tend to have with retaining facts. There was still a statistically significant difference between participants' beliefs about the two groups of students in relation to conceptual understanding as a goal, at the end of the program. The very significant difference in relation to Item 11 at the start of the program did not exist at the end, suggesting that participants finished the program less inclined to believe that 'survival' mathematics was the province of students experiencing difficulty learning mathematics.

Table 5 shows items for which there were significant changes from one administration of the survey to the next, in relation to either all students or to students with MLD. In this case items relating to students with MLD are italicised.

Table 5
*Items eliciting statistically different responses at the two administrations of the survey*

| | Initial Mean n=22 | Final Mean n=22 | Mean diff. (initial-final) | Std Dev. | Sig. (2-tailed) | Effect size |
|---|---|---|---|---|---|---|
| 4. Telling children the answer is an effective way of facilitating their mathematics learning. | 2.82 | 2.14 | 0.62 | 0.92 | 0.006** | 0.67 |
| *4. Telling children the answer is an effective way of facilitating their mathematics learning.* | 2.77 | 2.10 | 0.62 | 0.92 | 0.006** | 0.67 |
| *8. Students should not rely on concrete material rather than thinking for solving mathematics problems.* | 1.64 | 2.10 | -0.48 | 1.03 | 0.047* | 0.46 |
| 21. Explicit teaching in mathematics should focus on l task requirements, strategies, and highlighting significant mathematical learning. | 3.59 | 3.95 | -0.38 | 0.80 | 0.0.42* | 0.47 |

*p<0.05. ** p<0.01.

Following the program participants were less likely to believe that telling students answers was an effective way of teaching them. The change was significant and the effect size medium to large in relation to both students generally and those with MLD. Consistent with this was the change in relation to participants' opinions regarding what should be made explicit in mathematics teaching for all students. Care was taken in the delivery of the program to define explicit mathematics teaching in terms consistent with Item 21 (see Table 5) and not as prescribing procedures for solving problems or performing calculations. The participants were also more inclined, after the program, to reject the notion that students with MLD should use concrete materials as a substitute for thinking to get answers. The significant shift that occurred in relation to the relevant item (Item 8) for students with MLD between the two administrations of the survey brought the teachers' views in this regard in line with those concerning all students.

It was not possible to design a professional learning program within the constraints that existed (particularly in relation to time) that optimally satisfied all of Hawley and Valli's (1999) criteria for effective professional learning. Nevertheless the program was driven by student performance to the extent that the teachers were encouraged to focus on the needs of the particular students in their classrooms who were experiencing most difficulty in learning mathematics. The teachers had significant input into the content of the program, particularly in the second and third sessions, not only because the content was designed around the issues and topics that they nominated but because the content and direction of discussions was only loosely constrained by the researcher. The number of teachers involved made this more feasible than would have been the case with a larger group.

The program was not physically conducted in a school. In addition, 14 of the 22 teachers were the only participant from their school. In this sense the program was not school based, but considerable effort was made to assist the teachers to connect their learning in the program with their own classroom contexts. Allowing the teachers to determine content and to drive discussions was crucial in this respect and also contributed to the collaborative nature of the experience. Rather than presenting answers, the researcher attempted to provoke discussion and to engage with the participants in conversations that advanced collective understandings. Three sessions rather than a one-off event provided at least limited continuity and funding was available for the teachers' travel and relief. Multiple sources of information were provided in the form of presentations from the researcher, readings, input from participants, and indications of where further resources and information could be sought. The researcher presentations and readings included references to theoretical perspectives and the spacing of the sessions (three weeks between) allowed some time for learning to be consolidated and implemented. Implementation was specifically encouraged via requests to trial and share activities and approaches.

The fact that the participants were volunteers was likely to have been central to its effectiveness. Volunteering for the program can be seen as indicative of the teachers' desire to improve their students' achievement in mathematics, and of their willingness to change their practices. Teachers with these characteristics, identified respectively by Breyfogle and Van Zoest (1998) and Chapman (1996), are most likely to change in response to professional learning. The extent to which the teachers were relativistic thinkers (Arvold & Albright, 1995) was unknown, as was their propensity to reflect (Wilson & Cooney, 2002), but in attempting to embody Hawley and Valli's (1999) the program endeavoured to encourage such thinking.

## Conclusion

Answers to each of the research questions are provided in this section followed by a discussion of the study's implications.

### *Research Question 1*

The small number of participants in this study warrant caution in drawing conclusions from its results. Nevertheless, this study provides evidence that teachers do hold differing beliefs about mathematics teaching for students with MLD compared with other students. These differences related to the relative appropriateness of conceptual understanding as a goal of mathematics teaching, the role of concrete materials, the relative importance of basic mathematics skills, and the effectiveness of telling children the correct answers. In each case the difference in the teachers' beliefs was such that students with MLD would experience less broad, rich and challenging curricula than other students. Such a situation is counter to the aims of DETYA (2000) and DoET (2006a).

The result with respect to the use of concrete materials, specifically that their use for answer getting rather than for supporting conceptual development, was more likely to be sanctioned for students with MLD than others, is particularly important given the ongoing centrality of thinking in the Tasmanian curriculum (DoET, 2006b). It is also illustrative of the point made by Askew, Brown, Rhodes, Johnson, and Wiliam (1997) that superficially similar classroom practices may have different outcomes depending on the underlying beliefs of the teacher. It is certainly not sufficient to mandate the use of particular materials or approaches without addressing the relevant beliefs of teachers.

### *Research Question 2*

The study provided encouraging evidence that the beliefs of teachers in relation to mathematics, mathematics teaching, and students with MLD are susceptible to change. The professional learning program that formed the basis of this study was designed to comply, as far as possible, with what is known about the features of effective professional learning as summarised by Hawley and Valli (1999). Volunteers, as the participants in this study were, are more likely than others to be motivated to improve their students' learning and willing to change. The improved outcomes of professional learning for voluntary participants compared to non-volunteers have been noted elsewhere (e.g., Britt et al., 2001). The fact that the beliefs of these teachers did seem to change indicates that the relevant beliefs were evidentially held (Green, 1971). Non-volunteers, who may not be so convinced, would likely benefit from greater exposure to evidence of both the need for, and possibility of, change. Hawley and Valli (1999) suggest that evidence of students' achievement in the teachers' own classrooms could be most useful in this regard. A more sustained, school and classroom based approach would also be likely to be more effective regardless of the participants.

## Implications

While it seems that the program, despite its brevity, had some success in influencing the academic expectations of teachers in relation to students with MLD in ways likely to contribute to their improved achievement (Schoen et al., 2003), the extent to which the teachers actually changed practices is not known nor is the extent to which any such changes were sustained. Teachers were encouraged to engage in each of the sessions and activities with the needs of children in their own classes in mind in order to minimise the disjuncture between the contexts of the professional learning program and their classrooms and hence to maximise the chances that ideas discussed in the program would impact the teachers' classrooms (Beswick, 2003). Nevertheless, examination of both the impact of such programs on practice and the longevity of any changes requires a larger study.

It seems that the problem of translating policy concerning equity and inclusion into classroom practice that was identified by the Atelier Report (2004) is at least partly due, in the area of mathematics/numeracy, to beliefs that some teachers hold in regard to students with MLD and mathematics teaching and learning more generally. Attention will need to be paid to teachers' relevant beliefs if inclusive policy is to have a real impact on students with MLD.

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