Conceptualising Rehearsal Interactions Through an Examination of Rehearsals with In-service Teachers

Casey Hawthorne
Furman University

John Gruver
Michigan Technological University

Received: June 2022 ǁ Accepted: October 2023

Rehearsals have become an increasingly popular pedagogy to support preservice teachers in developing competency in enacting challenging instructional practices and communicating associated pedagogical commitments. To understand how best to support learners participating in rehearsals, researchers have begun to analyse the facilitation of teacher educators leading such pedagogy. These studies have revealed a variety of ways teacher educators can structure rehearsals. We contribute to this emerging body of knowledge by studying the facilitation of rehearsals in the context of professional development with experienced teachers. Our results expand researchers' understanding of the facilitation of rehearsals in three ways. First, our analysis revealed a range of new ways teacher educators can structure rehearsal interactions. This elaboration and refinement of earlier descriptions offers a variety of new tools for teacher educators to use when leading rehearsals. Second, we categorise these different structures based on their purpose and present three dimensions that contrast the different facilitation choices available. Such an organisation provides a framework that has the potential to aid teacher educators to be more purposeful as they consider the type of interactions they would like to support as well as researchers studying rehearsals to better understand the rationale and effect of such decisions. Third, our reflection about our choices in how we structured interactions when working with experienced teachers presents an example of how the facilitation of rehearsals might be adapted to better respond to the needs and strengths of in-service teachers.

Keywords • mathematics teacher education research • practice-based teacher education • rehearsals • pedagogies of enactment • professional development facilitation

Introduction

Ambitious teaching is based on the core principle that all students are sense-makers who bring valuable mathematical contributions to the learning community. As such, it is characterised by instruction that aims to develop flexible conceptual understanding by leveraging the contributions of a diverse learning community and positioning students' mathematical thinking at the center of decision making (Anthony, 2015b; Kazemi et al., 2016). While this view of teaching is at the heart of the mathematics instruction reform movement of the past several decades (Anthony et al., 2015b), supporting teachers in developing the ability to teach in such a way is quite challenging. For one, the very nature of ambitious teaching means it is necessarily dependent on context and the specific needs and ways of thinking within a particular learning community (Putnam & Borko, 2000). Therefore, teachers cannot simply be taught static skills to be implemented uniformly across contexts. Instead, teachers must develop adaptive expertise that enables them to "innovate when necessary, rethinking key ideas, practices, and values in order to respond to nonroutine inputs" (Lampert, 2010, p. 24). A second challenge is that teachers have very few examples to guide their understanding of ambitious mathematics teaching, as such a model differs significantly from dominant transmission forms of mathematics instruction found in most classrooms. Subsequently, any support must be designed thoughtfully to overcome the tendency of teachers to be socialised into the status quo (Anthony, 2015a).

Developing Ambitious Mathematics Teaching with Rehearsals

Given these challenges, several mathematics teacher education programs have begun to implement successfully a practice-based approach where instruction is the site of inquiry (e.g., Bailey & Taylor, 2015; Braseth, 2022). Among these different approaches, the use of what is commonly referred to as a rehearsal has become particularly popular. These are simulations of teaching situations in which a

teacher engages a group of peers playing the role of students (Kazemi, et al., 2009). Such a pedagogy is inherently relational and interactional, positioning novice teachers to engage in "the interactive work of teaching and not just to talk about that work" (Ball & Forzani, 2009, p. 503). This creates robust learning opportunities for students to simultaneously engage deeply with their peers' thinking and the big ideas in mathematics (Lampert, 2010). As the acting teacher strives to understand and adapt to needs of the learners, they develop firsthand experience grappling with the contextual nature of teaching, allowing them to explore and invent educative responses (Anthony, 2015a, 2018).

Moreover, a powerful characteristic of rehearsals is that during the enactment of a rehearsal a TE or other participants can pause instruction to highlight features, reflect on decisions, or explore alternatives (Lampert et al., 2013; Anthony, 2018). Such a structure not only makes public the work of teaching (Baldinger & Campbell, 2021), but also creates an environment where specific pedagogical decisions and actions are considered as a community. Those acting as teachers learn from their own decisions, but everyone learns by observing the instructional decisions, listening to the teacher educator's (TE's) interjections, and participating in the collective discussions around their peer's teaching (Kazemi & Wæge, 2015). Consequently, through the mutual navigation of instructional situations, participants not only gain first-hand experience in the details of how to respond to and elevate student thinking, but also develop a deeper understanding of the principles of teaching that guide such instructional practices. As such, they begin to internalise the professional commitments associated with ambitious teaching (e.g., the commitment to treat students as sense-makers; Ghousseini et al., 2015).

These affordances are particularly important in a mathematics learning context. The infusion of instructional practices and a professional identity that honour a diverse group of learners serves as a powerful force to challenge normative ways of teaching mathematics, such as the expectation for the teacher providing mathematical explanations (Anthony, 2018) and the tendency to react evaluatively to the correctness of student contributions rather than the mathematical thinking (Crespo, 2002). Furthermore, the affordances of rehearsals can help normalise methods of teaching that are not often found in mathematics classrooms such as sharing student thinking and learning from others. Already to date, studies have demonstrated the effectiveness of rehearsals to support teachers in enacting instructional practices that align with ambitious mathematics teaching, such as using representations to elevate student thinking and facilitate mathematical discussions (Wæge & Fauskanger, 2020) as well as provide models for how to facilitate a classroom community to engage in mathematical practices (Kobiela et al., 2022).

Despite the potential of rehearsals to support teachers in developing familiarity with and insight into challenging new instructional practices, researchers' understanding of how and why teacher educators structure interactions during rehearsals is still being developed (Kavanagh et al., 2020). Inevitably, without such understanding, teacher educators will struggle to adapt their facilitation as they attempt to use rehearsals in contexts not well reflected in literature on rehearsals (e.g., they are working with experienced in-service teachers). Thus, there is a need for researchers to specify various options for structuring interactions during rehearsals and outline the affordances of different choices. We contribute to this understanding by categorising the structure of interactions during a series of rehearsals with experienced in-service teachers (ISTs). Such a setting differs from that of a university methods course where most research around rehearsals has been grounded. This contrast offers potential insight into variations around facilitation in different contexts.

Literature Review

In this section, we first describe three common types of pedagogies that provide a theoretical framework often used to organise practice-based teacher education (PBTE). This background information serves to situate rehearsals in the broader research around PBTE. We then give an overview of the different ways and rationale TEs have structured rehearsals to highlight how our study contributes and builds on this growing body of research.

Types of Practice-based Pedagogies

Grossman et al. (2009) outlined three types of pedagogies common to many forms of PBTE: decompositions of practice, representations of practice, and approximations of practice. Decomposing

practice involves breaking down the practice of teaching and identifying the constituent parts. This allows teacher educators to highlight features of practice. Notably, this does not mean breaking down teaching into disconnected component skills. Rather, when decomposing practice, the constituent parts are still regarded as highly situational and fully embedded in the practice of teaching.

Representations of practices are depictions of teaching or a component of teaching (e.g., videos of teaching, narratives, teacher educator demonstrations). Different representations can make visible (or hide) different components of practice. For example, videos may show interactional features of teaching and learning in their full complexity but hide the teachers' rationale for their decisions, whereas written narratives can do the opposite (Grossman et al., 2009).

Approximations of practice are activities where teachers engage in teaching practices outside of the teaching context. Approximations can vary in terms of completeness and authenticity (Grossman, et al., 2009). At times, the inauthenticity of an approximation can be an affordance as it can allow for a greater focus on key components of teaching (Davis et al., 2017; McDonald, et al., 2013). As Grossman et al. (2009) noted, inauthenticity enables novice teachers to "focus on components of complex practice, allow[ing them] to hone their skills in a single element of [teaching] before they have to manage all the competing demands and conditions of uncertainty in actual practice" (p. 2092). However, approximations that lack some authenticity or completeness should retain the components of the target practice that are integral to that practice (Grossman & McDonald, 2008). For example, in the case of helping teachers learn to respond to student thinking, it would be reasonable to craft an approximation of teaching that limits some of the complexities of managing a classroom while continuing to ensure that teachers grapple with anticipating and responding to different ways students might reason.

Notably, approximations and representations can overlap. As expressed by Grossman and colleagues:

> Every approximation engages [teachers] in some element or version of practice, and so in that sense becomes a representation of practice for others. The distinction lies in the novice's role as observer or actor. A representation illustrates a facet of practice … whereas an approximation engages [teachers] in that practice. (2009, p. 2091)

Rehearsals lie on the borderline of approximation and representation. While rehearsals are often thought of as an approximation of teaching because the person acting as the teacher in the rehearsal enacts teaching practices, they can also function as a representation of practice for the rest of the community participating in the rehearsal.

Research on Structuring Rehearsals

While researchers' understanding of how TEs structure rehearsals is in its infancy, several studies provide insights into this process. One of the few studies that investigated the choices TEs make while facilitating rehearsals was a large-scale analysis of 90 rehearsals with preservice teachers (PSTs) conducted by Lampert and colleagues (2013), in which they coded the interactions of the teacher educator with the novice teacher. We refer to these as interruptions in the rehearsal, since these interactions often disrupt the flow of the lesson. While their coding of their interruptions focused on substance (i.e., what was being talked about), they also coded the structure of these interactions using four structure codes. Specifically, they found that the majority of interruptions included directive feedback (61%), with other interactions involving TEs providing evaluative feedback (28%), role playing the teacher or a hypothetical student (21%) or leading a discussion (17%). While this analysis presented an initial depiction of how TEs use interruptions to structure rehearsals, the four structure codes were not intended to describe the full range of options of how teacher educators might structure rehearsal interactions as well as the affordances of different choices. Other studies, however, help paint a more nuanced picture.

For example, some studies have emphasised how rehearsals can be opportunities for reflection in action (Schön, 1982). Davis et al. (2017) described their rehearsals this way, using the enactments as objects of inquiry, often directing participants' attention back to aspects of the performance that had gone unnoticed to motivate in-the-moment reflection. Similarly, Averill et al. (2016) argued that given the complexity of orchestrating a mathematical discussion, it can be productive to use interruptions to immediately reflect on details of instructional decisions. As such, they initiated 90% of their interruptions with a question, soliciting the rationale behind teachers' pedagogical decisions and

exploring alternative pedagogical choices. This questioning resulted in longer interactions, that engaged the whole class in discussions, which they believed allowed students to co-construct the meaning behind the instructional moves implemented.

Other studies have highlighted how TEs have used interruptions to solicit ideas from participants about how to productively move forward (e.g., Averill et al., 2016; Davis et al., 2017; Peercy & Troyan, 2020, Wæge & Fauskanger, 2020). For example, Davis et al. (2017) documented that the most prominent reason for TEs in their study to interrupt was to engage all participants in problem solving around how to effectively address problems of practice. Similarly, Averill et al.'s (2016) repeated questioning (as opposed to more direct feedback) served to engage the whole class in exploring alternative pedagogical choices. They argued that such a structure allowed students to co-construct the meaning behind the instructional moves implemented. Finally, Peercy and Troyan (2020) provided an example in which the TE facilitated a conversation among the participating teachers to debate various possible pedagogical options.

In addition to the various characterisations of how TEs have chosen to facilitate rehearsals, studies have also provided insight into the reasoning behind such choices. Peercy and Troyan (2020) provided a theoretical rationale for why TEs might foster problem-solving interactions, arguing that TEs must engage participants in discussions around pedagogical options to model responsive teaching and to align their facilitation with the tenets they are espousing. Conversely, Averill et al. (2016) offered up a practical reason for such a structure, describing how their focus on discussing possible pedagogical choices allowed more PSTs to take a prominent role in the rehearsal, easing some of the pressure to ensure that every PST had an opportunity to rehearse. Finally, Drake (2016) organised his rehearsals around the pedagogical rationale that encouraging participants to offer their suggestions would create an environment that would support risk-taking and innovation.

However, it is not just the TEs' goals and orientations that can affect why a TE might facilitate a rehearsal in a particular way. Researchers have also noted that the context of the rehearsal influences facilitation (Davis et al., 2017). Notably, Anthony et al. (2015b) described how their rehearsals were initially more directive in nature but became more reflective over time. This change seemed to be enabled by the establishment of norms surrounding the rehearsal process. As the purposes of the rehearsal became negotiated in the community of learners, the goals of the rehearsal could be supported by the wider group, rather than solely by the TE. Similarly, Drake (2016) noted that over time he intentionally relinquished some of his positional power as a teacher. To better facilitate discussion of practice during teaching, he tried to talk less in later iterations in an effort to draw on the combined intellect of his students.

While these studies have added depth to Lampert et al.'s (2013) initial description of how TEs might facilitate a rehearsal as well as insights into why TEs might make certain facilitation choices, researchers and teacher educators lack an organised framework of choices that TEs may make as they pause instruction and facilitate interactions during rehearsals. Notably, while many additional studies have discussed how they structured interactions, only Davis et al. (2017) specifically coded for the structure of their interactions. Clarifying such choices would support TEs' decision making as they consider the relative affordances of different types of interactions. This is imperative as the use of rehearsals expands and is implemented in different contexts. As TEs adapt to different programmatic factors, accommodate the needs of different participants, and target different practices, they need a wider variety of and more nuanced ways to structure rehearsal interactions to be more purposeful in their facilitation of rehearsals.

Focus of Current Research

The aim of this study is to clarify potential choices TEs need to make as they facilitate rehearsals. To do so, we examined the structure of rehearsals with experienced ISTs. We chose to examine rehearsals with ISTs because we anticipated that working with ISTs would increase the likelihood that the structure of our interactions would differ from those described in the research on rehearsals with PSTs and thus bring into greater relief the choices TEs make.

While one might argue that the simulation nature of rehearsals makes them better suited for PSTs who need a more structured learning environment, researchers have already presented initial empirical evidence highlighting the benefits of using rehearsals with ISTs (Wæge & Fauskanger, 2020). Although

experienced ISTs inevitably possess a deeper understanding of the content they teach, are more aware of common student struggles, and are more comfortable managing classroom interactions—all valuable resources in learning to teach ambitiously—some of the core practices in teaching ambitiously may still be new to them. Therefore, ISTs need a safe space to experiment and learn with colleagues while they are learning to teach ambitiously (Horn, 2010). They need environments where mistakes are expected, not threats to their professional identities as competent teachers. Rehearsals offer such opportunities (Lampert, 2010), providing a structure where a group of teachers, as a community, can explore and analyse new instructional practices, while also serving to elicit their current understandings about teaching.

Nonetheless, while researchers have noted the possible benefits in using rehearsals with ISTs, they also acknowledge the probable need for adaptations in how they are orchestrated (Kelley-Petersen, et al., 2018). In particular, instead of using rehearsals to develop teachers' initial ability to interact productively with students and content, rehearsals might be used to develop further their existing abilities to orchestrate classroom discussions. Specifically, teachers can learn to leverage the contributions of a diverse learning community to develop conceptual understanding and the ability to engage in complex mathematical practices.

To understand the potential differences in how rehearsals might be facilitated with ISTs we investigated the following research question,

> *How did we as teacher educators structure rehearsal interactions as we attempted to respond to the needs and resources presented by the experienced teachers with whom we worked?*

The purpose of investigating this question was not to characterise how TEs should facilitate rehearsals with ISTs, but rather to develop a more fine-grained synthesis of the types of interactions available to TEs as they endeavour to meet the needs and make use of the resources of the varying types of teachers with whom they might work.

Methods

Participants: Teachers and Facilitators

The authors of this paper led a 20-hour, optional summer professional development (PD) session with experienced middle school teachers. Nine Grade 8 and five Grade 7 teachers from 12 schools volunteered to participate in response to a district wide solicitation. While the state mandated teachers participate in a professional activity every five years for certification renewal, a weeklong PD exceeded the minimum requirements. These teachers' willingness to devote a longer period of time to professional development speaks to their interest in improving their craft. The demographics of their respective schools varied considerably, ranging from Title 1[1] institutions to those serving predominantly affluent student bodies. Notably, except for one teacher who had just completed her second year of teaching, the other teachers were quite experienced, with a range of 8 to 38 years in the classroom and an average of over 17 years of teaching. In addition, the teachers further distinguished themselves with an expertise leading mathematical discussions and a collective interest in students' mathematical thinking. Although the teachers possessed many strengths in leading student-centered discussions, we anticipated that their understanding of students' ways of reasoning might not be organised in ways that would enable them to strategically leverage student thinking during these discussions (c.f., Carpenter et al., 1988).

Conversely, while the first author had 15 years' experience teaching secondary mathematics in a K–12 setting, both facilitators were relatively new to teacher education. However, as a result of work in previous research studies, both facilitators had developed a strong understanding of teaching figural patterns, the mathematical focus of the PD. Thus, as leaders of the PD, we viewed our expertise as complementing that of the teachers. We felt we could offer the teachers a space where they could think

[1] The school received supplemental federal funding because of the high proportion of students from low-income homes.

deeply about how to teach figural pattern tasks as well as explore potentially new ways of teaching in this context.

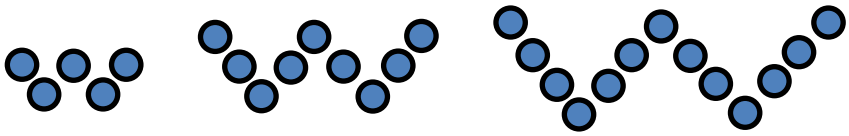
PD Structure

The PD lasted one week (5 days), with participants meeting each day for four consecutive hours. However, since none of the participants knew each other prior to volunteering for the PD (with the exception of two pairs), we organised a free lunch each day after the PD to help build community, which the 14 teachers attended regularly. Although the teachers were experienced, they were not familiar with rehearsals as a PD structure or the instructional activity (IA) we used to organise the rehearsals. We began the PD by familiarising the teachers with the IA we were going to use in our rehearsals. On Day 1 we modelled the IA by engaging the teachers as students in the lesson. This gave the participants firsthand experience in the instructional sequence and mathematical thinking at the heart of the IA. We then went through the different components of the IA, discussing the mathematical goals associated with each component, helping them anticipate student ways of reasoning that might emerge, and examining what contributions might be productive to leverage and why. On Day 2, we modelled the nature of rehearsals, with one author again leading the teachers through the IA while the other author interjected comments and suggestions from time to time. We also devoted time supporting teachers in leading and planning for a discourse rich lesson. On Day 3, the 14 teachers were placed in groups and given time to collectively plan a rehearsal, which one member of the group implemented as the teacher, who we called the acting teacher (AT). On Days 4 and 5, we videotaped the teachers as they collectively participated in six rehearsals.

The Instructional Activity

Working with a group of adept ISTs, we wanted to leverage their pedagogical expertise and competencies engaging with students' thinking. As such, we decided to tackle a more challenging instructional practice at the heart of student-centered instruction: the ability to leverage emergent student thinking and use classroom discussions to progressively build towards a complex mathematical goal. To do so, we created a new IA (see Table 1 for details), structured around figural patterns, with the mathematical goal of fostering a quantitative understanding of the associated algebraic expression. Thus, the IA was designed to support students in not only generalising the pattern, but interpreting how the resulting algebraic notation captures the different decomposed quantities identified within the figure (Knuth et al., 2005).

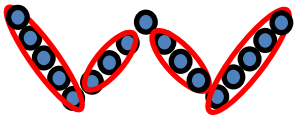
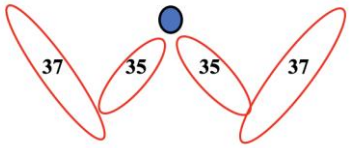
Not surprisingly, achieving such an instructional goal resulted in a longer than typical IA. To make each rehearsal a manageable length, we decomposed the overall IA into four smaller iterative phases, organised around the progressively more abstract representations (drawings, verbal descriptions, numeric expressions, and variable expressions), with which the students were asked to engage with the same figural pattern. Each of these phases served as the focus of a single rehearsal but were designed to follow the same lesson structure based on the five practices for orchestrating productive mathematics discussions (Stein, et al., 2008). We chose to use this framework because it offered teachers a way to think about how they might elicit and then leverage student thinking, a core tenant of ambitious teaching. During each phase of the IA, the teacher would first pose the task, then walk around the classroom probing student thinking to select and sequence student ideas, and finally lead a discussion, using discourse moves and board work to support students in making their thinking public. This design meant that each iteration involved the same general instructional practices, but differed in terms of the mathematical contributions that teachers might expect from students. Because of this self-similarity, each rehearsal could focus on the ability to adaptively support the development of the specific ways of reasoning associated with that phase. In Table 1, these phases are summarised and illustrated with possible student work on the example figural pattern task presented in Figure 1.



Stage 1 Stage 2 Stage 3

Figure 1. W task

Table 1
Four Phases of Instructional Activity

Phase	Representation	Teacher Actions	Possible Student Thinking
1	Drawing	*Introduce Problem:* Ask students to draw Stages 4 & 5. Select 2-3 students to share their picture and explain how they knew to draw it as such.	 It looked like a W with one new dot in each diagonal per stage.
2	Verbal description	*Encourage Students to Articulate Quantitative Relationships:* Ask students to describe in words what Stage 6 looks like. Select 2-3 students to share their understanding.	In Stage 6, the two outside diagonals had 7 and the two inside had 5 with the one dot in the center.
3	Numerical expression	*Foster Explicit Thinking*: Ask the class to write a numerical expression for a near (e.g., 6) and far stage (e.g., 37) that captures the structure of a particular way of decomposing the figure. Ask students to identify what quantity each number represents.	 37 + 35 + 1 + 35 + 37 The outside diagonals have one more than 36 and the inside have 1 less than 36.
4	Algebraic expression	*Link Quantitative Relationships to Symbols*: Ask students to write an algebraic expression for the n^{th} stage and then to identify what quantity each symbol represents.	$2(x + 1) + 2(x - 1) + 1$ Two outside diagonals each have 1 more dot than the stage number and the 2 inside diagonals have 1 less dot than the stage number. This leaves only the middle dot.

Data Analysis

We started by first identifying each interruption during the six rehearsals. We counted an interruption as beginning when the flow of the lesson stopped. This happened in several ways. A teacher educator could interject, a participating teacher could interject not as a student, or the AT could step out of her role as the teacher to offer an aside to other teachers and TEs. Generally, we counted the interruptions lasting until the teacher resumed teaching. However, there were also times when several discrete topics were discussed without the teacher resuming teaching in between. We counted these discrete discussions as separate interruptions when the substance changed notably.

We then coded each of these interruptions by who initiated it (TE or teacher) and in terms of its structure (i.e., the type of exchange that occurred). To code the structure, we began using the four a priori codes (directive, evaluative feedback, discussion, scaffold enactment) identified by Lampert et al. (2013). However, as we attempted to apply the codes, we found the descriptions for three of their

structure codes (Directive, Discussion, and Scaffold Enactment) to be quite broad. Therefore, in an effort to better characterise the exact nature of our interactions, we narrowed the definition of each of these codes. Specifically, we decided to define the directive code to denote only those interactions where the rehearsing teacher was to take up and try out the specific suggestion (as the name directive implies) and not include times when we speculated possibilities for a next move. Similarly, we used the discussion code to encompass only interactions when we reflected on general pedagogical issues without the purpose of deciding the next move. Finally, while Lampert et al. (2013) used the scaffold enactment code to refer to times when the TE took on the role of the teacher or a hypothetical student, no such moments involving us as TEs taking over the teaching duties that occurred during our rehearsals. Therefore, we used this code only to capture moments when we participated as a student in the rehearsal for the purpose of creating a particular problem of practice.

Ultimately, while we were guided by the framework introduced by Lampert et al. (2013), our goal of refining their codes and the resultant introduction of our modified codes meant we mostly used grounded theory (Strauss & Corbin, 1990) to formulate a set of inductive codes to capture the themes that emerged out of our interactions with the ISTs we observed. Our eventual codes emerged over several cycles of independently coding and then meeting to discuss the coding of each interruption. This process continued until we came to an agreement of the code for each interruption. This resulted in an additional seven codes, which are defined in Table 2 along with an example, which is representative of its use. Finally, we categorised the different structure codes by their function in the rehearsal, highlighting the different ways TEs can accomplish a similar purpose. Collectively, the pattern of our choices within and among these categories, illustrates our approach to facilitating and how we used the interactions to respond to the needs and resources presented by our participating teachers.

Table 2
The Structure Codes

Structure Codes (Lampert et al., 2013)	Description	Example
Evaluative feedback	Highlighting what was productive about a move or what could be improved on.	An AT represented student thinking incorrectly and a participating teacher corrected her, saying, "No, it's just one because the bottom dot is part of the second side."
Discussion	Engaging in a reflective discussion around general issues of instruction that emerged in rehearsal.	A participating teacher questions, "At what point would you ask if someone had another way? Would you have done that earlier? Would you do that later when you get a more common one?"
Refined structure codes		
Directive	Presenting a suggestion that was intended for the acting teacher (AT) to carry out.	A TE said, "I haven't heard [you] saying that we want an expression that captures that picture. I think making that clear [would be good]." The AT then launched the task again incorporating suggestion.
Scaffold enactment	Taking on the role of a student to increase the complexity of the ongoing engagement.	The AT tried to introduce a new interpretation of the figure, but the TE thought that students would need more support for this interpretation. Thus, he interjected as a confused student, saying, "I see why you drew the arrows there because you wanted to point out the five dots in the leg, but I'm not sure why we're talking about rows. There's no rows there."

Table 2 Cont.
The Structure Codes

New structure codes	Description	Example
Alternate move	A participant suggesting a specific teaching move but casting it as optional.	A TE said, "You gestured, but maybe … an arrow could be a way to accentuate the gesture. You also used the word column. So, I wonder if, [and] this is truly a wondering, if writing the word column up there might begin to support students in seeing something they're not seeing."
Negotiate the next move	Engaging in interactive discussion around an appropriate next pedagogical move.	An AT seemed unsure of what to do next and the TE paused instruction to ask the AT to share her thinking. In response, she explained that she liked a move that she had seen previously when another teacher represented a figural pattern abstractly, but she wasn't sure how to implement it in this case. A participating teacher suggested that she represent several boxes in the figural pattern with ovals. The TE then suggested what she might say as she drew these ovals.
Contextualising student thinking	Reflecting on authenticity of contributions relative to actual students.	A TE said, "You know, we've been talking about decompositions [interpretations of figures] for a week. But for your students, they're not even going to realise that … decomposing is a thing."
Highlighting	Drawing attention to a pedagogical act or decision made by AT.	A TE said, "I wanted to highlight [that] something we were grappling with was how does the teacher interact with the representations to highlight specific features. So, there are several ways that [Peg] did this."
Interpreting the situation	Making inferences about the meaning of an interaction or mathematical idea.	Students in the rehearsal seemed to be talking about a figural pattern in two similar but distinct ways. This distinction seemed to go unnoticed by the AT so a TE interjected, "We think there were two different ways people were thinking about this one decomposition."
Share thinking	Making public non-visible pedagogical actions such as professional noticing.	A TE asked, "Why did you pick these people [to share their work]?"
Managing the rehearsal	Reminding participants to maintain their role within the context of rehearsal.	A TE interjected "Let's all make sure…I would like us all to be students."

Note. With the exception of Managing the rehearsal, all codes could be initiated by the teacher educator or any participant.

Results

In this section we organise the codes and their frequencies into three categories. The first category, reflecting on pedagogy, describes situations in which participants reflected on how the pedagogical decisions shaped the instructional flow of the lesson. Within this category are the codes: discussion, highlighting, interpreting the situation, share thinking, and evaluative feedback. The second category, investigating subsequent teaching moves, includes interruptions that focused on what the teacher should do next and is comprised of the structures: alternate move, negotiate the next move, and directive. The final category, giving the student perspective, involves times when participants interjected features of student thinking into the rehearsal. It is comprised of the two codes: contextualising student thinking, and scaffold enactment. Of note, we did not include the code

managing rehearsal in any of the categories as such instances were not about pedagogy, but only included times when we reminded the participants of their role as students within the rehearsal.

In Table 3, we give the percentages of the interruptions with which each of the structure codes were given (individual column) along with the overall percentage of interruptions comprising of at least one of the different codes from that category (total column). In all, we identified 69 interruptions over the 6 different rehearsals. Two thirds of these interruptions were initiated by a TE and the remaining third were initiated by eight of the 14 participating teachers. Notably, in many cases the structure intended by whomever initiated the interruption was taken up differently by the other participants. For example, a TE might have introduced an alternative move, but the AT interpreted it as a directive, or the comment was followed by a whole group discussion. To account for these changes, we coded separately the structure of each interruption as it was initiated and taken up. While our analysis focuses on the initiated structure codes, we believe elevating this change provides a more accurate depiction of the interactions and helps to characterise the nature of the new codes as it shows authentically how they emerged. Furthermore, tracking how interruptions were taken up in addition to how they were initiated illuminated patterns in our interactions as well as revealed new codes. For example, the structure code, negotiate the next move, occurred almost exclusively as a result of how participants reacted to other queries, but was almost never initiated as such.

Table 3
Categorisation and Frequency of Structure Codes

		Initiated		Taken Up	
Category	Structure Codes	Individual	Total	Individual	Total
Reflecting on pedagogy	Discussion	19%	53%	29%	62%
	Share thinking	19%		23%	
	Interpreting the situation	9%		13%	
	Highlighting	4.5%		3%	
	Evaluative feedback	1.5%		1.5%	
Investigating subsequent teaching moves	Alternate move	22%	32%	13%	35%
	Directive	9%		13%	
	Negotiate the next move	1.5%		9%	
Giving the student perspective	Scaffold enactment	13%	20%	13%	19%
	Contextualising student thinking	7%		6%	

Several of the interruptions aligned with more than one structure code. Among the 69 interruptions, 77 codes were used to describe the initiated structure and 89 codes were used to describe how the structure was taken up. Since the percentage of each code was determined relative to the total number of interruptions (not relative to the number of total codes applied), the percentages in the individual columns add up to more than 100%. Similarly, since the percentages in the total columns reflect the percentage of interruptions receiving at least one code from that category and multiple interruptions received more than one code from the same category, the percentages in the total columns do not reflect the sum of the percentages of the corresponding codes.

Category 1: Reflecting on Pedagogy

Over half of the interruptions in our rehearsals (53%) were initiated to consider the affordances and limitations of pedagogical actions. An even greater proportion of interruptions was taken up in this way (62%). This illustrates that reflecting on pedagogy was a major focus of our rehearsals. Such a goal was most often pursued through discussions (19% of our interruptions were initially structured as a discussion and 29% were taken up as discussion) or share thinking (19% initiated, 23% taken up). As the example in Table 2 illustrates, participating teachers would sometimes pause the instruction themselves to invite discussions centered on pedagogical decisions. In fact, the most common reason for the participating teachers to initiate an interruption was to begin a discussion, with exactly half of

the discussions initiated by teachers. Similarly, asking a teacher to share her thinking after she had circulated around the classroom to examine student solutions emerged as a common structure. Teachers shared their thinking to make public non-visible practices such as the noticing of students' mathematical thinking (Jacobs et al., 2010) or the decision-making process behind the selecting and sequencing of student work. Again, such interruptions allowed us to discuss these decisions. The focus on discussions and sharing thinking, with the teachers themselves often initiating interruption to do so, illustrates our community's desire to not only develop the ability to enact particular pedagogical approaches, but also negotiate explicitly when, why, and how these approaches should be enacted.

In addition, the solicitation of teachers' insight also seemed to offer teachers space to share affective issues. Many of the teachers leading instruction began offering up their thinking voluntarily. At times, the AT would use this as an opportunity to share why a particular moment was difficult or how something did not go as she had expected. For example, after an exchange when an AT probed a student's thinking to make it public, she shared, "And I will tell you, just as an aside, this little part did not go the way I had planned. Because as I was looking around, I thought I was heading to a little more explicit [mathematical relationship] and it hasn't exactly worked out that way." As teachers were grappling with how to respond to and leverage student thinking, elevating these challenges appeared to be a way to learn and to lower their risk. These moves seemed to contribute to the creation of a community where rehearsals were viewed as a safe way to experiment with practice.

While less common than discussion and share thinking, two other structures in this category, highlighting (4.5% of interruptions as initiated, 3% as taken up) and interpreting the situation (9% of all interruptions as initiated, 13% as taken up), occurred with some frequency. These two structures are related, with the latter building on the former. Interpreting the situation refers to instances when the speaker shared how she understood an event or mathematical idea. This code differs from highlighting in that the speaker goes beyond simply elevating a particular moment and speculates about the affordances and limitations of such a move. Although interpreting the situation inherently requires highlighting, we did not apply both codes. Instances in which the speaker stopped instruction to note a particular move and consider the associated consequences were coded only as interpreting the situation, whereas highlighting was used in cases where the comment did not go beyond simply describing.

Finally, we note that evaluative feedback was rarely given (1.5% of interruptions as both initiated and taken up). Instead, more open forms of making observations became the norm, suggesting that we preferred structures that encouraged other members of the community to comment and collectively reflect on the pedagogy.

Category 2: Investigating Subsequent Teaching Moves

The most frequent reason for an interruption within our rehearsal was to suggest an alternative move (initiated in 22% and taken up in 13% of exchanges). Fifteen interruptions were initiated as alternate move, with at least one occurrence in each of the six rehearsals. Of the 15 interruptions that were initiated in this way, nine were initiated by a TE and six were initiated by a participating teacher. In contrast only six interruptions were initiated as directive (9%). However, many more interruptions were taken up as such (13%). Not surprisingly, of the few exchanges that were structured as directive, all but one was initiated by the TE.

While Lampert et al.'s (2013) previous code of directive most likely included instances of alternative move, we believe the distinction between these two codes is important in characterising the nature of our rehearsals. The two types of interaction, although both focus on exploring what a teacher could do next, are qualitatively different. In directive exchanges, the expectation is that the teacher will try out the suggestion, usually because the move is viewed as productive. This can elevate issues the teacher should consider while making decisions, including professional commitments or critical features of the lesson. In the example presented in Table 2, the TE highlighted the need to communicate clearly to students that numerical representations should be strongly tied to the quantitative relationships they found in previous phases. In contrast, suggesting an alternative move is cast as an opportunity to explore the affordances of a move rather than a prescription for the teacher. In fact, in our rehearsals, the optional nature was made explicit at times. For example, at one point during a rehearsal, a TE interrupted to say, "Another idea, … in fact I don't think this is a good idea, … is to say, 'Jessica just wrote this up. Can someone explain how Jessica might be thinking about it?'" Since the TE explicitly

said he did not think his suggestion was a good idea, it appeared that he did not intend to tell the teacher what to do next, but rather was inviting discussion about the affordances of different options.

The much higher frequency of interruptions involving a TE or teacher suggesting an alternative move rather than to make a directive implies that as a community, we preferred structuring the rehearsals to initiate conversations about what to do next rather than focusing on expanding the teachers' images of how one might respond to students. This characterisation is consistent with the finding that several interruptions (9%) were taken up as negotiate the next move. While it did not become a norm to stop instruction to explicitly solicit ideas about what the teacher should do next, as only one interruption (1.5% of all interruptions) was initiated in this way, the six instances that turned into negotiating the next move suggest teachers felt comfortable giving input about what to do next during general discussions about pedagogy or as the AT was sharing her thinking about an instructional decision. In the one instance that was initiated as negotiate the next move, the AT stopped her instruction and asked for help as to what to do next. Multiple teachers provided possible options before the conversation converged on a particular way forward.

Category 3: Giving the Student Perspective

Another feature of our rehearsal, as illustrated by our interactions, was our desire to introduce specific details of student thinking into the rehearsal. One way we did this was through the structure scaffold enactment, in which the TE participated in the rehearsal as if he were a student in the classroom. This occurred in five of the six rehearsals and in a total of nine interruptions (13% of total). In addition, we used the structure contextualising student thinking, where we offered up commentary of details from our experience about common student challenges, to further support participants in reflecting on the nuances of student thinking. There were five different instances (7% of interruptions), which were initiated as contextualising student thinking, appearing in three of the six rehearsals.

The need for multiple structures to elevate student thinking stemmed from the multiple occasions throughout our rehearsals when the instructional demands were lowered because the teachers, acting as students, provided more detailed and sophisticated thinking than would likely be immediately available in typical classrooms. Such inauthentic contributions often captured the full trajectory of the mathematical thinking the lesson was to engender, lowering the demand on the teacher to leverage less articulate and emergent thinking. Similarly, on other occasions, the AT, through her choices of selecting and sequencing, was able to avoid examples of less articulate student thinking. Consequently, the AT was not tasked with grappling with how to respond to authentic, more emergent, student thinking in discussions. To counteract this tendency, the TEs would scaffold enactment, by interjecting authentic student thinking so that the AT was forced to respond to it. Such student contributions were often dismissed, characterised as only coming from a problematic student who was intentionally creating havoc for teacher, rather than a genuine form of reasoning that students struggling with the material might offer up. Therefore, we as the TEs felt the need to further elevate details of student thinking by offering commentary to contextualise the student thinking, which involved describing how student reasoning in the rehearsal compares to authentic student thinking we had experienced in the classroom.

Discussion

The three categories of structure codes documented in our analysis illustrate the nature of interruptions we facilitated during our rehearsals and how we responded to the interpreted needs of our participating ISTs. Undoubtedly, the choices we made in how to structure our rehearsals were shaped by our orientation as educators. However, the trends in our decision-making do not seem to be simply artifacts of our personal idiosyncrasies as TEs as many of the structures we used have been documented in the literature, highlighting that we were leveraging, to some degree, general approaches. Below, we identify three dimensions, each aligning with one of the three categories we identified and capturing a different aspect of how facilitation choices can vary. These dimensions emerged as we reflected on how repeated choices of how to structure interruptions within each of the three categories shaped the nature of our rehearsal. We then illustrate how two contextual features, stemming from our particular situation of working with experienced ISTs, shaped our decisions about how to structure interruptions and thus where our rehearsals fell along these dimensions. In addition, we discuss ways this emergent

framework might be used more generally, informing both TEs and researchers alike of how to structure rehearsals to best meet the needs of their audience and pedagogical goals.

Dimension 1: Enactment Versus Reflection

The first dimension, enactment versus reflection, describes how a rehearsal can vary as a TE makes choices about which structures to use within the category, reflecting on pedagogy. As Table 3 shows, the majority of our interruptions featured a code from this category. Such a high frequency of reflection suggests that our moves structured the rehearsals in such a way that participants were not only able to gain experience trying out new instructional techniques, but also allowed us as a community to discuss these particular episodes as they occurred. As Grossman et al. (2009) described, an activity can function as both an approximation of practice and a representation of practice. While our rehearsals served as an approximation of teaching for the acting teacher, our real time reflection meant that our rehearsals often functioned as a representation of practice as well. As such, we note that TEs, through their facilitation, can modify the way teachers experience the rehearsal. Such a shift is embodied in Dimension 1, which is characterised by the level to which the rehearsal focuses on the firsthand enactment of instructional practices or the contemporaneous analysis of instructional practices.

To highlight the differences along this dimension, we note that during our rehearsal we chose to stop and collectively reflect on particular situations to elicit participants' professional understanding and motivate conversations about critical pedagogical issues. This is similar to the way Averill et al. (2016) described their facilitation. They believed that open reflections would support PSTs' grappling to make meaning of the different situations. They saw such a structural change as particularly important given that not every student teacher had the chance to rehearse as an AT due to constraints of their teacher preparation program. In contrast, other TEs have focused on giving participants experience navigating different pedagogical issues through the enactment of particular practices, minimising the level of reflection during the actual rehearsal. While such models often include reflection within the overall rehearsal cycle (e.g., McDonald et al., 2013), shifting it outside of the rehearsal structure itself inevitably modifies that nature of reflection, altering how participants process the pedagogical choices in the moment and what information they draw on to make decisions moving forward.

Thinking about facilitation through this continuum, we see affordances with structuring rehearsals both towards enactment and reflection. While in-the-moment discussions can leverage details of instruction and support participants to attend to and grapple with pedagogical nuances, such reflection can also overwhelm them as they attempt to simultaneously manage instruction and analyse the affordances of their decisions. Therefore, a focus on enactment might be more appropriate with participants who are struggling to implement challenging new practices and need more hands-on experience before they are ready to reflect on the details of possible ramifications of decisions. Regardless of the ultimate choice, we believe it is important for TEs to understand that their choices in structuring interruptions shape the degree to which participants experience moments within the rehearsal primarily as an opportunity to enact details of new practices or as objects of reflection.

Dimension 2: Nature of Instructional Guidance-Scaffolded Versus Exploratory

Our choices within the second category, investigating subsequent teaching moves, capture how we attempted to use rehearsals as a tool to collectively explore possible instructional options. While we gave directives at times, the majority of the interruptions were used to encourage the community to negotiate possible paths forward (23.5% of the interruptions were initiated as either alternative move or negotiate the next move versus only 9% being initiated as directive, see Table 3), often highlighting the affordances of the different choices. Throughout our PD, when offering details about enacting specific teaching moves, we tended to cast our suggestions as optional, positioning them as choices to be explored. This invited our community of ISTs to share alternative choices, fostering a culture where teachers felt more comfortable voicing their ideas and serving to draw in their expertise. In fact, many of the teachers leading instruction began offering up their thinking voluntarily. At times, the AT would use this as an opportunity to share why a particular moment was difficult or how something did not go as she had expected. As teachers were grappling with how to respond to student thinking, elevating these challenges appeared to be a way to learn and to lower their risk. These moves seemed to

contribute to the creation of a community where rehearsals were viewed as a safe way to experiment with practice.

As described in the literature review, our facilitation of rehearsals aligns with the problem-solving approach taken by other TEs (Davis et al., 2017; Peercy & Troyan, 2020; Wæge & Fauskanger, 2020), who used interruptions to elicit suggestions from participants about possible responses to various problems of practice. It also contrasts with other studies who seemed to have constrained the decision-making role of participants. For example, the TEs in Lampert et al.'s (2013) study played a prominent role in guiding the in-the-moment decisions of the ATs through directive feedback, the most prevalent interaction, and by taking on the role of the teacher. Such specific guidance serves to introduce new ways of interacting with students and allows participants to grapple with the meaning and implementation of these new instructional moves without being overwhelmed with the task of coming up with the ideas themselves. We see both approaches as appropriate in certain contexts. While less structure can help engage teachers in problem-solving and position participants as responsible and contributing members of the community, evidence suggests that a more scaffolded facilitation approach can be quite effective with novice teachers. Kavanagh (2020), working with early career teachers, found that limiting participants' options and providing specific moves to implement supported teachers in focusing on details of how best to respond to student ideas. These examples illustrate that facilitation can shape the role of participants within the community in terms of negotiating instructional choices.

Dimension 3: The Need to Interject Student Thinking

Our attempts to consistently interject student thinking into our rehearsals, as reflected in the category Giving the student perspective, highlights how important we perceived authentic student thinking was in achieving our instructional goals. Twenty percent of our interruptions involved different ways in which we infused student thinking into the rehearsal. In addition to playing the role of students, we repeatedly felt the need to offer commentary to contextualise student thinking and at times substantiate its authenticity, as certain contributions were avoided or dismissed.

The need to infuse authentic student thinking in rehearsals, while described in a few projects, has not been identified as a universal challenge associated with rehearsals. Most studies seem to function well with the occasional role playing by the teacher to introduce problems of practice (Lampert et al., 2013). Such examples are often accompanied with other artifacts of student thinking (student work, videos, etc.) introduced during the full rehearsal cycle (e.g., McDonald, et al., 2013). This combination seems sufficient for most contexts as issues around authentic student thinking have not been overwhelmingly raised. In contrast, Campbell et al. (2020) noted that PSTs in their study, as a result of their limited experience, struggled to come up with interjections that captured the essence of learners' emergent thinking. Rather than formulate misconceptions grounded in sense making, PSTs, acting as students in their rehearsal, introduced simple mistakes or superficial miscalculations that merely needed to be corrected, not examined and deliberated. To overcome this challenge, they introduced detailed planted errors that included the exact reasoning necessary to overcome their misconception. These examples suggest that the level to which authentic student thinking is necessary within a particular rehearsal can vary, depending on the context. This range is encapsulated by Dimension 3. In our rehearsal, because our targeted practice relied heavily on responding to nuances of student contributions, the necessity of authentic student thinking was particularly prominent.

Illustrating Context's Role in Facilitation

We now illustrate how context can shape where rehearsals fall on these dimensions by reflecting on how two contextual features seemed to contribute to our decision-making working with ISTs. First, we consider how the experience and identity of our participating teachers affected our decision making. Second, we examine how our pedagogical goal influenced our responses. Delineating how we structured the interactions in response to these features highlights how TEs might adjust their facilitation to accommodate the differences in their unique contexts. While we do not suggest that how we chose to orchestrate our rehearsals be followed universally in all PD settings, we do highlight how our perception of the strengths and attributes of the participating ISTs influenced our decision making and make connections to other studies that have made similar observations. Furthermore, we see these

contextual factors as a productive lens to look at rehearsals more generally, whether for TEs to consider as they decide where best to position themselves on these dimensions or for researchers as they analyse and make sense of rehearsal work.

Experience and identity of participants

We see specific ways that the experience and identity of the participating ISTs in our study influenced how we structured our rehearsals. These teachers had a wealth of instructional expertise and an interest in learning even more about teaching and learning. At the same time, we also found that their years in the classroom had resulted in well-formed views about pedagogy and student thinking that were not wholly compatible with our instructional goals as well as robust teaching practices that were likely resistant to change. This dynamic affected how we facilitated our rehearsals. Their openness to explore new ways of teaching allowed us to approach the rehearsals in the spirit of experimentation. Furthermore, their professional identities allowed us to offer alternatives without them becoming defensive, as long as we took care to ensure these suggestions were not seen as questioning their underlying competency as an educator. Their expertise also provided productive resources as we navigated different pedagogical dilemmas, which we wanted to honour and take advantage. Below we consider how this dynamic affected our decisions in relation to the three dimensions identified above.

In terms of Dimension 1, our facilitation choices focused less on enactment and structured the rehearsals more as a representation of practice. Given our teachers' expertise as educators, we viewed our work less about developing particular practices, and more about helping them understand when and how to apply the instructional tools they possessed already. As such, we wanted to offer them opportunities to explain their nuanced decision-making processes. Reflecting on instructional decisions in the moment allowed us to analyse in detail the rationale and outcome of such decisions. While a rehearsal that focuses on enactment can serve to ground theoretical considerations in concrete images through firsthand participation, we saw reflection, especially situated in the details of instruction, as a useful tool to help our teachers reposition their current practices and apply them towards new pedagogical goals. Furthermore, we believed incorporating more reflection during our rehearsals would serve to draw out the current beliefs and understandings of the ISTs. Allowing the teachers to share their interpretation of the instructional ideas helped make visible their interpretation of instructional situations as well as supported them in integrating their new learning with their current image of instruction. Notably, Valenta and Wæge (2017), who also reported a preference for more discussion in their study of facilitation with ISTs, attributed their shift in interactions to the teachers' experience. They speculated that the teachers already possessed many skills and wanted to use the rehearsals to understand this new form of pedagogy.

In addition, our teachers' experience and identity as competent educators affected the nature of the social dynamics, which in turn shaped our facilitation choices along Dimension 2. As compared to a university setting, where the professor brings much more knowledge and experience to the situation, teachers in our study had more developed identities. Therefore, in an attempt to establish our relationship more as equals, we chose to make our rehearsal less scaffolded and used our facilitation to position our work more as an exploratory activity. We believed a less scaffolded enactment would allow our teachers to draw on their experience to bring a range of options into the rehearsal and enrich the overall experience. Furthermore, we thought that a less directive approach that encouraged the teachers to engage more in the negotiation process would be met with a more receptive stance, allowing the norms of our community to shape their understanding. Similar to how reflection elicited teachers' current thinking about instruction, less scaffolded enactment drew out teachers' routinised ways of reacting. As such, first eliciting teachers' current practices as a starting point for negotiation allowed for a deeper integration of new practices and rationales into the teachers' ways of teaching.

In addition, the teachers' identities as experienced and knowledgeable educators meant that at times, they were resistant to changing their view of what a student might say. This became problematic when our views of what was authentic and their views of what was authentic did not align. Consequently, we felt a heightened need to put forward our understanding of authentic student thinking (Dimension 3). Such a situation highlights that depending on the identity of the participants and the degree to which their views about learning align with the goals of the PD, TEs working with ISTs will need to consider how necessary the introduction of different forms of authentic student thinking is. For example, it might be that more artifacts of authentic student thinking need to be incorporated into the overall cycle of investigation and enactment (McDonald et al., 2013). This can be

achieved through the use of representations of practice or by asking participants to enact the IA with actual students. Regardless, we believe intentional facilitation will be important for disrupting the tendency of ISTs to draw on narratives formed from years of experience when reasoning about the productivity of particular teaching moves.

Pedagogical goal

Given the experience of the ISTs in our PD, we chose to focus our work on developing the complex practice of leveraging emergent student thinking. Although many of the consequences of this practice are visibly discernible, the details of the actual thought process are not. Most of the work of this practice occurs internally as teachers observe and grapple with different mathematical and pedagogical options presented. Consequently, we often paused to elicit and make public the noticing and decision-making process in which our ATs engaged. Rather than focusing on the enactment of instructional details to implicitly guide participants' understanding of this practice, we chose to elevate the use of reflection as a tool to discuss and make sense of elements of this practice that might otherwise go undetected. Such facilitation choices shifted our work in relation to Dimension 1, so that our rehearsal functioned simultaneously as an approximation of practice and representation of practice.

In addition, our pedagogical goal elevated the importance of authentic student thinking. Truly leveraging student thinking to advance the mathematical agenda of the classroom is challenging. To engage in this practice in its full complexity, initial ideas must be emergent and incomplete. In contrast, our participants tended to give more developed and articulate answers. Such productive contributions meant that the AT simply needed to make public the ideas presented, rather than work to notice and distil out the pedagogically useful ideas embedded in more ambiguous statements. Thus, we felt the need to use interruptions to reflect on the degree to which responses were realistic and interject authentic student thinking ourselves during our rehearsals.

Conclusion

In this study, we examined how we structured interactions during rehearsals with a community of in-service teachers. We chose to work with experienced ISTs because we anticipated the adaptations needed to use rehearsals with experienced educators would likely reveal a difference in facilitation than had previously been identified in the literature. Building off the work of Lampert et al. (2013), we developed an expanded and detailed coding scheme that captured the different ways we attempted to leverage the expertise of this group of experienced math educators as well as respond to their needs as they engaged with challenging instructional practices associated with ambitious teaching. We grouped these codes into three categories: reflecting on pedagogy, investigating subsequent teaching moves, and giving the student perspective. Based on these categories, we conceptualised three dimensions: enactment versus reflection, nature of instructional guidance-scaffolded versus exploratory, and the need to interject student thinking, which highlight different ways in which a TE could choose to structure interactions and thus shape the nature of the rehearsal. Finally, we reflected on how the context in which we worked likely influenced our choices about how to structure the rehearsals.

While our data stem from only a small number of rehearsals with our unique community, our study contributes to researchers' understanding of rehearsals in three ways. First, the identification of different ways a TE can structure interruptions provides TEs a range of new facilitation tools they can use to better meet the needs of their participants and overall learning context. Second, the three categories and associated dimensions offer a framework of how rehearsals can be structured. This organisation has the potential to aid TEs in being more purposeful with their facilitation as they consider the type of interactions that would best accommodate the needs and strengths of their participants. Likewise, researchers studying facilitation can use these dimensions to better understand how trends in decisions about how to structure interruptions can shape the nature of the rehearsal and offer differing affordances. Finally, our reflection about our own choices in how we structured interactions within the context of professional development with experienced teachers illustrates that TEs' choices are influenced by the history of the group (e.g., the competencies they already possess, the familiarity they have with the instructional focus, and the social dynamics of the group, etc.). It also provides an example of how the facilitation of rehearsals might change when using this pedagogy with ISTs and offering further evidence that rehearsals are a productive format to be used flexibly in a range of contexts, including with experienced teachers.

References

Anthony, G., Hunter, J., & Hunter, R. (2015a). Prospective teachers' development of adaptive expertise. *Teaching and Teacher Education, 49,* 108–117.

Anthony, G., Hunter, J., & Hunter, R. (2015b). Learning to professionally notice students' mathematical thinking through rehearsal activities. *Mathematics Teacher Education and Development, 17*(2), 7–24.

Anthony, G. (2018). Practice-based initial teacher education: Developing inquiring professionals. In G. Kaiser, H. Forgasz, M. Graven, A. Kuzniak, E. Simmt & B. Xu, B. (Eds.), *Invited lectures from the 13th International Congress on Mathematical Education* (pp. 1–18). Springer International Publishing.

Averill, R., Drake, M., Anderson, D., & Anthony, G. (2016). The use of questions within in-the-moment coaching in initial mathematics teacher education: enhancing participation, reflection, and co-construction in rehearsals of practice. *Asia-Pacific Journal of Teacher Education, 44*(5), 486–503. https://doi.org/10.1080/1359866X.2016.1169503

Baldinger, E. E., & Campbell, M. P. (2021). Making learning visible: Cases of teacher candidates learning to respond to errors through multiple approximations of practice. *Mathematics Teacher Education and Development, 23*(4), 119–142.

Ball, D. L., & Forzani, F. (2009). The work of teaching and the challenge for teacher education. *The Journal of Teacher Education*, 60(5), 497–511.

Bailey, J., & Taylor, M. (2015). Experiencing a mathematical problem-solving teaching approach: Opportunities to identify ambitious teaching practices. *Mathematics Teacher Education and Development, 17*(2), 124. https://mted.merga.net.au/index.php/mted/article/view/260

Braseth, E. A. (2022). Mathematics teachers' perceptions of teaching practices alignment with ambitious teaching. *Mathematics Teacher Education and Development, 24*(1), 23–38.

Campbell, M. P., Baldinger, E. E., & Graif, F. (2020). Representing student voice in an approximation of practice: Using planted errors in coached rehearsals to support teacher candidate learning. *Mathematics Teacher Educator, 9*(1), 23–49. https://doi.org/10.5951/MTE.2020.0005

Carpenter, T. P., Fennema, E., Peterson, P. L., & Carey, D. A. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for Research in Mathematics Education, 19*(5)385–401. https://www.jstor.org/stable/749173

Crespo, S.: (2002), Praising and correcting: Prospective teachers investigate their teacherly talk. *Teaching and Teacher Education 18,* 739–758. https://doi.org/10.1016/S0742-051X(02)00031-8

Davis, E. A., Kloser, M., Wells, A., Windschitl, M., Carlson, J., & Marino, J. C. (2017). Teaching the practice of leading sense-making discussions in science: Science teacher educators using rehearsals. *Journal of Science Teacher Education, 28*(3), 275–293. https://doi.org/10.1080/1046560X.2017.1302729

Drake, M. R. (2016). Learning to coach in practice-based teacher education: A self-study. *Studying Teacher Education, 12*(3), 244–266.

Ghousseini, H., Beasley, H., & Lord, S. (2015). Investigating the potential of guided practice with an enactment tool for supporting adaptive performance. *Journal of the Learning Sciences, 24*(3), 461–497. https://doi.org/10.1080/10508406.2015.1057339

Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P. (2009). Teaching practice: A cross-professional perspective. *The Teachers College Record, 111*(9), 2055–2100. https://doi.org/10.1177/016146810911100905

Grossman, P., & McDonald, M. (2008). Back to the future: Directions for research in teaching and teacher education. *American Educational Research Journal, 45*, 184–205. https://doi.org/10.3102/0002831207312906f

Horn, I. S. (2010). Teaching replays, teaching rehearsals, and re-visions of practice: Learning from colleagues in a mathematics teacher community. *Teachers College Record, 112*(1), 225–259. https://doi.org/10.1177/016146811011200109

Jacobs, V. R., Lamb, L. L., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169-202. https://www.jstor.org/stable/20720130

Kavanagh, S. S., Metz, M., Hauser, M., Fogo, B., Taylor, M. W., & Carlson, J. (2020). Practicing responsiveness: Using approximations of teaching to develop teachers' responsiveness to students' ideas. *Journal of Teacher Education, 71*(1), 94–107. https://doi.org/10.1177/0022487119841884

Kazemi, E., Franke, M., & Lampert, M. (2009). Developing pedagogies in teacher education to support novice teachers' ability to enact ambitious instruction. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia* (Vol. 1, pp. 12–30). Adelaide, SA: MERGA.

Kazemi, E., Ghousseini, H., Cunard, A., & Turrou, A. C. (2016). Getting inside rehearsals: Insights from teacher educators to support work on complex practice. *Journal of Teacher Education, 67*(1), 18–31. https://doi.org/10.1177/0022487115615191

Kazemi, E., & Wæge, K. (2015). Learning to teach within practice-based methods courses. *Mathematics Teacher Education and Development, 17*(2), 125–145.

Kelley-Petersen, M., Davis, E. A., Ghousseini, H., Kloser, M., & Monte-Sano, C. (2018). Rehearsals as examples of approximation. In P. Grossman (Ed.), *Teaching core practices in teacher education* (pp. 85–101). Harvard Education Press.

Knuth, E. J., Alibali, M. W., McNeil, N. M., Weinberg, A., & Stephens, A. S. (2005). Middle school students' understanding of core algebraic concepts: Equality & variable. *Zentralblattfiir Didaktik der Mathematik – International Reviews on Mathematical Education, 37*(1), 68–76.

Kobiela, M., Iacono, H., Cho, S., & Chandrasekhar, V. (2022). Examining the potential of rehearsal interjections to support the teaching of mathematical practice: The case of mathematical defining. *Journal of Mathematics Teacher Education, 26*, 759–784. https://doi.org/10.1007/s10857-022-09545-4

Lampert, M. (2010). Learning teaching in, from, and for practice: What do we mean? *Journal of Teacher Education, 61*(1–2), 21–34. https://doi.org/10.1177/0022487109347321

Lampert, M., Franke, M. L., Kazemi, E., Ghousseini, H., Turrou, A. C., Beasley, H., & Crowe, K. (2013). Keeping it complex using rehearsals to support novice teacher learning of ambitious teaching. *Journal of Teacher Education, 64*(3), 226–243. https://doi.org/10.1177/0022487112473837

McDonald, M., Kazemi, E., & Kavanagh, S. S. (2013). Core practices and pedagogies of teacher education: A call for a common language and collective activity. *Journal of Teacher Education, 64*(5), 378–386. https://doi.org/10.1177/0022487113493807

Peercy, M. M., & Troyan, F. J. (2020). "Am I doing it wrong?" Critically examining mediation in lesson rehearsal. *Teaching and Teacher Education, 93*, 103082. https://doi.org/10.1016/j.tate.2020.103082

Putnam, R., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher, 29*(1), 4–15.

Schön, D. (1982). *The reflective practitioner.* Basic Books.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*(4), 313–340. https://doi.org/10.1080/10986060802229675

Strauss, A. L., & Corbin, J. (1990). *Basics of qualitative research: Grounded theory procedures and techniques.* SAGE Publications.

Valenta, A., & Wæge, K. (2017). Rehearsals in work with in-service mathematics teachers. In T. Dooley & G. Gueudet, *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education* (CERME10). http://hdl.handle.net/11250/2489648

Wæge, K., & Fauskanger, J. (2020). Teacher time outs in rehearsals: In-service teachers learning ambitious mathematics teaching practices. *Journal of Mathematics Teacher Education*, *24*, 563–586. https://doi.org/10.1007/s10857-020-09474-0

Corresponding author

Correspondence to Casey Hawthorne
[casey.hawthorne@furman.edu]

Ethics Declarations

Ethical approval

Ethical approval for the research was granted by the Furman University Institutional Review Board and informed consent was given by all participants for their data to be published.

Competing interests

The authors declare there are no competing interests.