Developing Preservice Teachers' Self-efficacy and Growth Mindset for Teaching Mathematics: Practices from a Mathematics Methods Course

Yao Fu
University of Wisconsin-Whitewater

Ozgul Kartal
University of Wisconsin-Whitewater

Received: November 2022 ‖ Accepted: June 2023

This study investigated the change in preservice teachers' (PSTs) mathematics teaching self-efficacy through a mathematics methods course and revealed the best practices that helped the PSTs develop a higher self-efficacy and a growth mindset for teaching mathematics. We collected data through a scaled-response questionnaire to measure PSTs' mathematics teaching self-efficacy, and an open-response survey to explain what, in the methods course, facilitated the development of PSTs' (N = 92) self-efficacy and mindset for teaching mathematics. We conducted paired samples t-tests and descriptive analysis to examine the scaled-response questionnaire data and used an open coding process to analyse the open-response survey data. We found that the PSTs significantly improved their mathematics teaching self-efficacy throughout the methods course; and identified five practices that contributed most to the development of self-efficacy and growth mindset in teaching mathematics, which included inductive teaching, differentiation, supporting productive struggle, problem solving, and probing thinking questions. This study sheds light on how to strengthen teacher education programs to better prepare PSTs in teaching mathematics to elementary and middle school students.

Keywords • mathematics teaching self-efficacy • growth mindset for teaching mathematics • mathematics methods course • preservice teachers • productive struggle • inductive mathematics teaching

Introduction

Research shows that elementary education majors have the highest rate of mathematics anxiety of any college major (e.g., Bursal & Paznokas, 2006; Gresham, 2007; Hall & Ponton, 2005; Harper & Daane, 1998; Hembree, 1990; Kelly & Tomhave, 1985; Vinson, 2001), and preservice teachers' (PSTs) mathematics anxiety is negatively correlated with their self-efficacy for teaching mathematics (Brady & Bowd, 2005; Bursal & Paznokas, 2006; Gresham, 2008; Swars et al., 2006). Many PSTs have a fixed mindset about mathematics learning and teaching and often possess low self-efficacy in teaching mathematics (Bates et al., 2013; Cutler, 2020). Given that teachers' mindset and self-efficacy can significantly influence teaching performance and student learning outcomes (Anderson et al., 2018; Huangfu, 2012; McKinney, 2018; Ramirez et al., 2018; Skaalvik & Skaalvik, 2010), it is critically important to help PSTs develop a high self-efficacy and a growth mindset for teaching mathematics throughout teacher preparation programs.

Even though growing research has evidenced the interconnectedness between self-efficacy and growth mindset (Palazzolo, 2016; Tassell et al., 2020), little is known about what could help PSTs improve self-efficacy and growth mindset in teaching mathematics. In this study, the Self-Efficacy for Teaching Mathematics Instrument (SETMI; McGee & Wang, 2014) was used to determine the change in PSTs' mathematics teaching self-efficacy through a mathematics methods course and explored what aspects of the course facilitated development of self-efficacy and growth mindset in teaching mathematics.

Literature Review

Mathematics Teaching Self-efficacy

Self-efficacy in teaching mathematics or mathematics teaching self-efficacy is defined as one's belief in his/her ability to teach mathematics effectively (Enochs et al., 2000). Researchers have widely studied the effects of in-service teachers' self-efficacy on teaching and learning, and abundant literature has corroborated the influence of self-efficacy on teachers' burnout, job satisfaction, and instructional practices. For example, teachers with lower self-efficacy beliefs were more likely to suffer from emotional exhaustion and depersonalisation (i.e., emotionally detached attitudes toward students), and become dissatisfied with their current work; they were also less likely to apply instructional strategies that engaged students in learning (Huangfu, 2012; Maslach et al., 1996; Viel-Ruma et al., 2010). Hence teacher self-efficacy beliefs have been found to be positively related to students' academic achievement in mathematics and English (Khan, 2012; Maguire, 2011), as well as to class level interaction quality (Perera & John, 2020). Apart from its effects on teaching and student achievement, studies that investigated the interactions between PSTs' mathematics self-efficacy, mathematics teaching self-efficacy, and mathematical performance found positive relationships (e.g., Bates et al., 2011; Bursal & Paznokas, 2006; Johnson et al., 2018; Skaalvik & Skaalvik, 2010; Zuya et al., 2016). Additionally, there has been a growing interest in studying how mathematics content or mathematics methods courses in teacher education programs could influence PSTs' self-efficacy in teaching mathematics. Although researchers (e.g., Althauser, 2018; Giles et al., 2016; Looney et al., 2017) found positive impact of such courses increasing elementary/middle school PSTs' confidence in their own mathematics abilities and efficacy in teaching mathematics; there is limited understanding about specific instruction and teaching approaches that help increase mathematics teaching self-efficacy.

Growth Mindset

A growth mindset is the belief that intelligence, abilities, and talents are learnable and capable of improvement through effort, whereas a fixed mindset holds that these traits are inherently stable and unchangeable over time (Dweck, 2006). The effects of fixed and growth mindsets have been investigated in numerous educational research studies in relation to increased self-efficacy, persistence, motivation, and greater academic achievement (e.g., Bedford, 2017; Blackwell et al., 2007; DeBacker et al., 2018; Esparza et al., 2014; Hochanadel & Finamore, 2015; O'Rourke et al., 2014; Rhew et al., 2018; Yeager & Dweck, 2012). Research found moderate (e.g., Aronson et al., 2002; DeBacker et al., 2018; Yeager & Dweck, 2012) to significant (e.g., Bedford, 2017; Blackwell et al., 2007) impact of growth mindset interventions on student achievement, engagement, and motivation in particular subjects such as science and mathematics (Aronson et al., 2002; Bedford, 2017; Blackwell et al., 2007, Yeager & Dweck, 2012). Although strong teacher effects (i.e., teachers' support for or undermining of growth mindset beliefs) on students' growth mindset were observed among general student populations, similar teacher effects were not observed among students who are identified as gifted and talented (Esparza et al., 2014), and growth mindset intervention had a significant difference in the motivation but not self-efficacy of adolescents with special needs (Rhew et al., 2018). There have been ongoing debates about the growth mindset studies around the world such as dismissing contextual factors, diverging interpretations of the results, small intervention effects, lack of generalisability, and non-significant results in replication studies. For example, in a large-scale experimental study (Foliano et al., 2019), in the United Kingdom, in which teachers were trained to deliver growth mindset lessons; they found no evidence of an impact of the changing mindsets intervention on literacy or numeracy skills; while means and standard deviations were very similar between the treatment and control group on non-cognitive traits (i.e., intrinsic value, self-efficacy, test anxiety, and self-regulation). Sisk et al. (2018) found overall weak effects for their meta-analysis, with some results supporting specific tenets of the theory, such as that students with low socioeconomic status or who are academically at risk might benefit from mindset interventions. The ongoing critics and debates lie in theoretical questions about the effect of mindset interventions in educational settings, to what extent and under what conditions implicit theories of growth mindsets are important to academic achievement and non-cognitive traits, how a focus on growth mindset masks the biases and inequities that the students are experiencing, and how realistic it is to expect equality to happen (i.e., closing the achievement gap) in growth mindset classrooms without any comprehensive reform (Miller, 2019). In this respect, mindset scholars noted that growth mindset interventions may complement the effects of high-quality educational reforms, but do not replace them (Yeager et al., 2013). To sum up, an abundance of research has been conducted on the effects of growth mindset and its interventions in the context of teaching and learning, as well as in relation to affective domains. In those studies, the growth mindset was studied as teachers' beliefs about their students' ability or students' beliefs about their own ability to learn the school subject at hand. Differently in this study, we investigated the role of particular approaches and practices in fostering PSTs' growth mindset for teaching mathematics, not PSTs' beliefs about their intelligence or ability to understand the mechanics of mathematics.

Dweck (2006) described growth mindset in teaching as a mindset that asks, "*How* can I teach them?" and "*How* will they learn best?" instead of asking "*Can* I teach them?" and "*Can* they learn?"—which are indicators of a fixed-mindset in teaching (p. 64). Sun's (2018) Math Teaching for Mindset Framework (MTMF) describes growth mindset for teaching mathematics as:

- having high expectations for all students;
- multidimensional perspective of mathematical success;
- using comparative structures (i.e., public postings of a range of student work);
- adopting explicit mindset messaging, valuing the process over product,
- engaging in mistakes at a deep level with students;
- viewing struggle and failure as an opportunity to learn;
- encouraging and valuing student risk taking;
- promoting students' generation and experimentation;
- implementing tasks with multiple entry points and solution paths;
- having students do most of the mathematical work;
- giving verbal praise for effort and process;
- providing written and additional feedback; and
- offering multiple opportunities for assessment (pp. 341–343).

Although teachers' self-efficacy beliefs, perceived capabilities, and mindset in teaching mathematics are mostly derived from their training experiences and not their actual ability to understand the mechanics of mathematics (Hull et al., 2016), a few studies investigated the role of particular teaching approaches and practices in fostering elementary students' and teachers' growth mindset (e.g., Laurian-Fitzgerald & Fitzgerald, 2016; Rissanen et al., 2019), and the impact of these approaches on sustained change to teacher mindset and practice (e.g., Seaton, 2018). For example, Hull et al. (2016) worked with 332 teachers, and found that professional development in structured mathematics inquiry resulted in an increased self-efficacy belief in teachers about their instructional practices and student engagement. Rissanen et al. (2019) conducted a case study of an experienced mixed-mindset classroom teacher and presented core features of growth mindset pedagogy actualised in the teacher. They observed that the teacher supported each student's individual learning processes, promoted mastery orientation, and fostered process-focused thinking in her students. They also observed critical instances where the influence of the teacher's fixed mindset became apparent. The critical points they identified in the teacher's practice were lack of persistence in teaching some of her students, relying on the motivating power of success, protecting some of her students from challenges instead of teaching them how to cope with mistakes and failures, and implementing trait-focused pedagogy for academically competent students.

Apart from studies on in-service teachers, Waid (2018) explored six preservice mathematics teachers' beliefs about mathematics and mindset and their assessment practices by using a mixed-methods approach. The results showed that PSTs' growth mindset beliefs were related to their use of growth mindset assessments which were defined as assessments that measure strands of mathematical proficiency such as procedural fluency, conceptual understanding, problem solving, and reasoning, rather than measuring students' ability to reproduce the procedural skills or factual knowledge learned in class. Waid (2018) suggested that teacher educators should provide PSTs with more training in growth mindset assessments and ask PSTs to frequently reflect on their beliefs and practices on growth mindset. More recently, Thurmond (2020) examined the impact of a core mathematics course on 30 preservice mathematics teachers' mindset beliefs. Analysis of quantitative and qualitative data evidenced the participants' increase in growth mindset beliefs and decrease in fixed mindset beliefs over the duration of the course. The findings indicated that the growth mindset strategies, such as reflective practice and Number Talks—i.e., a 5–15-minute classroom conversation around purposefully crafted problems that are solved mentally (Parrish & Dominick, 2016)—could help improve PSTs' mindset beliefs.

Research Questions

The constructs used in this study are self-efficacy and growth mindset in teaching mathematics. The literature review provides a foundation that self-efficacy and mindset are interconnected; and they influence PSTs' future instructional practices, which subsequently may affect their future students' mathematics outcomes (Tassell et al., 2020). This connection and combined effect of mindset and self-efficacy on teaching practices framed the design of this study, which examined both self-efficacy and mindset.

Although preservice teacher preparation was found to be a critical factor that contributes to teachers' self-efficacy (Hull et al., 2016), the practices that would potentially develop PSTs' self-efficacy and a growth mindset for teaching mathematics remain understudied. To fulfil this gap, this study investigated the change, if any, in PSTs' self-efficacy in teaching mathematics through a mathematics methods course and revealed the practices and activities that helped develop their self-efficacy and growth mindset in teaching mathematics. Specifically, the following three research questions guided the study:

1. *How does the mathematics methods course influence the PSTs' self-efficacy, if any, in teaching mathematics?*
2. *What aspects of the mathematics methods course influence the PSTs' development of self-efficacy, if any, in teaching mathematics?*
3. *What aspects of the mathematics methods course influence the PSTs' development of growth mindset, if any, in teaching mathematics?*

Methods

Research Context and Participants

The context for this research was an undergraduate mathematics methods course at a mid-sized, regional, Mid-Western university teacher education program in the United States. The participants of this study were 92 PSTs enrolled in the mathematics methods course who did not have any mathematics teaching experience before. The PSTs were elementary and middle school education majors who would be licensed to teach Grades 1–8; and the vast majority of them were Caucasian females in their junior or senior years, with only seven male PSTs. This course was the only mathematics methods course the PSTs take in their program; they completed two undergraduate maths content courses (i.e., Mathematics for the Elementary Teacher 1 and 2) before taking the methods course. Typically, PSTs take this course the semester before their student teaching experience. The mathematics methods course adopted the Teaching for Robust Understanding (TRU) Framework (Schoenfeld, 2014) and its five dimensions: a rich and connected mathematics content, high cognitive demand, equitable access, agency/ownership/identity, and formative assessment. This framework is also in alignment with Sun's (2018) MTMF, and hence although there was not an explicit teaching of growth mindset in the course, the cultivation of a growth mindset was ingrained within the course itself. The primary topics of the course was mathematical content and practice standards in the United States, the eight effective mathematics teaching practices stipulated in National Council of Teachers of Mathematics' (NCTM, 2014) *Principles to Action* (PtA), teaching mathematics with meaning and representations, teaching models for maths, creating assessment for learning, and differentiated instruction. The PSTs learned how to read and unpack mathematics content standards (i.e., *Common Core State Standards for Mathematics* [CCSI, 2010]), and how to identify and plan for development of Standards for Mathematical Practices (SMP) that are elaborated in CCSSM. The SMPs outline different areas of expertise that teachers of mathematics should aim to develop in their students at all levels (see Table 1).

Table 1
CCSI (2010) and NCTM (2014) Mathematics Learning and Teaching Practices

Standards for Mathematical Practices	Effective Mathematics Teaching Practices
1. Make sense of problems and persevere in solving them	1. Establish mathematics goals to focus learning
2. Reason abstractly and quantitatively	2. Implement tasks that promote reasoning and problem solving
3. Construct viable arguments and critique the reasoning of others	3. Use and connect mathematical representations
4. Model with mathematics	4. Facilitate meaningful mathematical discourse
5. Use appropriate tools strategically	5. Pose purposeful questions
6. Attend to precision	6. Build procedural fluency from conceptual understanding
7. Look for and make use of structure	7. Support productive struggle in learning mathematics
8. Look for and express regularity in repeated reasoning	8. Elicit and use evidence of student thinking

These practices are built upon essential processes and proficiencies that have long been recognised as significant in mathematics education. The first four of these practices comprises the NCTM (2000) process standards, which include problem solving, reasoning and proof, communication, representation, and connections. The second four encompasses the strands of mathematical proficiency as defined in the National Research Council's (2001) report, "Adding It Up." These strands consist of adaptive reasoning, strategic competence, conceptual understanding (grasping mathematical concepts, operations, and relationships), procedural fluency (being skilled in executing procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (having a natural inclination to perceive mathematics as sensible, useful, and valuable, combined with a belief in diligence and one's own capability). In support of SMPs, NCTM (2014) described and illustrated eight research-informed teaching practices that support the mathematics learning of all students, listed in Table 1. The PSTs were consistently engaged in the practice of reading, unpacking, and planning for content standards and seeking to develop SMPs in their prospective students throughout the methods course within all the course topics. An equal amount of time was allocated for each of the eight effective mathematics teaching practices throughout the course.

The instructor discussed the eight effective teaching practices through chapters of PtA together with additional activities and teaching video clips. More specifically, the PSTs read the chapter on the particular practice; watched and analysed a teaching video clip (from the NCTM PtA tool kit) that focused on the teaching practice; discussed how the practice was emerged in the video clip and how it was supported by the teacher referring to the teacher and student actions; and practised the skill in the context of a given task, scenario, or mathematics content as appropriate. For example, for "posing purposeful questions," they were given a variety of student work in response to a problem, which they analysed and formulated purposeful questions for each work. For "implement tasks that promote reasoning and problem solving," PSTs created multiple different solutions for the given problem-solving tasks and discussed the array of solutions in connection to each other, as well as mistakes, difficulties, and struggles. For the "facilitate meaningful mathematical discourse," they determined the discourse level in a given classroom video.

Teaching mathematics with meaning and with representations involved exploring the variety of contexts, representations, tools, and manipulatives in the mathematics content topics across elementary and middle grades through hands-on activities. For example, they created a word problem and meaning for a given fraction division, and represented and solved the problem using fraction manipulatives and non-algorithmic thinking, without referring to the standard algorithm. Teaching mathematics with meaning and representations activities were evenly distributed throughout the course. For the topic of teaching models for mathematics, the PSTs were introduced to a variety of inductive teaching styles, in comparison to the direct teaching model.

Inductive teaching is defined as an umbrella term for teaching models that set up experiences to induce students to construct knowledge for themselves (Prince & Felder, 2006) and present new information in the context of situations and problems to which students can relate to their existing cognitive structures (Bransford et al., 2000). This is in contrast to traditional (i.e., direct) teaching where the focus is on a teacher telling students a new procedure or skill, and then the students simply practise that same procedure or skill in repetitive iterations in order to consolidate their proficiency in it (Kartal & Tillett, 2021). The instructor of the course demonstrated various inductive teaching models (e.g., integrative, concept attainment, teaching through problem solving, and discovery) through sample lessons; the PSTs, in groups, designed lessons employing one of the introduced models and implemented in class, and then, they individually designed and peer-taught a lesson in their preferred model. They were required to target multiple SMPs in their lesson, and design and implement the lesson to demonstrate the eight effective mathematics teaching practices as they taught the lesson to their peers.

The topic of creating assessment for learning took two class meetings, in which students learned and practised how to design powerful assessments, analyse student work, provide feedback, and design further instruction. Adopting NRC's (2001) definition for mathematics proficiency, powerful assessments were designed to evaluate students' procedural fluency, conceptual understanding, strategic competence, adaptive reasoning, and productive disposition. This type of assessment was also referred to as growth mindset assessments in the literature (e.g., Waid, 2018). For the topic of differentiated instruction, the PSTs learned and discussed the idea of equitable pedagogy in the context of teaching mathematics, and learned and practised a variety of differentiation techniques through hands-on activities. They were asked to integrate multiple of these differentiation techniques in their individually designed peer-taught lessons. The topic of differentiation took two class meetings.

Data Sources and Analysis

We collected data using two instruments: (1) the Self-Efficacy for Teaching Mathematics Instrument (SETMI) (McGee & Wang, 2014); and (2) an open-ended survey. The SETMI was used to measure the PSTs' self-efficacy in teaching mathematics at the beginning and end of the course. Though SETMI was originally used with in-service teachers, researchers (e.g., McCampbell, 2015; McGuire, 2016) have adopted the instrument to assess PSTs' self-efficacy. One PST did not complete the SETMI, therefore we gathered 91 PSTs' responses. Engagement in any mathematics methods course could potentially augment the students' self-efficacy. Nonetheless, when coupled with the utilisation of open-ended questions, our intention was to foster a more coherent comprehension of the essential practices that contribute to this enhancement. Therefore, in addition to the SETMI, all 92 PSTs completed the open-ended survey, which was used to identify practices that helped the PSTs develop a growth mindset and self-efficacy in teaching mathematics over the course of the study.

The SETMI consists of 22 items on a Likert-type 5-point scale for which the participants rated their agreement from "none at all" to "a great deal." The instrument measures two underlying factors of self-efficacy: efficacy for pedagogy in mathematics (EPM) and efficacy for teaching mathematics content (ETMC). Specifically, the EPM contains the first seven SETMI items that assess self-efficacy in everyday demands of a mathematics classroom, including motivating students, asking questions, providing explanations, implementing assessment strategies, and applying alternative teaching strategies. An example item for the EPM subscale is "To what extent can you motivate students who show low interest in mathematics?" (McGee & Wang, 2014, p. 7). The ETMC includes the last 15 SETMI items and asks the participants how well they can teach particular mathematics topics to students (e.g., integers, fractions, decimals, data interpretation, and measurement of area and perimeter). An example item for the ETMC subscale is "How well can you teach students to perform strategies for composing and decomposing numbers by manipulating place value in addition and subtraction" (McGee & Wang, 2014, p. 8). Content validity and a strong internal consistency reliability was reported for the SETMI, with Cronbach's alpha of $.86$ for EPM and $.93$ for ETMC, in the original study (McGee & Wang, 2014); and a Cronbach's alpha of $.86$ for EPM and $.95$ for ETMC were computed in the current study. We analysed the participants' responses to the SETMI at the beginning and end of the course, using paired samples t-tests and descriptive analysis to compare the PSTs' mean scores of the SETMI (i.e., 22 items), EPM, and ETMC in pre- and post-survey data.

At the end of the course, the PSTs responded to two open-ended survey questions which uncovered how the methods course impacted their mindset and self-efficacy for teaching mathematics. The first survey question revealed what helped them develop a growth mindset in teaching mathematics: Which aspects of mathematics methods course, if at all, helped you develop a mindset that asks, "How can I teach them?" and "How will they learn best?" instead of asking "Can I teach them?" and "Can they learn?" The second survey question revealed what helped them increase self-efficacy in teaching mathematics: Which aspects of mathematics methods course, if at all, helped you develop self-efficacy in teaching mathematics (i.e., belief in your ability to successfully teach mathematics in elementary and middle grades)? The first survey question was formulated based on Dweck (2008), and the second survey question used Enochs et al.'s (2000) definition of self-efficacy in teaching. Both survey questions were formulated in a non-directive way so that participants could respond freely. That is, we used the words "aspects" so that the participants could choose to talk about particular instances, practices, experiences, delivery methods, interactions, resources, or such about the methods course; or they could explain how and why any aspect of the course did not help them. In other words, there was not any clear hypothesis, initially, about what would help PSTs develop self-efficacy and growth mindset; and we wanted to prioritise the voices and perspectives of PSTs by capturing their interpretations of the experiences. Therefore, we used grounded theory to develop contextually relevant theories that emerged from the data itself (Glaser & Strauss, 1967), and are grounded in the realities and experiences of the participant PSTs. The close connection between theory and practice in grounded theory enhanced our findings' applicability (Creswell, 2013) by informing the design and development of our maths methods course. Responses to the two open-ended survey questions were coded using open coding procedure in grounded theory (Strauss & Corbin, 1990). More specifically, we collected data from students in one section; and two researchers independently coded the data and determined some initial categories. The categories from both researchers were merged to establish an initial coding scheme. Then, more data were collected from all sections of the course and analysed independently by the two researchers, in the following semesters until no new categories appeared to be necessary. The categories were grounded in the data set. After establishing a final coding scheme (see Table 2), two researchers coded the entire data independently with a coding agreement of 89%— percentage of judgments on which coders agree (Scott, 1955). The data were re-examined to clarify any ambiguities and reach a consensus. First, written responses to those two open-ended questions were divided into meaningful segments of varying lengths, depicting a different aspect of the course. A total of 389 codes were generated including nine categories as shown in Table 2.

Table 2
Coding Scheme

Categories	Description
Differentiation	Mentioned one or more of the mathematics content specific differentiation techniques that they learned and applied; or includes "all students can learn" or "access for all" ideas in relation to differentiation, differentiated instruction, and or equitable teaching.
Problem Solving	Mentioned problem solving task assignment, multiple entry-exit points, different ways of solving problems, different ways of explaining reasoning and solution; or reflects on going through problem solving process and in-class problem solving activities and discussions.
Inductive Teaching	Mentioned learning about inductive teaching style or a particular teaching model that falls into inductive style, variety of ways of teaching mathematics, designing an inductive style lesson, group or peer-teaching activities.
Productive Struggle	Mentioned the concept and idea of "productive struggle" or the effective teaching practice "support productive struggle in learning mathematics." If they mention only the term "struggle" it does not fall into this category.
Probing Thinking Questions	Explicit mention of "probing thinking questions" "probing questions" "thinking questions" "assessing questions" "questions to help students think" or "how to ask good questions" or "what questions to ask"
Manipulatives and Concrete, Semi-concrete, Abstract sequences (CSA)	Mentions tools, manipulatives, CSA sequence, or representations.
Effective Mathematics Teaching Practices	Mentioned the term "effective math teaching practices" in general without referring to any specific one among the eight effective mathematics teaching practices in NCTM'S (2014) PtA.
Standards for Mathematical Practices (SMP)	Mentioned the term "SMP," "CCSSM practice standards," or "practice standards" without referring to any specific one.
Analysing Student Work and Misconceptions	Mentioned "misconceptions" "student work," or reflects on the understanding students' thinking assessment assignment.

Findings

We conducted paired samples *t*-tests and descriptive analysis to scrutinise survey results collected from 91 PSTs, and to quantitatively describe the change in their self-efficacy in teaching mathematics through the mathematics methods course. Before data analysis, normality checks were carried out and assumptions were met. We also coded 92 PSTs' written responses to further examine the course impact and identify the elements of the methods course that facilitated the development of PSTs' growth mindset and self-efficacy in teaching mathematics. The PSTs responded to both questions with the practices they learned and applied in the methods course, rather than resources, delivery methods and structure, interactions, or such. All participants' names in this study have been changed to pseudonyms. In the section below, the results are presented in response to the three research questions.

Change in the PSTs' Self-efficacy in Teaching Mathematics

In response to the research question, "How does a mathematics methods course influence the PSTs' self-efficacy in teaching mathematics?" the statistical tests provided evidence of an increase in PSTs' self-efficacy in teaching mathematics. Furthermore, the qualitative data elucidated the impact of the methods course, specifically highlighting the acquisition of alternative mathematics teaching methods and the development of problem-solving skills, encompassing various approaches for solution explanation.

The paired samples t-tests showed a significant difference between the PSTs' mean scores for the SETMI pre-survey ($M = 3.52,\ SD = 0.64$) and post-survey ($M = 4.04,\ SD = 0.52$); $t\ (90) = -7.25, p < 0.001,\ d = -.76$. There were also significant differences between mean scores for EPM pre-survey ($M = 3.73,\ SD = 0.65$) and post-survey ($M = 4.12,\ SD = 0.54$); $t\ (90) = -5.26,\ p < 0.001,\ d = -.77$ and ETMC pre-survey ($M = 3.42,\ SD = 0.71$) and post-survey ($M = 4.01,\ SD = 0.56$); $t\ (90) = -7.33,\ p < 0.001,\ d = -.55$. That is, the PSTs, on average, reported an increase in their overall self-efficacy in teaching mathematics. Specifically, they became more efficacious in using mathematics pedagogy and teaching mathematics content.

The qualitative data corroborated the PSTs' increased self-efficacy in teaching mathematics, especially their self-efficacy beliefs related to using mathematics pedagogies. For instance, Kate described how the course increased her self-efficacy in implementing alternative teaching strategies for mathematics.

> I believe that the course helped me gain more confidence in my skills in teaching mathematics. Both the group lesson and the inductive lesson implementation helped increase my confidence in myself as a teacher. These activities helped me practice what I will be doing in the classroom. The inductive teaching lesson made me think about and create a different way of learning for students. Rather than using direct instruction, it had students exploring ideas on their own, which ultimately, is more beneficial for students. I believe that this class helped me realise that I am able to teach mathematics and that I need to be thinking about how to make it meaningful for my students. This class challenged me to think about my teaching style and how to best support my students. Overall, I have more confidence in my abilities compared to when I started the class.

In her comment below, Jessica reflected on how she became more confident in helping all students learn. Before taking the course, she only had knowledge about the direct-teaching approach. Through the course, she learned about inductive teaching and gained confidence in using this student-centred teaching approach to enrich learning for all.

> Learning about all of the [inductive] approaches to teaching math effectively helped me to believe in myself as a teacher. For a while I was getting discouraged because it seemed as though math teaching I've observed often took the approach of direct teaching. Although I knew this is not how I wanted to teach my future classroom, I didn't know how to approach other styles effectively. Learning about and getting to practice an inductive style of teaching that is focused on discovery and is student-centered really gave me the confidence that I can actually help all students learn rather than reverting to direct-teaching.

In addition, Alex enhanced her self-efficacy in providing alternative explanations to students when they are confused. The following quote showed how she developed such self-efficacy through group work and in-class presentations on different mathematics topics.

> In class work and presentations helped me develop my self-efficacy in teaching mathematics because it showed me that I can explain what I am thinking in a way that students will understand. Before this class, I found it very difficult to explain my thinking for a math problem and explain why the approach I used worked. I knew my answer was correct and I knew why it was correct, but I didn't know how to put that into words. By presenting different topics to the class and through the group work, I had to force myself to put my thoughts into words and explain it in a variety of different ways if they were not understanding.

Practices that Developed Self-efficacy for Teaching Mathematics

In response to the second research question, seven categories emerged (see Table 3). Learning about inductive teaching approaches, embracing and learning to support productive struggle, engaging in problem solving, practising differentiation, using questioning techniques, analysing students' mathematics work and misconceptions, and using manipulatives helped PSTs' development of self-efficacy in teaching mathematics. The frequencies and sample quotes for each category are presented in Table 3.

Table 3
What Helped PSTs Develop Self-Efficacy for Teaching Mathematics? (N = 92)

Categories	Sample Quotations	Frequency (Percent)
Inductive teaching	"Learning about all of the [inductive] approaches to teaching mathematics effectively helped me to believe in myself as a teacher. For a while I was getting discouraged because it seemed as though mathematics teaching I've observed often took the approach of direct teaching. Although I knew this is not how I wanted to teach my future classroom, I didn't know how to approach other styles effectively. Learning about and getting to practice an inductive style of teaching that is focused on discovery and is student-centered really gave me the confidence that I can actually help all students learn rather than reverting to direct-teaching." "Before coming into this class, I did not have high expectations for teaching mathematics because it is a subject that I have always struggled with because we were never given a clear answer as to why we follow a specific step. After completing the lesson plans using the inductive style of teaching, I feel more confident that I am able to instruct students by having them explore the content rather than just following the direct instruction format."	65 (71%)
Productive struggle	"... I think supporting productive struggle is a huge part in being able to successfully teach mathematics, I think this because in mathematics there is always going to be some sort of struggle for the students, it's a subject that doesn't really come easy to many. Having an environment that supports this type of ideology can be super beneficial to the students. This can help with the students taking more risks in the classroom and not just giving up when they are struggling on something. This will only help deepen their critical thinking abilities and help give them more of a drive to want to learn more about math. This is definitely not an easy task to do but something I think will be super beneficial in helping me develop self-efficacy." "... that helped me the most in developing self-efficacy in teaching mathematics is knowing that it's okay to struggle, even as a teacher. With struggle comes learning as this is true with both teachers and students. I want my students to know that it's okay to struggle and that there will be times that I will engage them in productive struggling …"	20 (22%)
Problem solving	"One of the very first classes we all had the same problem, but we found 10 different ways to solve the problem. This opened my eyes to see that there is not only one way to solve a problem and definitely not one way to teach a problem." "... look at problems in ways that other people might not. I've also learned that I have to be very adaptive and flexible because every student will do a problem differently and I need to be able to interpret it."	20 (22%)
Differentiation	"... differentiated instruction made me feel more confident and develop self-efficacy, because although I may not necessarily have a mathematics brain ... boundaries and differentiation for student needs. This has shown me that if I am creative and persistent, I will still be able to teach students math …"	16 (17%)

Table 3 Cont.
What Helped PSTs Develop Self-Efficacy for Teaching Mathematics? (N = 92)

Categories	Sample Quotations	Frequency (Percent)
Probing thinking questions	"... asking good probing questions to get a better understanding of how they are learning." "... more confident in knowing what questions to ask …" "... ask the right questions to get our students thinking about mathematics that will help them build stronger connections."	13 (14%)
Analysing student work and misconceptions	"I have realised that I know how to recognise misconceptions in student work and can anticipate those misconceptions when planning a lesson. I would feel uncomfortable going into teaching without having to go through these exercises." "... address their misconceptions-and my own, that I will be able to confidently teach mathematics."	8 (9%)
Manipulatives and CSA	"Something I really enjoyed was learning how to use manipulatives to teach all kinds of mathematical concepts …" "... how we can use manipulatives to get a better understanding when we are physically seeing and touching the objects ... more so than just memorising an algorithm."	8 (9%)

The majority of the PSTs reported that they felt more confident to teach mathematics from learning how to have their prospective students explore the content (i.e., inductive teaching) rather than just following the direct instruction format; and became more efficacious in creating meaningful and engaging lessons. For example, one PST commented, "It helped me see that I can make a difference in how students view mathematics and that I actually can create meaningful lessons in mathematics class that engage students and relate to their lives …" The PSTs often mentioned their mathematics learning experiences as students and how they learned mathematics without understanding; and explained that learning how to teach mathematics with understanding through inductive teaching styles helped them develop a self-confidence for teaching mathematics. PSTs reported that learning about benefits of productive struggle, how to support productive struggle, and themselves experiencing productive struggle in the course led to their increased self-efficacy in teaching mathematics. In addition, they explained that engaging in problem solving and exploring many ways to solve and explain a problem was helpful in developing self-efficacy. Learning to differentiate instruction and use of manipulatives supported the idea of being persistent in teaching for all, and questioning skills and analysing student work and misconceptions helped them better understand their students' thinking.

Practices that Developed Growth Mindset in Teaching Mathematics

In response to the third research question, six categories emerged (see Table 4) related to the practices that helped develop a growth mindset in teaching mathematics. Survey responses indicated that all 92 PSTs left the course with an improved growth mindset in teaching mathematics. Similar to self-efficacy results, learning about inductive teaching models and differentiation, the idea of supporting productive struggle, learning to ask questions to probe into student thinking, using manipulatives, and engaging in problem solving helped PSTs develop a growth mindset in teaching mathematics.

Table 4
What Helped PSTs Develop a Growth Mindset for Teaching Mathematics? (N = 92)

Categories	Sample Quotations	Frequency (Percent)
Inductive teaching	"... learning about the various inductive teaching models helped change my thinking because it showed me that there are a wide variety of ways to teach the same lesson. Therefore, if a student is not understanding it, I know different ways that I can explain the material to them or different activities. This transferred from can I find the right way to teach them to how can I teach them in a different way if they are not understanding."	55 (60%)
Differentiation	"I think the differentiation topics and learning the different strategies, multiple means of engagement, accessibility, and representation really highlights the idea that every concept can be taught in a multiple of ways so that every child has the opportunity to learn."	38 (41%)
Productive struggle	"I really enjoyed the push for the productive struggle, because for most teachers it's easy to jump in, and learning that struggle can actually benefit them was eye opening." "Another aspect that I will take away with me is the use of productive struggle. In elementary school, I was not taught ways to work through my misunderstandings in mathematics. By encouraging my students that it is alright to struggle through an exercise, they are able to not only feel comfortable with me as their teacher, but also comfortable in the idea that they can learn mathematics given the proper tools."	30 (33%)
Probing thinking questions	"... how to ask meaningful probing questions that will guide students in thinking critically which can help support and foster their learning." "... not to give the answer away to students but to instead ask them probing and thinking questions to help them work through their struggle."	22 (24%)
Problem solving	"I was able to see different ways of solving the problem that I wouldn't have thought of initially." "We learned many different ways to solve one problem in this class which I felt was valuable …" "... learning different ways to teach similar problems really helped me expand my understanding on both mathematics as well as my own thinking."	21 (23%)
Manipulatives and CSA	"... develop my mindset is providing a variety of teaching tools that meet students' needs. I think the tool that I will use the most is manipulatives. I think manipulatives can be beneficial for many different learners. Furthermore, I like the CSA sequence for helping students learn best."	20 (22%)

The PSTs explained that their mindset changed when they learned that there are a wide variety of ways to teach mathematics (i.e., inductive teaching models), and they started thinking about how a student would learn instead of how they can just talk at the students and give them good explanations. Furthermore, learning and practising a variety of differentiation techniques helped them with the mindset that mathematics can be taught to all students. They related differentiation with the idea of "all students can learn mathematics," and developed a growth mindset for teaching mathematics as they learned a variety of differentiation strategies. They reported that exploring the idea of "support productive struggle" helped them grow their mindset. More specifically, they discussed supporting productive struggle practice as embracing a view of students' struggles as opportunities for delving more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions, as described in the PtA (NCTM, 2014). The PSTs often reflected on their own struggles with mathematics as learners and explained that embracing and viewing struggle as a necessary component of learning helped them develop a growth mindset for teaching mathematics.

Other practices that the PSTs found most helpful in growing their mindset was learning how to ask probing thinking questions; exploring the idea of problem solving and engaging in the problem-solving process; and practising the use of manipulatives and building Concrete, Semi-Concrete, and Abstract sequences (CSA). The PSTs explained that learning to ask probing thinking questions helped them develop a growth mindset as they learned that questions can be used to further assist student learning. Similar to asking probing questions, the PSTs viewed manipulatives and CSA sequences as tools to further assist student learning. The PSTs reflected on their own challenges of looking for different ways to solve problems during the methods course, as they reported that problem solving helped them develop a growth mindset.

In summary, the findings of the study indicated that the PSTs' self-efficacy and mindset for teaching mathematics grew through the course, as evidenced in their responses to the survey and open-ended questions. The results showed that learning about and practising a variety of inductive teaching styles of mathematics, differentiation techniques, and supporting productive struggle most helped the PSTs develop a growth mindset in teaching mathematics. Planning, writing, and implementing an inductive style lesson, going through productive struggle in class, learning to support productive struggle, and exploring many ways of solving mathematics problems most helped them improve their self-efficacy in teaching mathematics.

Discussion and Implications

Change in the PSTs' Self-efficacy in Teaching Mathematics

The first research question was designed to explore how a mathematics methods course could influence the PSTs' self-efficacy in teaching mathematics. As described in the literature review (e.g., Althauser, 2018; Giles et al., 2016; Looney et al., 2017), researchers have found that mathematics methods courses in teacher education programs helped enhance PSTs' self-efficacy in teaching mathematics, using a variety of instruments. The qualitative and quantitative data generated through the current study do not only confirm the substantial impact of a mathematics methods course on the PSTs' overall self-efficacy in teaching mathematics, but also expand the literature by explaining what type of experiences, in a mathematics methods course, most helped PSTs in becoming more efficacious in applying various mathematics pedagogies and teaching mathematics content. This finding supports Johnson et al.'s (2018) conclusion that the beliefs of elementary PSTs concerning their ability to deliver effective mathematics instruction to students are influenced not by the extent of their mathematical content knowledge, but rather by their proficiency in applying that knowledge in the form of pedagogical content knowledge for teaching mathematics.

Practices that Developed Self-efficacy and Growth Mindset in Teaching Mathematics

The second and third research questions explained what type of practices enhance the PSTs' self-efficacy and growth mindset in teaching mathematics. Although research supports the interactive relationship between self-efficacy and growth mindset (e.g., Palazzolo, 2016; Tassell et al., 2020), little is known about what could prepare PSTs in developing their self-efficacy and growth mindset in teaching mathematics. By synthesising the results of the last two questions, we identified five practices that contributed most to the development of both constructs, which include inductive teaching, differentiation, productive struggle, problem solving, and probing thinking questions.

Inductive teaching

We found that learning about, designing, and practising inductive mathematics teaching styles most helped the PSTs grow their mindset and improve self-efficacy in teaching mathematics. This finding builds on Hull et al.'s (2016) results that training on teacher-led mathematics inquiry (i.e., structured inquiry) instruction increased teachers' self-efficacy in student engagement and their instructional practices. In the context of our study, inductive mathematics teaching was used as an umbrella term that described a variety of possibilities involving open and or structured inquiry processes as described in Kartal and Tillett (2021). Hull et al. discussed that teacher-led mathematics inquiry instructional practice led to an improved self-efficacy because teachers observed a greater student responsiveness and engagement in this new approach which supported a deeper understanding of the content. Similarly, the PSTs in our study pointed out greater student understandings, increased critical thinking opportunities, higher levels of engagement, and meaningful learning opportunities that use of inductive mathematics teaching may offer for their prospective students.

Differentiation

Differentiating instruction is one of the core features and basis of growth mindset pedagogy in supporting students' individual learning processes, helping them overcome the barriers, and not giving up on students (i.e., persistence) (Rissanen et al., 2019); and was found to be one of the most helpful activities/topics that helped PSTs develop a growth mindset for teaching mathematics, as well as improve self-efficacy. The view of differentiation in this mathematics methods course adopted the *equitable access to the mathematics* dimension of the TRU framework (Schoenfeld, 2014), that invites active engagement of all students with the core mathematical content, as an integral part of inductive style mathematics teaching. That is, the differentiation techniques that were discussed and practised in this methods course aimed to preserve problematic aspects of the mathematics tasks and support the learning of the targeted mathematics concepts without declining the expectations. The PSTs linked differentiation to the idea of being persistent, that every concept can be taught in multiple ways, and that every child can learn, and hence found it helpful in developing a growth mindset and self-efficacy in teaching mathematics. In addition, it is also important to note that many PSTs discussed differentiation in relation to productive struggle in their responses as in "... I now have many differentiation strategies that I can use with struggling students to make their struggles productive rather than unproductive."

Productive struggle

Teaching the positive role of failures, mistakes, and challenges in learning, and not protecting students from challenges were identified as core features of a growth mindset pedagogy in a mathematics classroom (Rissanen et al., 2019), which are also practices that support productive struggle in students. Handling mistakes and struggles as learning opportunities was also noted as a component of growth mindset for teaching mathematics in MTMF (Sun, 2018). In line with Rissanen et al.'s framework for growth mindset pedagogy and Sun's MTMF, the PSTs in our study made connections to their own struggles, mistakes, and challenges in learning mathematics in the past, and their experience of productive struggle in the methods course that moved them towards understanding, reasoning, and sense making of mathematics (Warshauer, 2015). Learning about the productive struggle as a phenomenon (i.e., struggle is a natural part of learning and an integral part of doing mathematics) led to an improved self-efficacy in the PSTs for teaching mathematics. They also perceived supporting productive struggle practice as an avenue for cultivating a growth mindset in their prospective students. Supporting productive struggle is identified as one of the eight effective mathematics teaching practices by NCTM's (2014) PtA. There is limited literature, however, that documents the introduction of this teaching practice at the prospective level (Warshauer et al., 2021). The results of this study suggest that the phenomenon of productive struggle and the practice of supporting productive struggle should be further investigated in teacher education programs, especially in the mathematics methods courses.

Problem solving

The need to focus on a wide range of problem-solving strategies when educating mathematics teachers has been highlighted in the research (e.g., Bruun, 2013; Hallman-Thrasher, 2017), and identified as a component of MTMF by Sun (2018). The PSTs' responses supported that engaging in problem solving and looking for a variety of approaches to solve a problem helped them improve self-efficacy and growth mindset. The PSTs indicated that learning and practising problem-solving had a two-fold impact on their self-efficacy and growth mindset for teaching mathematics. First, they pointed out their difficulty and struggle to find multiple approaches to the given problem-solving tasks in the methods course—such difficulty was also documented in Kartal et al. (2020). As evidenced in the excerpts in Tables 3 and 4, the PSTs indicated that being able to solve a problem in many different ways expanded their own thinking and flexibility, which in turn helped them increase their self-efficacy and growth mindset. Second, the PSTs recognised problem solving as a portal for exploration, discovery, mathematical reasoning, and productive struggle for their prospective students, as well as a tool to access how students think. For example, PSTs said, "... allowing them to explore it on their own and struggle along the way to find the way they learn the concept best," "... how important it is for students to problem solve and struggle to find an answer," and "... by working through a problem in a way that a specific student might, I feel I have a better understanding of how they think."

Probing thinking questions

Learning to ask probing thinking questions was studied in relation to *pose purposeful questions* for effective mathematics teaching practice (NCTM, 2014) in the mathematics methods course, which the PSTs found helpful for improving growth mindset, as well as for self-efficacy—albeit less frequently than it was reported for growth mindset. The PSTs reported that asking probing thinking questions helped them develop a growth mindset for teaching mathematics, as it is a way to foster student learning, critical thinking, and productive struggle, as evidenced in Table 3. For self-efficacy, the PSTs reported that by means of asking "the right" questions (i.e., probing thinking), they can now access how students learn, think, and connect (see Table 4). Teachers in the United States ask fewer probing questions that support the deep levels of student understanding than teachers in other high-achieving countries (Stigler & Hiebert, 2009 a, b); and thus, it is critically important to put emphasis on the practice of *posing purposeful questions* in teacher preparation programs. The results of this study confirm the importance of engaging PSTs in asking probing thinking questions in the mathematics methods courses. Given that explicit teaching of questioning skills helps PSTs cultivate their questioning skills (Morrissey et al., 2020), teacher educators should engage PSTs in such practices in the mathematics methods courses to equip them with the tools that help them build self-efficacy and growth mindset for teaching mathematics.

Conclusions and Future Directions

This study was conducted in the mathematics methods course, in which the elementary PSTs were taught pedagogical practices, and were not directly taught mathematics content; however, they utilised their mathematics content knowledge and skills within the context of teaching mathematics. The goal of our study was to explore any change in the PSTs' self-efficacy in teaching mathematics through a mathematics methods course and identify the practices that helped them develop self-efficacy and growth mindset in teaching mathematics. The results showed that the course significantly increased the PSTs' overall self-efficacy in teaching mathematics; and felt they became more efficacious in applying appropriate pedagogies and teaching mathematics content. Among the many topics, activities, and experiences over the course of a semester-long mathematics methods class, five practices that PSTs learned and applied became prominent in helping them develop a growth mindset and self-efficacy in teaching mathematics: inductive teaching, differentiation, supporting productive struggle, problem solving, and asking probing thinking questions. Among these five practices, three of them (i.e., productive struggle, problem solving, and asking probing thinking questions) overlap with the three of the eight NCTM's (2014) effective mathematics teaching practices, listed in Table 1 (i.e., support productive struggle in learning mathematics, implement tasks that promote problem-solving and reasoning, and pose purposeful questions).

This study suggests that the PSTs should be given opportunities to discuss, observe, and experience these five practices in mathematics methods courses. Future studies can investigate these practices and topics more in-depth in relation to PSTs' and in-service teachers' self-efficacy and growth mindset for teaching mathematics. Research efforts could be geared towards creating modules in which these five practices are interwoven and examine the impact on PSTs and or in-service teachers. Follow-up studies should be conducted with PSTs in their student teaching and or beginning years of teaching to explore how their growth mindset and self-efficacy evolve hand in hand with actualisation of these five practices. It would be informative to observe how students respond to inductive teaching, differentiation, productive struggle, problem solving, and probing thinking questions. It is possible that there could be other factors or explanations (e.g., school-based experiences) for changes in self-efficacy among PSTs that were not specifically addressed in this study. In this study, we focused on capturing PSTs' subjective experiences and perceptions regarding specific aspects of the methods course.

It is important to note that the results of this study rely on self-reported data, which is susceptible to social-desirability bias. This bias suggests that PSTs may have been motivated to present themselves more positively or align their responses with societal expectations when answering the survey questions. Nevertheless, the utilisation of self-report instruments allowed for the direct assessment of PSTs' self-efficacy and growth mindset in teaching mathematics, offering insights into their personal perspectives instead of relying on indirect assessments through researchers' observations and interpretations.

References

Althauser, K. L. (2018). The emphasis of inquiry instructional strategies: Impact on preservice teachers' mathematics efficacy. *Journal of Education and Learning, 7*(1), 53–69.

Anderson, R. K., Boaler, J., & Dieckmann, J. A. (2018). Achieving elusive teacher change through challenging myths about learning: A blended approach. *Education Sciences, 8*(98), 1–33.

Aronson, J., Fried, C. B., & Good, C. (2002). Reducing the effects of stereotype threat on African American college students by shaping theories of intelligence. *Journal of Experimental Social Psychology*, *38*(2), 113–125.

Bates, A. B., Latham, N. I., & Kim, J. (2011). Linking preservice teachers' mathematics self-efficacy and mathematics teaching efficacy to their mathematical performance. *School Science and Mathematics*, *111*(7), 325–333.

Bates, A. B., Latham, N. I., & Kim, J. (2013). Do I have to teach math? Early childhood preservice teachers' fears of teaching mathematics. *Teacher Attributes*, 5, 1–10.

Bedford, S. (2017). Growth mindset and motivation: A study into secondary school science learning. *Research Papers in Education*, *32*(4), 424–443.

Blackwell, L. S., Trzesniewski, K. H., & Dweck, C. S. (2007). Implicit theories of intelligence predict achievement across an adolescent transition: A longitudinal study and an intervention. *Child Development*, *78*(1), 246–263.

Brady, P., & Bowd, A. (2005). Mathematics anxiety, prior experience and confidence to teach mathematics among pre-service education students. *Teachers and Teaching*, *11*(1), 37–46.

Bransford, J. D., Brown, A. L., & Cocking, R. R. (Eds.). (2000). *How people learn: Brain, mind, experience, and school.* National Academy Press.

Bruun, F. (2013). Elementary teachers' perspectives of mathematics problem solving strategies. *The Mathematics Educator, 23*(1), 45–59.

Bursal, M., & Paznokas, L. (2006). Mathematics anxiety and preservice elementary teachers' confidence to teach mathematics and science. *School Science and Mathematics*, *106*(4), 173-180.

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics*. https://learning.ccsso.org/wp-content/uploads/2022/11/ADA-Compliant-Math-Standards.pdf

Creswell, J. W. (2013). *Qualitative inquiry and research design: Choosing among five approaches* (3rd ed.). SAGE Publications.

Cutler, C. S. (2020). Preservice teachers' mathematical mindsets during pandemic-induced pivot to online learning. *Frontiers in Education, 5*, 1–6. https://doi.org/10.3389/feduc.2020.595264

DeBacker, T. K., Heddy, B. C., Kershen, J. L., Crowson, H. M., Looney, K., & Goldman, J. A. (2018). Effects of a one-shot growth mindset intervention on beliefs about intelligence and achievement goals. *Educational Psychology, 38*(6), 711–733.

Dweck, C. S. (2006). *Mindset: The new psychology of success.* Random House.

Enochs, L. G., Smith, P. L., & Huinker, D. (2000). Establishing factorial validity of the mathematics teaching efficacy beliefs instrument. *School Science and Mathematics*, *100*(4), 194–202.

Esparza, J., Shumow, L., & Schmidt, J. A. (2014). Growth mindset of gifted seventh grade students in science. *NCSSSMST Journal*, *19*(1), 6–13.

Foliano, F., Rolfe, H., Buzzeo, J., Runge, J., & Wilkinson, D. (2019). *Changing mindsets: Effectiveness trial.* National Institute of Economic and Social Research.

Giles, R. M., Byrd, K. O., & Bendolph, A. (2016). An investigation of elementary preservice teachers' self-efficacy for teaching mathematics. *Cogent Education, 3*(1), 1160523. https://doi.org/10.1080/2331186X.2016.1160523

Glaser, B. G., & Strauss, A. L. (1967). *The discovery of grounded theory: Strategies for qualitative research.* Aldine Publishing Company.

Gresham, G. (2007). A study of mathematics anxiety in pre-service teachers. *Early Childhood Education Journal*, *35*(2), 181–188.

Gresham, G. (2008). Mathematics anxiety and mathematics teacher efficacy in elementary pre-service teachers. *Teaching Education*, *19*(3), 171–184.

Hall, J. M., & Ponton, M. K. (2005). Mathematics self-efficacy of college freshman. *Journal of Developmental Education*, *28*(3), 26–32.

Hallman-Thrasher, A. (2017). Prospective elementary teachers' responses to unanticipated incorrect solutions to problem-solving tasks. *Journal of Mathematics Teacher Education, 20*(6), 519–555. http://dx.doi.org/10.1007/s10857-015-9330-y

Harper, N. W., & Daane, C. J. (1998). Causes and reduction of mathematics anxiety in preservice elementary teachers. *Action in Teacher Education*, *19*(4), 29–38.

Hembree, R. (1990). The nature, effects, and relief of mathematics anxiety. *Journal for Research in Mathematics Education*, *21*(1), 33–46.

Hochanadel, A., & Finamore, D. (2015). Fixed and growth mindset in education and how grit helps students persist in the face of adversity. *Journal of International Education Research*, *11*(1), 47–50.

Huangfu, W. (2012). Effects of EFL teachers' self-efficacy on motivational teaching behaviors. *Asian Social Science, 8*(15), 68–74.

Hull, D. M., Booker, D. D., & Näslund-Hadley, E. I. (2016). Teachers' self-efficacy in Belize and experimentation with teacher-led mathematics inquiry. *Teaching and Teacher Education, 56*, 14–24.

Johnson, T., Byrd, K., & Giles R. (2018). Mathematics self-efficacy and content knowledge: Exploring the connection for elementary preservice teachers. *Research Issues in Contemporary Education, 3*(1), 30–38.

Kartal, O., Morrissey, S. G., & Popovic, G. (2020). Analysis of elementary preservice teachers' identification of mathematical problem-solving tasks and anticipated student solutions. *The Mathematics Educator, 29*(1), 85–111.

Kartal, O., & Tillett, W. (2021). Transforming preservice teachers' conceptions and lesson plan designs for teaching mathematics from a direct-teaching approach to a reform-based approach. *Investigations in Mathematics Learning, 13*(2), 107–124.

Kelly, W. P., & Tomhave, W. K. (1985). A study of mathematics anxiety/mathematics avoidance in preservice elementary teachers. *The Arithmetic Teacher*, *32*(5), 51–53.

Khan, S. A. (2012). The relationship between teachers' self-efficacy and students' academic achievement at secondary level. *Language in India, 12* (10), 436–449.

Laurian-Fitzgerald, S., & Fitzgerald, C. (2016). Engaging undergraduate pre-service teachers in the development of growth mindset and grit. In Ş. Olah, G. Roşeanu, S. Bodogai & L. Coturbaş (Eds.), *Current challenges in social sciences* (pp. 138–154). Presa Universitară Clujeană.

Looney, L., Perry, D., & Steck, A. (2017). Turning negatives into positives: The role of an instructional math course on preservice teachers' math beliefs. *Education, 138*(1), 27–40.

Maguire, K. (2011). *The role of teacher efficacy in student academic achievement in mathematics* (Publication No. 3449809) [Doctoral dissertation, Walden University]. ProQuest Dissertations and Theses.

Maslach, C., Jackson, S. E., & Leiter, M. P. (1996). *Maslach burnout inventory manual* (3rd ed.). Consulting Psychologists Press.

McCampbell, S. (2015). *Pre-service teachers' self-efficacy for teaching mathematics*. [Doctoral dissertation, University of New Mexico]. UNM Digital Repository. https://digitalrepository.unm.edu/educ_ifce_etds/28/

McGee, J. R., & Wang, C. (2014). Validity-supporting evidence of the self-efficacy for teaching mathematics instrument. *Journal of Psychoeducational Assessment, 32*(5), 390–403.

McGuire, J. (2016). Preservice teacher self-efficacy for teaching mathematics. [Doctoral thesis, The University of Maine]. DigitialCommons@UMaine. https://digitalcommons.library.umaine.edu/honors/40

McKinney, E. (2018). *The impact of teacher growth mindset on student self-efficacy and math performance for all students and for English learners* (Publication No. 13421529) [Doctoral dissertation, University of Texas at San Antonio]. ProQuest Dissertations and Theses Global.

Miller, D. I. (2019). When do growth mindset interventions work? *Trends in Cognitive Sciences, 23*(11), 910–912.

Morrissey, S., Kartal, O., & Popovic, G. (2020). Explicit teaching of questioning in math methods course: Preservice teachers' attempts to ask probing questions. In A. I. Sacristan, J. C. Cortes-Zavala & P. M. Ruiz-Arias (Eds.), *Mathematics education across cultures* (Proceedings of the 42nd Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Mexico, pp. 1662–1666).

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.*

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.*

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, & B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. National Academy Press.

O'Rourke, E., Haimovitz, K., Ballweber, C., Dweck, C., & Popović, Z. (2014, April). Brain points: A growth mindset incentive structure boosts persistence in an educational game. In *Proceedings of the SIGCHI conference on human factors in computing systems* (pp. 3339–3348). ACM.

Palazzolo, S. D. (2016). *The relationship between mindset and self-efficacy in pre-service elementary teacher candidates teaching science, and its implications on science teaching*. [Doctoral thesis, University of Windsor].

Parrish, S., & Dominick, A. (2016). *Number talks: Fractions, decimals, and percentages.* Math Solutions.

Perera, H. N., & John, J. E. (2020). Teachers' self-efficacy beliefs for teaching math: relations with teacher and student outcomes. *Contemporary Educational Psychology, 61*, Article 101842. https://doi.org/10.1016/j.cedpsych.2020.101842

Prince, M. J., & Felder, R. M. (2006). Inductive teaching and learning methods: Definitions, comparisons, and research bases. *Journal of Engineering Education, 95*(2), 123–138. https://doi.org/10.1002/j.2168-9830.2006.tb00884.x

Ramirez, G., Hooper, S. Y., Kersting, N. B., Ferguson, R., & Yeager, D. (2018). Teacher math anxiety relates to adolescent students' math achievement. *AERA Open, 4*(1), 1–13. https://doi.org/10.1177/2332858418756052

Rhew, E., Piro, J. S., Goolkasian, P., & Cosentino, P. (2018). The effects of a growth mindset on self-efficacy and motivation. *Cogent Education*, *5*(1), Article 1492337.

Rissanen, I., Kuusisto, E., Tuominen, M., & Tirri, K. (2019). In search of a growth mindset pedagogy: A case study of one teacher's classroom practices in a Finnish elementary school. *Teaching and Teacher Education*, *77*, 204–213.

Schoenfeld, A. H. (2014). What makes for powerful classrooms, and how can we support teachers in creating them? A story of research and practice, productively intertwined. *Educational Researcher, 43*(8), 404–412. https://doi.org/10.3102/0013189X14554450

Scott, W. A. (1955). Reliability of content analysis: The case of nominal scale coding. *Public Opinion Quarterly*, *19*(3), 321–325.

Seaton, F. S. (2018). Empowering teachers to implement a growth mindset. *Educational Psychology in Practice*, *34*(1), 41–57.

Sisk, V. F., Burgoyne, A. P., Sun, J., Butler, J. L., & Macnamara, B. N. (2018). To what extent and under which circumstances are growth mind-sets important to academic achievement? Two meta-analyses. *Psychological Science, 29*(4), 549–571.

Skaalvik, E. M., & Skaalvik, S. (2010). Teacher self-efficacy and teacher burnout: A study of relations. *Teaching and Teacher Education, 26*(4), 1059–1069.

Stigler, J. W., & Hiebert, J. (2009a). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom.* Simon and Schuster.

Stigler, J. W., & Hiebert, J. (2009b). Closing the teaching gap. *Phi Delta Kappan, 91*(3), 32–37.

Strauss, A., & Corbin, J. (1990). *Basics of qualitative research: Grounded theory procedures and techniques.* SAGE Publications.

Sun, K. (2018). Brief report: The role of mathematics teaching in fostering student growth mindset, *Journal for Research in Mathematics Education JRME, 49*(3), 330–335.

Swars, S. L., Daane, C. J., & Giesen, J. (2006). Mathematics anxiety and mathematics teacher efficacy: What is the relationship in elementary preservice teachers? *School Science and Mathematics*, *106*(7), 306–315.

Tassell, J., Gerstenschlager, N. E., Syzmanski, T., & Denning, S. (2020). A study of factors impacting elementary mathematics preservice teachers: Improving mindfulness, anxiety, self-efficacy, and mindset. *School Science and Mathematics, 120*(6), 333–344. https://doi.org/10.1111/ssm.12425

Thurmond, K. M. (2020). *Exploring the impact of growth mindset strategies on mathematics pre-service teachers' beliefs.* [Doctoral dissertation, University of Houston].

Viel-Ruma, K., Houchins, D., Jolivette, K., & Benson, G. (2010). Efficacy beliefs of special educators: The relationships among collective efficacy, teacher self-efficacy, and job satisfaction. *Teacher Education and Special Education, 33*(3), 225–233.

Vinson, B. M. (2001). A comparison of preservice teachers' mathematics anxiety before and after a methods class emphasizing manipulatives. *Early Childhood Education Journal*, *29*(2), 89–94.

Waid, B. E. (2018). *Preservice mathematics teacher beliefs and growth mindset assessment practices* [Doctoral dissertation, Columbia University].

Warshauer, H. K. (2015). Productive struggle in middle school mathematics classrooms. *Journal of Mathematics Teacher Education, 18*(4), 375–400. https://doi.org/10.1007/s10857-014-9286-3.

Warshauer, H. K., Starkey, C., Herrera, C. A., & Smith, S. (2021). Developing prospective teachers' noticing and notions of productive struggle with video analysis in a mathematics content course. *Journal of Mathematics Teacher Education*, *24*, 89–121. https://doi.org/10.1007/s10857-019-09451-2

Yeager, D. S., & Dweck, C. S. (2012). Mindsets that promote resilience: When students believe that personal characteristics can be developed. *Educational Psychologist, 47*, 302–314.

Yeager, D., Walton, G., & Cohen, G. L. (2013). Addressing achievement gaps with psychological interventions. *Phi Delta Kappan, 94*(5), 62–65.

Zuya, H. E., Kwalat, S. K., & Attach, B. G. (2016). Pre-service teachers' mathematics self-efficacy and mathematics teaching self-efficacy. *Journal of Education and Practice*, *7*(14), 93–98.

Corresponding author

Correspondence to:

Yao Fu
fuy@uww.edu

Funding

This study was funded by the University of Wisconsin-Whitewater Assessment Grant.

Ethics Declarations

Ethical approval

Ethical approval for the research was granted by the University of Wisconsin-Whitewater and informed consent was given by all participants for their data to be published.

Competing interests

The authors declare there are no competing interests.