Pre-service Mathematics Teachers' Views on an Inquiry-based Learning Environment Emphasising Mathematical Communication

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This study examined pre-service mathematics teachers' views on a communicative learning environment based on an inquiry-based learning approach. In the research, the phenomenological research model was used. The research was carried out with 24 pre-service teachers studying in the Primary Education Mathematics Teaching program of a state university located in Istanbul. The data were collected using a semi-structured interview protocol comprised of eight items prepared by the researcher. In the analysis of the data, content analysis was conducted using a qualitative data analysis computer software package. Within the scope of the research, pre-service teachers were asked to evaluate the inquiry-based learning approach, the course they took with this approach, the learning environment based on mathematical communication, and their level of readiness for the process. The results showed that pre-service teachers had difficulties in adapting to the inquiry-based learning approach and the mathematical communication environment, but they found the course process useful and efficient. It was determined that pre-service teachers attribute the difficulties they experience during the process to some factors, such as teachers, past learning experiences, and the education system.

Keywords • mathematics teacher education research • mathematics teaching • inquiry-based learning • pre-service teachers • mathematics communication

Introduction

Scientific and technological advancements have profoundly impacted society, influencing educational expectations. The primary goal of education is to cultivate individuals with relevant skills and knowledge. Mathematics plays a pivotal role in acquiring these essential qualities. The effective teaching of mathematics necessitates the utilisation of active learning approaches within educational environments. Effective mathematics teaching requires learning approaches that actively engage students, emphasising their participation and involvement (Prince, 2004).

Active learning is rooted in constructivist principles, forming its foundation. Constructivist learning entails students exploring complex scenarios instead of routine exercises, actively engaging in the learning process rather than being passive recipients, adopting a questioning perspective to acquire relevant information, and assuming responsibility for their own learning (PRIMAS, 2011). This approach encourages deeper and more meaningful learning experiences and communicative learning environments (Ahmedi et. al., 2023; Mikroyannidis et al., 2015). Inquiry-based learning, aligned with the constructivist approach, has garnered considerable attention in recent years. This study examines the pre-service teachers views on inquiry-based learning and mathematical communication on the premise that the strategies enrich the learning of pre-service mathematics teachers, who are future practitioners of the inquiry-based learning approach.

Theoretical Framework

The inquiry-based learning approach, pioneered by Dewey and Vygotsky (Taylor & Bilbrey, 2012), is a pedagogical strategy that enables students to construct knowledge by following scientific research processes (Keselman, 2003). It is a practical method that establishes a connection between students' existing knowledge and scientific principles (Panasan & Nuangchalerm, 2010). This approach encourages learners to explore causal relationships, test hypotheses, and enhance problem-solving skills through research and observation (Pedaste et al., 2012; Pedaste & Sarapuu, 2006). Additionally, active participation and assuming responsibility for learning foster the development of students' critical thinking abilities (Ikpeze & Boyd, 2007; Llewellyn, 2001).

According to Oğuz Ünver and Arabacıoğlu (2011), people naturally generate questions in their minds while observing and making sense of the events around them. As a result, inquiry-based learning relies on questions arising from real observations. In terms of the features of the inquiry-based learning method, Gordon and Brayshaw (2008) presented a concise summary illustrated in Figure 1.

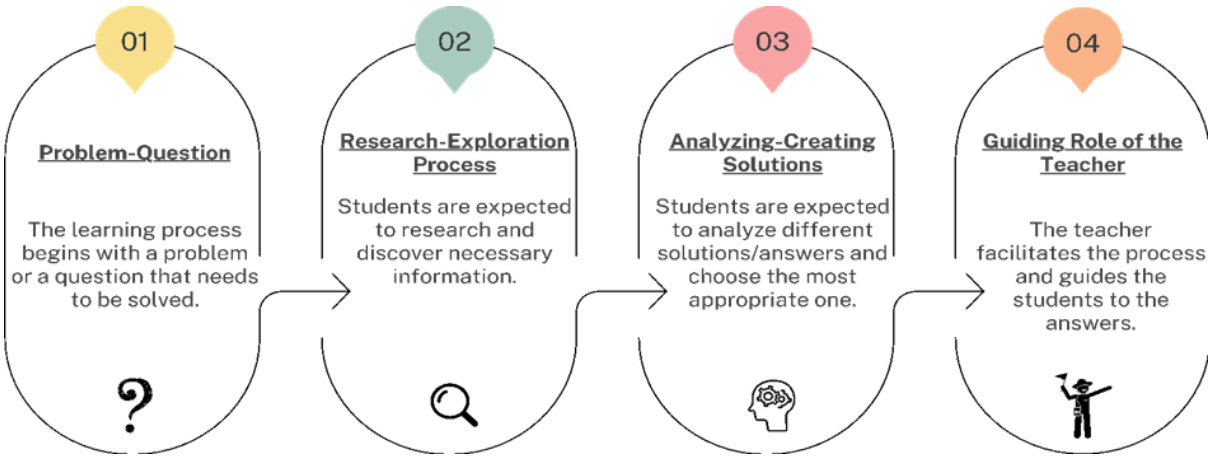

Figure 1. Characteristics of inquiry-based learning method.

Incorporating inquiry-based learning into mathematics education appears to be effective in fostering students' higher-order thinking skills. By encouraging students to explore, question, and discuss mathematical concepts, this approach not only enhances cognitive abilities but also promotes deeper engagement with the material. Consequently, such active engagement is closely tied to improved mathematical communication skills. Marshall and Horton (2011) highlighted the important link between research-based teaching and enhanced mathematical communication. To establish a classroom environment centered on mathematical communication through inquiry-based learning, understanding its impact on critical thinking and communication skills is crucial. This approach not only improves students' critical thinking skills (Duran & Dökme, 2016) but also enhances mathematical communication skills in problem-solving (Istimuryani et al., 2023). Emphasising student-centered exploration and communication, inquiry-based learning deepens understanding by challenging students to reinvent or create mathematics (Hussein, 2023). Furthermore, the development of mathematical discussions and communication supports teachers in creating more inquiry-based mathematics classes (McCrone, 2005).

Inquiry-based learning environments foster intense discussions among students as they embark on problem-solving and seek understanding through questioning. These discussions play a vital role in the attainment of meaningful outcomes. Within the framework of inquiry-based learning, students delve into a chosen subject as a phenomenon, conducting thorough investigations, synthesising their findings, and presenting and discussing their results with peers (Arini et al., 2019). Throughout this process,

students require proficient mathematical communication skills to effectively convey their findings and engage in fruitful discussions that yield meaningful outcomes (Goldston et al., 2010).

In the literature, numerous research findings demonstrate the favourable impact of inquiry-based learning on various aspects of students, encompassing affective attributes such as motivation, self-regulation skills, positive attitudes, and beliefs, as well as cognitive factors like academic achievement and scientific process skills (Correia & Harrison, 2020; Gomez et al., 2015; Laursen et al., 2016; Mutlu, 2020).

Background of the Study

Today's expectations for education have challenged traditional teaching methods and highlighted constructivist understanding and active learning. Academics argue that the constructivist learning paradigm, which emphasises facilitating the student through active learning, problem solving and interactive methods, is more suitable for raising individuals who can adapt to contemporary learning conditions (Biesta, 2011; Tomljenović & Vorkapić, 2020). In these approaches, teachers play an important role in effective mathematics teaching. The Basic Law of National Education No. 1739 emphasises the criteria of general knowledge, specialised field knowledge and pedagogical field knowledge for both in-service and pre-service teachers (NEF, 2014). To improve the quality of teaching, teachers need to have solid content knowledge (Brown & Borko, 1992; Davis & Simmt, 2006; Hill et al., 2005; Tchoshanov, 2011). Cohen et al. (1993) supported this perspective by revealing that teachers with strong content knowledge develop more activities and strategies to improve their lessons. As highlighted by Brown and Borko (1992) and An et al. (2004), inadequate content knowledge among teachers can have detrimental effects on their pedagogical expertise.

Teacher education programs play a pivotal role in shaping future educators by taking a constructivist learning approach and fostering their motivation to teach. These programs serve as exemplars of communicative learning environments, providing pre-service teachers with hands-on experiences to internalise these methodologies. The exposure to such environments is crucial for pre-service teachers to seamlessly adapt to the active participation and collaborative learning demands of constructivist principles. Inquiry-based learning, inherent in its nature, serves to steer prospective teachers away from rote learning practices, encouraging a research-oriented and inquiry-based educational mindset. Furthermore, this pedagogical method has the potential to enhance pre-service teachers' skills, including effective communication and advanced mathematical thinking.

Despite the limited application of research-based learning in pre-service teacher education studies, the literature underlines the important contributions of educators to both affective and cognitive aspects (Celep Havuz & Karamustafaoğlu, 2016; Ecevit & Kaptan, 2019; Gürbüz & Bostan Sarıoğlan, 2022; Preston et al., 2015; Silm et al., 2019). Preston et al. (2015) pointed out that participation and success in teacher education increases through research-based learning and emphasised the importance of this in assessing the knowledge and deficiencies of teacher candidates. Although existing research has tended to focus on science teacher candidates, this study aims to provide mathematics teacher candidates with an inquiry-based learning environment in a communicative learning environment and to investigate their perspectives on this pedagogical approach, which is considered suitable for the conceptual structure of mathematics. Evaluating the impact of inquiry-based learning courses on pre-service teachers' content knowledge and teaching pedagogy is crucial and leads to the formulation of the following research questions.

- *RQ1 What are pre-service mathematics teachers' perceptions regarding the inquiry-based learning approach?*
- *RQ2 How do pre-service mathematics teachers evaluate the course process?*
- *RQ3 What are the perspectives of pre-service mathematics teachers regarding the communicative learning environment provided?*
- *RQ4 What are pre-service mathematics teachers' opinions regarding their readiness for inquiry-based learning and communication-based learning environments?*

Method

Model of the Research

The phenomenological research model was selected as the appropriate approach for this study. Phenomenology is a qualitative research model that seeks to understand the perceptions and thoughts of individuals who have experienced a specific phenomenon or event (Creswell et al., 2007). This research is consistent with the characteristics of phenomenological research as it aims to understand and interpret the lived experiences and perceptions of the participants regarding the inquiry-based learning methodology.

Study Group

The study involved 24 pre-service teachers enrolled in the Primary Education Mathematics Teaching program at a state university in Istanbul. The participants were selected among 80 first-grade students enrolled in the "Fundamentals of Mathematics" course. The pre-service teachers taking the course originate from diverse regions across Turkey and predominantly belong to lower-middle socioeconomic backgrounds. The selection process employed ensured a balanced representation based on gender and varying levels of in-class achievement (high, medium, low). The consent of the chosen pre-service teachers was obtained, and those who did not consent were replaced with other suitable candidates. Participants were assured that their involvement or lack thereof would not influence their grades.

Purposive sampling, specifically maximum variation sampling, was employed to select participants for the study. Maximum variation sampling seeks to include people with a variety of relevant traits (Grix, 2010). The aim was to select a heterogeneous sample that captured perspectives that were typical as well as those that were possibly more extreme in nature. In this research, factors such as gender, academic performance, and the type of high school graduated were considered as potential influences on the perspectives of pre-service teachers. Therefore, an equal number of participants were selected based on these variables. Table 1 lists participant details. In addition, the academic performances of pre-service teachers were determined by looking at their last term weighted grade point averages in their academic transcripts.

Table 1
Demographic Characteristics of Participants

Variable		Participants
Gender	Female	T1, T2, T3, T4, T5, T6, T7, T10, T12, T15, T22, T24
	Male	T8, T9, T11, T13, T14, T16, T17, T18, T19, T20, T21, T23
Academic Performance	Low	T1, T2, T13, T16, T18, T19, T20, T22
	Medium	T5, T6, T7, T8, T11, T12, T14, T23
	High	T3, T4, T9, T10, T15, T17, T21, T24
High School Graduation	Science and Anatolian High Schools	T4, T5, T6, T8, T9, T10, T15, T17, T21, T24
	Plain and Vocational High Schools	T1, T2, T3, T7, T11, T12, T13, T14, T16, T18, T19, T20, T22, T23

Descriptive Information on the Learning Environment

In this study, a communication-based learning environment was established using an inquiry-based learning method, considering the roles of both teacher and student. This approach aligns with Kersaint (2015) perspective and is illustrated in Figure 2.

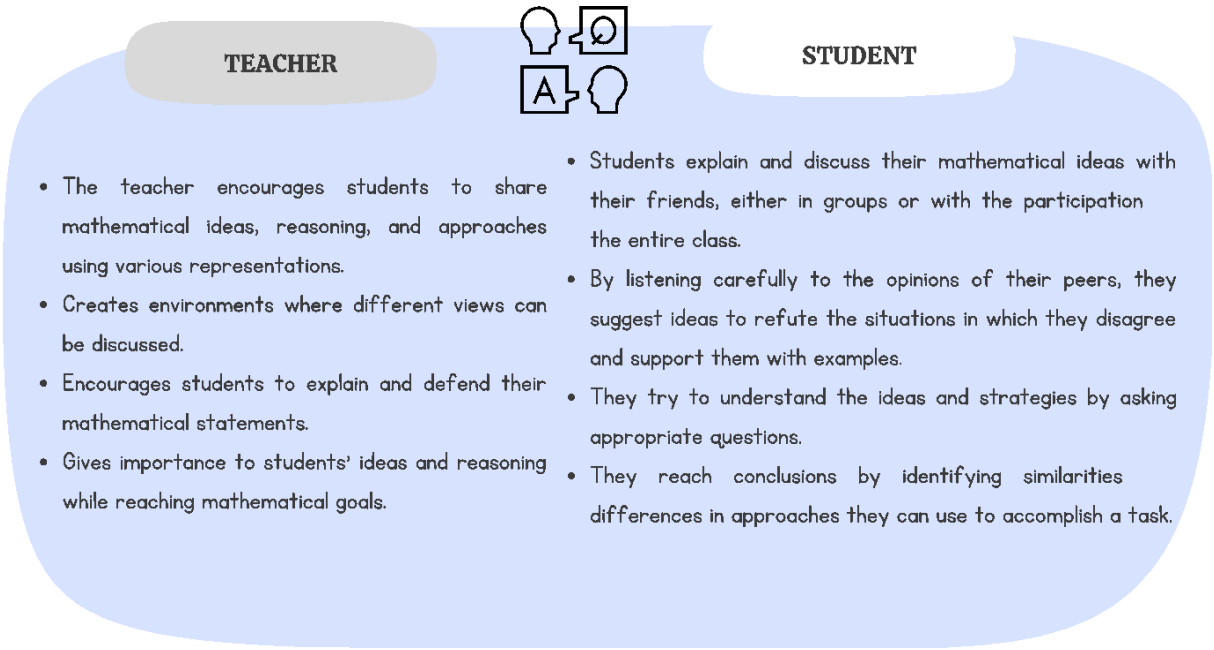

Figure 2. Teacher and student roles in communication-rich learning environments.

The learning environment designed within the scope of the research was applied in a course called "Fundamentals of Mathematics" conducted by the researcher. Weekly, participants received questions to connect core concepts with upcoming topics. They conducted research, drew conclusions, and engaged in guided discussions with the researcher, aiming for meaningful outcomes. During the implementation phase, the researcher encouraged active participation from the pre-service teachers to enhance interaction between the researcher and the participants. Additionally, interactions were facilitated through the virtual classroom on the CANVAS learning management system that the university utilised, particularly during extracurricular activities. This approach helped participants feel more at ease during interviews, promoting more genuine and candid responses. An illustrative lesson example follows.

Sample of a course

Problem stage. The inquiry process began with the research question, "What sets fractions apart from rational numbers?" The pre-service teachers' existing knowledge was evaluated without biasing their perspectives. Initial responses indicated that most participants perceived the two concepts as synonymous. Some defined fractions as positive rational numbers. To guide their research, specific questions were introduced. Examples of these questions included:

- How would you define the concept of a fraction?
- Can it be classified as a fractional number?
- Are the numbers 1/2, 4/8, and -3/5 all fractions?

Research-discovery phase. During a one-week research and discovery process, pre-service teachers defined fractions as equal parts of a whole and realised they are not a number. They also understood that negative numbers do not represent fractions. Although their understanding of fractional numbers improved through discussions and guidance, some still struggled to connect fractions with rational numbers. To address this, redirecting questions were introduced, initiating another one-week research process. Examples of these questions included:

- What does the concept of equivalent fractions signify?
- What is the definition of a set, and what are its characteristics?
- Can fractional numbers such as 1/2, 2/4, 3/6, etc., be grouped together in the same number set, despite representing different fractions?

- Can a relationship be established between equivalent fractions and the concept of equivalence relation?

Analysing and creating appropriate solutions phase. After one-week, pre-service teachers retained their understanding of equivalent fractions and sets. They concluded that equivalent fraction numbers like 1/2, 2/4, 3/6 cannot be considered distinct elements in a set due to the rule against repetition. They also recognised that equivalent fractions form an equivalence class and acknowledged the presence of a number set containing integers, including positive and negative fractional numbers.

Conclusion stage. After the process, pre-service teachers concluded that fractions involve equal divisions of a whole, while rational numbers encompass a set including fractional numbers. They recognised that each equivalence class formed by equivalent fractions has a representative element belonging to the set of rational numbers. Thus, the set of rational numbers can be obtained as follows:

$$\{a/b: a, b \in Z, b\neq 0 \text{ and } a, b \text{ are relatively prime}\}$$

Data Collection and Analysis

In-depth interviews, a widely accepted qualitative research method, were used in the research. During face-to-face interviews, a semi-structured interview protocol comprised of eight questions developed by the researcher was used to investigate perspectives on the research-based learning method, course, communication-centered learning environment and the readiness of teacher candidates. The protocol incorporated questions such as "What benefits do you think the courses given using the research-based learning method provide you?", "How do you evaluate the impact of the course on your communication skills?" The interviews, which lasted 15–30 minutes and took place in the researcher's room, were audio recorded with the consent of the participants.

Guba and Lincoln's (1994) four criteria—credibility, dependability, confirmability, and transferability—were used to validate the interview process and interview protocol. In this study, measures were implemented to ensure all these criteria were met. Credibility was established by fostering meaningful interactions between the participants and the researcher, whilst the selection of diverse participants and direct quotations from their perspectives expressed ensured transferability. To increase the confirmability and transferability of the data, participant confirmation was also sought to verify the results reached by the researcher based on the views of the participants. In this context, the findings derived from the interviews were subsequently presented to the participants for validation and confirmation. To ensure reliability, an external analyst other than the researcher also evaluated and interpreted the interviews.

Raw data were recorded using a voice recorder to preserve all information. The interview question responses were systematically organised into four separate categories: evaluations of the research-based learning approach, reflections on the course process, evaluation of communication-based learning environments, and evaluation of the readiness levels of pre-service teachers. By classifying the raw data according to predefined criteria and representing the data in each subcategory with codes, subcategories emerged during the categorisation process. In line with the suggestion of Lincoln and Guba (1985), direct quotations are included in the presentation of the research findings for the confirmability of the data.

Expert opinions were consulted using Lawshe's (1975) technique to measure the suitability and adequacy of the questions for the research topic, that is, content validity. This technique involves grading the questions based on their appropriateness and the need for corrections or removal. It is recommended to involve five to 40 experts to determine the content validity (Ayre & Scally, 2014; Lawshe, 1975). This study surveyed eight mathematics education experts. All experts rated the items as appropriate, resulting in a Content Validity Ratio (CVR) of 1.00. The average CVR value for all items determines the Coverage Validity Index (CGI) for the entire form, which in this case was 1.00. This confirmed the suitability of the interview form and its items.

To enhance the reliability of the analysis, two researchers independently coded the data into categories, and the agreement percentages were calculated using the following formula (Miles & Huberman, 1994):

Reliability = (Number of Concordant Categories)/(Number of All Concordant and Incompatible Categories)

According to Miles and Huberman (1994), a consensus rate of 70% or higher is deemed satisfactory. In this study, the researchers achieved a 95% agreement, indicating a high level of reliability in the analysis.

This study employed content analysis to transcribe and analyse the opinions of pre-service teachers. Qualitative data analysis software package (*NVivo*™) was utilised to process the data, which involved the establishment of predetermined categories, sub-categories, dimensions, and codes. For each code, frequency and percentage values were calculated, and the findings were presented visually using figures and graphics. Given that a participant's opinions within a category may correspond to multiple codes, the study did not enforce equal summation of frequencies for codes within a subcategory to the number of participants.

Results

The pre-service teachers' opinions are discussed in four major categories: inquiry-based learning approach, course process, communicative learning environment, and readiness levels. The views on the inquiry-based learning approach are presented in Figure 3.

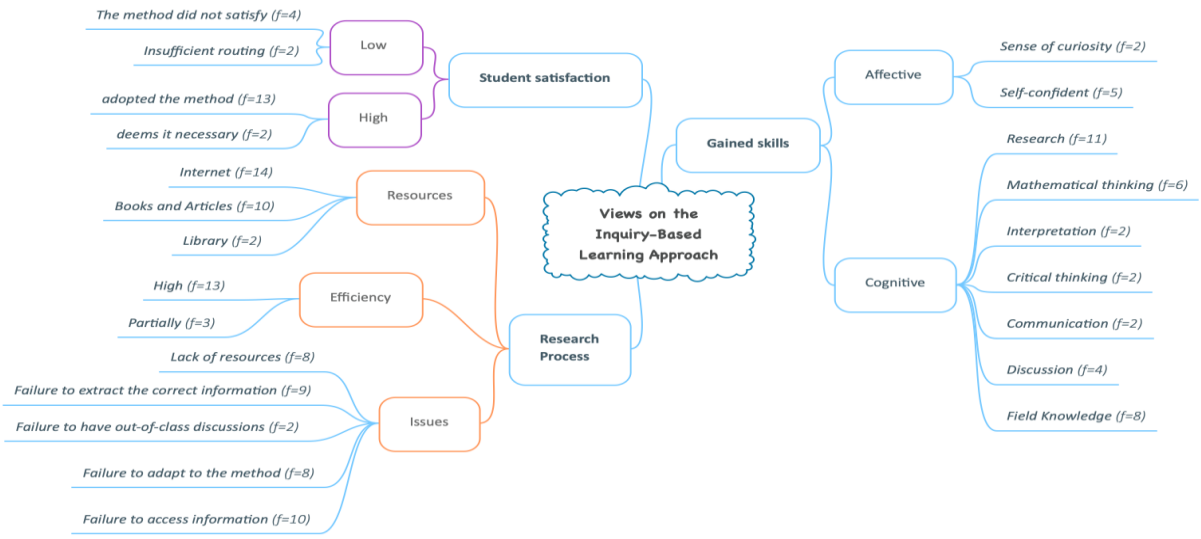

Figure 3. Views on the inquiry-based learning approach.

In the research phase of the inquiry-based learning approach, which is a crucial component of the research process, pre-service teachers predominantly relied on internet resources (*f* = 14), books, and articles (*f* = 10). One pre-service teacher articulated the strategy adopted:

> T_6: *I searched for mathematics articles on specific websites related to the topics we studied. Additionally, I conducted book research, specifically using a general mathematics book that I purchased.*

Due to the COVID-19 pandemic, some pre-service teachers expressed a preference against using the library option, citing various challenges. These included difficulties in accessing resources (*f* = 10), extracting accurate information (*f* = 9), limited availability of resources (*f* = 8), and adapting to the method (*f* = 8). Here are some examples of their views:

T_7: *Most of the resources I found were in English. My English is not very good. In most of the Turkish resources I found, I could not find the answers I was looking for.*

T_3: *In our internet research, some sites' results were different from others. As students, we didn't know which one was true. Already in the lesson, different results were obtained.*

T_1: *Not very satisfying. Because we did not have the resources to reach the answers to the questions you gave. Therefore, we could not do excellent research and reach the results we wanted.*

T_5: *We encountered uncertainty in task duration and lacked guidance. Additionally, the teaching format differed significantly from what we were accustomed to. In our usual system, information was provided, and problem-solving relied on that information. However, this method emphasised the significance of independently acquiring information.*

Despite all these problems, many of the pre-service teachers (f = 13) stated that the course process was productive for them. The opinion of a pre-service teacher is as follows:

T_7: *I was confused initially. Because I felt like I was coming to class and leaving without doing anything. Later, I started to like this work. Although we came up with a lot of testing logic, we felt like we had moved to a different stage.*

The pre-service teachers believed that the inquiry-based learning approach equipped them with skills in both cognitive and affective domains. They considered research skills (f = 11) as the most important cognitive skill acquired through this approach. Furthermore, they acknowledged its contribution to their content knowledge (f = 8), mathematical thinking skills (f = 6), and discussion skills (f = 4). Here are some examples of their opinions:

T_{13}: *This method has improved my research ability, taught me to find things on my own and synthesised results from different sources and arrive at a conclusion.*

T_{10}: *I learned new things because I have never gone into the details of mathematics so much before, and yes, it was satisfying with the way you were taught in the lesson.*

T_{15}: *The biggest contribution of this method to me may be mathematical thinking. I learned that even if you can't get results in some things, it is valuable to think about it. Maybe I'm wrong, but it helps because it makes you think.*

T_3: *We researched the weekly questions and discussed the obtained information in class. I would say that it improves our discussion skills.*

Pre-service teachers identified self-confidence (f = 5) as the primary affective skill fostered by the inquiry-based learning approach. Illustrating this is the following example:

T_2: *Before university, I lacked confidence in teaching and interacting with students. However, this method restored my self-assurance. In our first lesson, when you asked why natural numbers are named as such, I wouldn't have known how to answer. But now, I can confidently provide a response.*

Pre-service teachers evaluated their satisfaction with the inquiry-based learning approach in two dimensions: high or low. They attributed their satisfaction to their adoption of the method (f = 13). One pre-service teacher stated:

T_{17}: *I strongly believe that this method has greatly benefited me. Previously, integral was taught without a clear understanding, only focusing on rules. However, this method emphasises conceptual understanding, shifting my perspective to prioritise logic over memorisation.*

It was observed that a few pre-service teachers with low satisfaction with the method thought that the method did not satisfy them (f = 4), or they found the guidance insufficient (f = 2). Some opinions were as follows:

T_{11}: *The questions you gave were logical things, but unfortunately, the way the course was taught did not satisfy me.*

T_{18}: I think your guidance is insufficient. If you had guided us correctly at the beginning of the process, we would have turned to the right resources afterward. We could better distinguish between true and false information.

Pre-service teachers' views on the course process were evaluated in terms of content, processing, and evaluation. The findings are presented in Figure 4.

Figure 4. Views on the course process.

In the content sub-category, opinions were examined regarding relevance, attractiveness, scope, and contribution. Many pre-service teachers (f = 19) found the course process suitable for its purpose, while some found it interesting (f = 8). Examples of opinions were as follows:

T_{20}: *The content was relevant as it aligns with our future teaching in secondary school. These topics are significant, and I find it more meaningful to explore them rather than focusing solely on limits, derivatives, and integrals.*

T_{6}: *What I learned in the course were things that I did not think of in general. I learned most of it. I never thought about why this was so. In this respect, the course was very interesting.*

It was seen that there were many people (f = 14) who found the content sufficient in terms of scope, while others found the content narrow (f = 10). Some views expressed were:

T_{7}: *The course content covered the required curriculum for abstract mathematics and analysis courses this semester, with no shortage of subject matter or knowledge. However, the specific questions posed in the lessons could be further expanded.*

T_{17}: *I did not find the scope very sufficient. Since we did not have much time, we covered the topics very blindly. We could go into a little more detail.*

Pre-service teachers evaluated the contribution of the course to them as high (*f* = 14). An example follows:

> T_{18}: *You addressed fundamental topics that will be beneficial to us, which greatly contributed to our learning. For example, the questions you posed regarding rational numbers were particularly insightful and led to valuable outcomes.*

The pre-service teachers discussed the teaching of the course across four dimensions: course environment, duration, instructor, and methods/techniques. They found the learning environment productive (*f* = 18) but expressed that the lesson duration was insufficient (*f* = 18). Here are some example opinions:

> T_{13}: *The course enhanced my research skills, teaching me to independently seek information and combine* findings *from various sources to draw conclusions.*
>
> T_{8}: *I think it's not enough time. For example, not everyone can express themselves. The downside is that the time is limited.*

The pre-service teachers listed the characteristics they observed in the lecturer teaching the course as being a guide (*f* = 11), displaying a positive approach (*f* = 11), and having strong communication (*f* = 7). Examples of opinions are as follows:

> T_{7}: *It was a refreshing experience for us to engage in guided teaching instead of the traditional teacher-led approach. Unlike other lessons where we passively receive instruction, your class introduced a new and enjoyable method.*
>
> T_{3}: *You communicated well with students throughout the course. It wasn't just a lesson that you talked about, and we listened to. It was beautiful in that respect.*
>
> T_{13}: *I liked your way of speaking and your class dominance, I saw that you dominate the class without using a loud voice.*

Regarding the methods and techniques employed in the course, pre-service teachers placed emphasis on conceptual learning (*f* = 13). They also mentioned the utilisation of participatory learning (*f* = 6), question-answer (*f* = 5), and peer learning (*f* = 4) techniques. Some opinions expressed were:

> T_{18}: *I believe this course is beneficial for teacher preparation as it focuses on the logical foundations and origins of concepts. Teaching this course made me feel prepared and confident in becoming a teacher.*
>
> T_{15}: *Your participatory approach, contrasting with traditional teacher-centered methods, was the right choice. The interactive discussion environment enabled us to recognise and learn from our mistakes effectively.*
>
> T_{7}: *You were asking very different questions in the lessons, and we were able to reach very interesting results through those questions.*
>
> T_{8}: *I think the discussions we had during the lesson were very useful. Thus, we had the chance to evaluate the issues that we could not understand from many perspectives.*

In the evaluation sub-category, a considerable proportion of pre-service teachers (*f* = 12) thought that the lessons conducted with the research-based learning approach were challenging, with some thinking that they increased their success. Examples are as follows:

> T_{4}: *I can say that the questions in the exams were challenging for me. Because you asked production-based questions in the exam as I did in the lesson.*
>
> T_{21}: *I think that the course was beneficial for me and increased my success in the exams.*

The pre-service teachers evaluated the questions they encountered both in the course and in the exams as mostly generating (*f* = 10), difficult (*f* = 6), interpretation-based (*f* = 6), and high-level thinking (*f* = 5). Some of these opinions were as follows:

T_2: *In our lessons and exams, you preferred questions that we need to produce in addition to our constantly existing knowledge.*

T_4: *Before entering the mid-term exam, I thought that more terminological questions would be asked, but they were asked in a more complex, interpretive, and overlapping way. This may be better.*

Pre-service teachers' views on the learning process based on communication were discussed in two sub-categories: students' perspectives and their contributions to the learning process. The findings are presented in Figure 5:

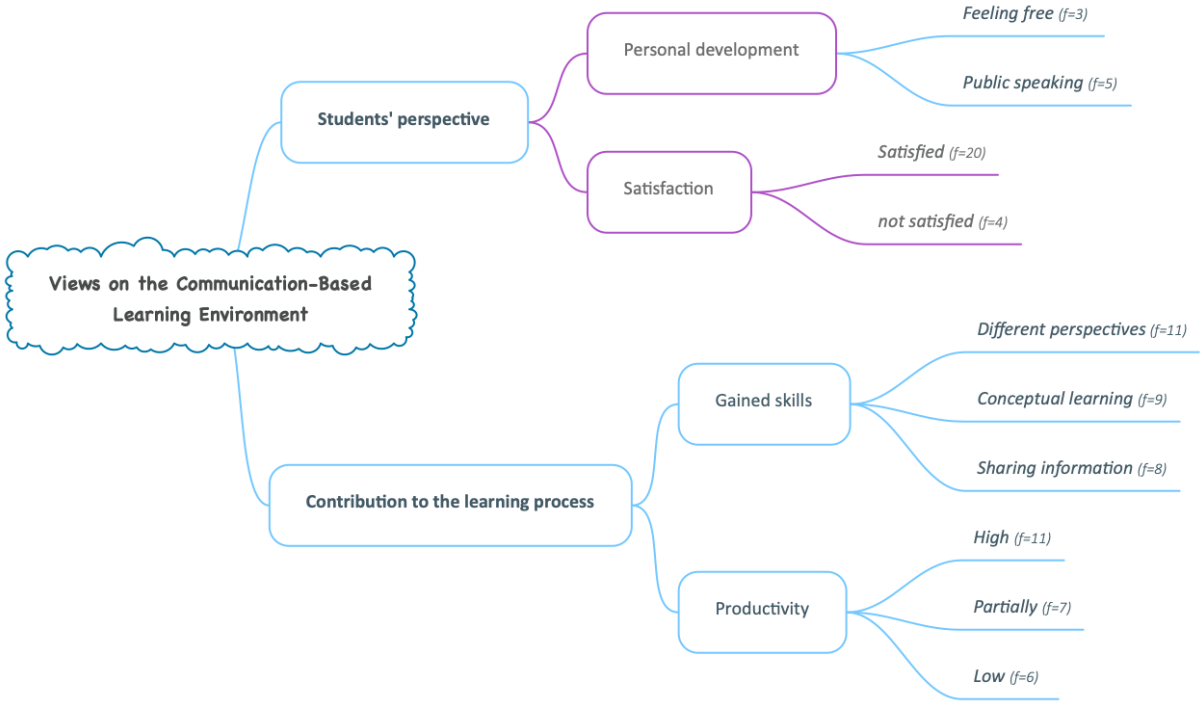

Figure 5. Views on the communication-based learning environment.

The pre-service teachers were mostly (f = 20) satisfied with the communicative learning environment. While some of them (f = 5) stated that the environment contributed to their speaking skills. Others (f = 3) stated that it made them feel free. Some of the ideas expressed were:

T_1: *I was pleased with the communication-based nature of the course. It enhanced my ability to provide comments and express mathematical ideas effectively.*

T_6: *Previously, I had a phobia of speaking in public. The course's communication-based teaching significantly contributed to overcoming my phobia.*

Pre-service teachers analysed the impact of a communicative environment on their learning process in terms of skill development and productivity. They noted that the environment offered them diverse perspectives (f = 11), aided their conceptual learning (f = 9), and facilitated knowledge sharing (f = 8). As a result, they believed that the communication-based course was moderately (f = 7) or highly (f = 11) effective. Several examples are presented below:

T_{19}: *I gained new insights by exploring different perspectives and examining logical foundations. The communication process in the lessons expanded my understanding and broadened my perspectives.*

T_{17}: *My perspective on concepts changed as I prioritise understanding over memorisation. I loved this lesson because observations, discussions, and research fused together in my mind, resulting in a lasting understanding.*

T_{10}: *This class had the highest attendance compared to other courses we've taken. It was engaging, with active participation and ample learning opportunities. We conduct research, discuss among ourselves, and share our thoughts on what we've learned and found interesting.*

Finally, the pre-service teachers' opinions about the preparedness level of the applied method and the communication process were taken. Opinions were examined in the sub-categories of preliminary information and affecting factors. The findings are presented in Figure 6.

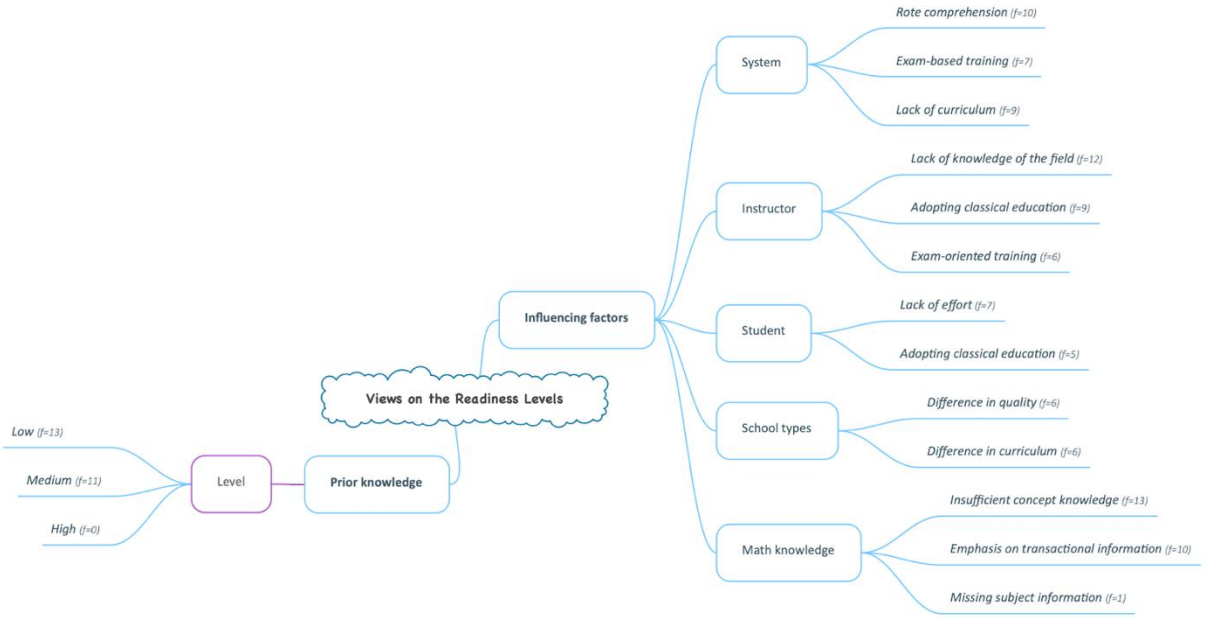

Figure 6. Views of pre-service teachers on their readiness levels.

Pre-service teachers found their prior knowledge of the lesson taught with the inquiry-based learning approach at a low (f = 13) or medium level (f = 11). Two pre-service teachers said:

T_{24}: *I cannot say that my prior knowledge is very sufficient. There were subjects that I did not know or fell short of. But I don't think I'm too weak either.*

T_{10}: *I thought my prior knowledge was sufficient, but actually, it was insufficient. We studied natural numbers and sets in the lesson. Normally, I thought I knew all of them, but I couldn't answer any of the questions.*

In the research, pre-service teachers were asked about the factors influencing their readiness levels. According to their responses, the primary systemic issues in mathematics education in Turkey are rote learning (f = 10), problematic curricula (f = 9), and exam-centered education (f = 7). Here are some examples of their opinions:

T_{3}: *I think our shortcomings are caused by the system. Most of us have the logic that whatever the teacher says is true. We do not believe that we can research and find the right one ourselves.*

T_{19}: *I attribute this to the curriculum rather than the teacher. Because the curriculum is too intense, the teacher does not spare time for conceptual learning.*

T_{12}: *There is the reality of exams in Turkey, and teachers are turning to it. Whatever is in the scope of the exam is limited to that content.*

Pre-service teachers emphasised the influential role of teachers in shaping their readiness for teaching. They raised concerns about teachers' limited subject knowledge (*f* = 12), reliance on traditional teaching methods (*f* = 9), and emphasis on exam-oriented instruction (*f* = 6), which they identified as key barriers to their preparedness. Examples of their viewpoints were:

> T_5: *Unfortunately, teachers do not have enough information. I asked my teachers some of the questions you asked us, but I could not get satisfactory answers.*
>
> T_{20}: *The subject was taught to us only as memorised knowledge. Why and how this information came about, we did not learn in any way.*
>
> T_7: *Students are successful in the exam if their procedural knowledge is good. Teachers also consider their students' knowledge of operations to be sufficient.*

The pre-service teachers also attributed their insufficient readiness to themselves and stated that the lack of effort (*f* = 7) and their adoption of classical education (*f* = 5) were the most important reasons for this. Some of the views are as follows:

> T_2: *My poor prior knowledge is partly due to me. We were not very curious about such things, and even if we did, the teachers would not meet it.*
>
> T_{22}: *In our middle school and high school years, teachers were talking, and we were listening and taking notes. We didn't even think of things like asking questions or doing research.*

Pre-service teachers stated that another factor affecting their readiness level is school type. The two prominent factors related to school types were the difference in quality (*f* = 6) and the difference in the curriculum (*f* = 6). The samples are as follows:

> T_3: *I initially studied at a medical high school where mathematics was taught superficially. Later, I transferred to an Anatolian high school for my final years. Despite attending a cram school and preparing for the university entrance exam, I couldn't bridge the gaps in my mathematics education from medical high school. The university exam solely emphasised results, leading me to rely on formula memorisation. Overall, my school-based math education was deficient.*
>
> T_{10}: *I attended Anatolian High School, but I noticed significant differences in the curriculum compared to other school types. Science high schools offer more advanced education, while plain high schools lack similar opportunities.*

Finally, pre-service teachers expressed the mathematics knowledge presented to them as a factor affecting their readiness level. In this regard, the inadequacy of conceptual learning (*f* = 13) and the emphasis on procedural knowledge (*f* = 10) came to the fore. Some views expressed were:

> T_{12}: *Math teachers often state that dividing a number by zero is undefined, but students may not fully grasp why. Exams prioritise practice rather than conceptual understanding. In summary, we possess knowledge without true comprehension.*
>
> T_{11}: *We prioritised practicality over conceptual understanding, resulting in limited learning outcomes through memorisation and practice.*

Interpretation and Discussion

Pre-service Teachers' Views on the Inquiry-based Learning Approach

The most important finding in this category is insufficient research and questioning undertaken by the pre-service teachers. The pre-service teachers encountered substantial difficulties in the process of searching for resources, which is a basic requirement of the research-based learning approach. Although they mostly relied on internet searches, they expressed that they had difficulties in accessing information and extracting the correct data. In today's information age, teacher candidates' inability to effectively use the vast amount of information on the internet is an indication of under-developed resource scanning skills. Additionally, struggles to obtain accurate information can be attributed to inadequate

mathematical knowledge and conceptual development. Coşkun Ögeyik and Köksal (2014) emphasised the importance of developing prospective teachers' research skills to foster an investigative classroom environment throughout their teaching careers. The findings of this study are consistent with the existing literature supporting these results (Letina & Canjek-Androić, 2021).

Despite facing challenges in adapting to the inquiry-based learning method, the pre-service teachers expressed a strong belief in its effectiveness. The most demanding aspect for them was the shift in the lecturer's role from providing ready-made answers to acting as a guide, a challenge that ultimately prompted increased research engagement and communication among themselves. Despite difficulties, the pre-service teachers acknowledged that the method enhanced their research skills, contributed to their content knowledge, and improved their mathematical thinking abilities. Numerous studies in the relevant literature support these findings, emphasising the positive impact of the inquiry-based learning method on students' thinking skills and academic achievement (Abdi, 2014; Aktamış et al., 2016; Attard et al., 2021; Duran & Dökme, 2016; Wale & Bishaw, 2020; Wilson, 2020). Additionally, research outcomes demonstrate the method's support in developing pre-service teachers' content knowledge and conceptual understanding (Aydin, 2020; Güler & Sahin, 2018).

Initially lacking in research and questioning skills, pre-service teachers demonstrated improvement in adopting the inquiry-based learning method over time. Their positive perception of the method's utility for academic development and personal growth is noteworthy. The development of research skills is crucial for prospective teachers, enabling them to make informed decisions in their teaching practice by staying abreast of the latest educational theories and strategies (Brew & Saunders, 2020). These skills empower student teachers to implement evidence-based methods, enhancing teaching effectiveness and contributing to improved student outcomes (Makar, 2013). This perspective is expected to cultivate a research-oriented culture, benefiting them in their future careers and professional lives.

Pre-service Teachers' Views on the Course Process

Many pre-service teachers found the inquiry-based learning environment, centered on mathematical communication, well-suited for their educational goals. They acknowledged its positive impact on developing both research skills and communication abilities. Recognising gaps in their foundational knowledge of mathematical concepts, they expressed a desire to improve their mathematical thinking and conceptual understanding for effective teaching. Throughout the course, they grasped that acquiring these skills necessitated active engagement with the inquiry-based learning approach. Additionally, they acknowledged the importance of communication in the constructivist learning approach and expressed satisfaction with the emphasis on a communication-based learning environment.

The pre-service teachers placed more importance on the contribution of the inquiry-based learning process and the communicative environment to their comprehension rather than the quantity of content covered. A major proportion of them, however, felt that a considerable amount of time was dedicated to this approach and the process of mathematical communication, which hindered further exploration of the subject matter. Consequently, many of them found the duration of the course insufficient. These findings are consistent with the results of studies conducted by Tatar et al. (2009) and Açıkgöz and Uluçınar Sağır (2020).

The pre-service teachers perceived the course environment, which was based on inquiry-based learning and communication, as highly productive. They recognised that the course contributed meaningfully to both their subject knowledge and teaching skills. It is important to note, however, that the level of participation in course activities and discussions was relatively low. This low participation can be attributed to their unfamiliarity with the method and learning environment. Due to the challenges they faced in adapting to the new environment, the pre-service teachers hesitated to actively engage. Nevertheless, towards the end of the course, it was observed that they gradually embraced the method and became more enthusiastic about participating actively in the learning process.

The pre-service teachers thought that the instructor who conducted the course process made an extensive contribution to the learning environment by taking on the role of a guide, implementing

positive teaching approaches, and using strong communication skills. They highlighted that the course, which was based on inquiry-based learning and communication, particularly enhanced their conceptual development, and fostered participatory learning. Numerous research studies have provided evidence supporting these observations (Laksana et al., 2019; Menteş, 2012; Şimşek & Kabapınar, 2010).

A notable proportion of the pre-service teachers expressed challenges with examination and evaluation practices in the research-based learning course, particularly with questions requiring interpretation and originality. Aligned with the approach's nature, the researcher intentionally employed question types that fostered original idea generation instead of merely providing ready-made information. Such assessment methods proved demanding for candidates accustomed to multiple-choice or memorisation-based questions. This difficulty can be linked to the limited preference for diverse assessment approaches among teachers in Turkey (Menteş, 2012). Furthermore, Uymaz and Çalışkan (2019) observed that teachers in Turkey predominantly rely on questions at the remembering and understanding levels of Bloom's taxonomy in exams, with minimal emphasis on higher-order thinking skills. The findings of this study align with these observations.

Pre-service Teachers' Views on the Communication-based Learning Environment

The pre-service teachers were highly satisfied with the classroom communication process, emphasising that it provided different perspectives, supported conceptual learning and facilitated knowledge sharing. Considering the unique language of mathematics, the correct use of the language is considered very important to understand mathematical concepts. Mathematical communication enables students to identify and correct misconceptions, creating an environment that allows for diverse perspectives and improved conceptual understanding. This communicative approach allows pre-service teachers to shape their teaching methods through effective reflective practices. Numerous research studies (Brendefur & Frykholm, 2000; Harianja et al., 2020; Hirschfeld-Cotton, 2008; Lomibao et al., 2016) have confirmed the positive impact of mathematical communication on these dimensions. The pre-service teachers in this study attributed the effectiveness of the course to the enriching contributions of the mathematical communication process, especially to actively establishing meaningful dialogues, and leaving the classroom with a feeling of satisfaction and enthusiasm for the course.

Pre-service Teachers' Readiness Levels

Lessons employing the inquiry-based learning approach and mathematical communication provide valuable insights when assessing the prior learning experiences and readiness of pre-service teachers. Following Bloom's (1995) definition, readiness encompasses general and specific abilities, cognitive and affective knowledge, and skills necessary for acquiring new knowledge, emphasising the student's existing knowledge base. Adequate readiness levels are crucial for acquiring new information and participating in meaningful discussions (Harman & Çelikler, 2012). The pre-service teachers acknowledged their generally insufficient prior knowledge concerning the course content, which posed a weighty challenge during the initial weeks of the course. This inadequacy hampered their ability to search for and select relevant information from various sources, limiting their active engagement in classroom discussions and information sharing.

The pre-service teachers attributed their inadequate prior knowledge and limited readiness to various factors, including a predominant rote learning approach, an exam-centric educational emphasis, curriculum deficiencies, and shortcomings in the Turkish educational system. Despite the nationwide adoption of a constructivist approach in Turkey, educational practices still heavily rely on rote learning, impacting the central exams used for higher education admissions. The current system prioritises exam preparation over students' conceptual understanding, with procedural questions dominating central exams and diverting attention from student-centered learning and communication—essential elements of constructivist pedagogy. This issue aligns with findings from prior studies by Baştürk and Doğan (2010), Özdemir and Kaplan (2017), Yeşil and Şahan (2015), and Yılmaz and Altınkurt (2011). Pre-service

teachers identified the curriculum, particularly its dense nature in Turkey, as a major contributor, posing challenges for implementing constructivist learning that demands more time—a concern also emphasised by Dağdelen and Ünal (2017).

The pre-service teachers highlighted the influence instructors had on their readiness levels, pointing to traditional teaching methods, exam-oriented instruction, and insufficient content knowledge as pivotal factors. The National Council of Teachers of Mathematics (2000) underscores teacher competencies, particularly in content knowledge, as crucial for quality mathematics learning. However, challenges persist in Turkish mathematics education, with teachers often lacking essential content knowledge, as noted in various research studies (Gürbüz et al., 2013; Konyalıoğlu et al., 2012; Pirpiroğlu & Doğru, 2015; Tanışlı, 2013). Central exams, serving as a determinant for students' higher education transition in Turkey, further impact education quality, given that success in these exams is central to student and parental expectations. The prevalence of procedural knowledge-based questions in these exams incentivises teachers to prioritise practice over conceptual teaching, reinforcing a traditional, rote-based approach rather than embracing a constructivist understanding (Kartal, 2016). This situation, as highlighted by Özdaş (2019), profoundly affects the educational landscape, particularly due to the negative consequences of central exams.

The pre-service teachers acknowledged their insufficient readiness, attributing it to a lack of effort and a preference for traditional education. Scholars such as Sarıer (2016) and Steinmayr et al. (2019) emphasised the pivotal role of students' effort and motivation in academic success, underlining the principle that learning thrives through active engagement. According to the pre-service teachers, students often lean towards traditional education, relying on readily available information, primarily driven by a strong motivation for exam success. Consequently, this inclination results in inadequate conceptual learning and knowledge retention.

Another concern the pre-service teachers raised is the disparity in quality of education and curriculum among different types of schools. In Turkey, various types of schools exist, such as Science High Schools, Anatolian High Schools, Vocational and Technical High Schools, and Plains High Schools. Among these, Science High Schools are particularly renowned for their educational quality, attracting high-achieving students. Conversely, many other high school types exhibit a wide range of educational quality, often falling below satisfactory standards. The absence of standardisation in this regard leads to broad disparities in students' knowledge, skills, and educational foundations.

Conclusion

The findings indicate that the pre-service teachers exhibited limited familiarity and adaptation to research, questioning, and discussion processes. Furthermore, their past educational experiences seemed to have fostered a preference for rote learning, resulting in difficulties in engaging in mathematical communication. Over time, however, the pre-service teachers gradually embraced the inquiry-based learning approach and gained proficiency from its implementation. These research outcomes hold importance in highlighting the problematic aspects of the Turkish education system. Pre-service teachers who struggled to meet the demands of the inquiry-based learning approach and encountered challenges in establishing mathematical communication attributed these difficulties to the prevalence of rote learning, teacher competencies, and systemic issues within the Turkish educational context.

It is crucial for pre-service mathematics teachers to acquire and apply instructional methods such as the inquiry-based learning approach, to equip them with relevant skills and enable them to adopt a teaching approach that aligns with contemporary educational demands. Therefore, it is recommended to implement such methods and establish appropriate models specifically designed for pre-service teachers, particularly within undergraduate education programs. Moreover, it is essential for pre-service teachers to recognise the importance of effective communication, which is an indispensable component of mathematics education and the teaching profession in today's learning contexts. To enhance the

quality of teachers in Turkey, it is necessary to implement reforms that align elements such as the education system, teaching curricula, and teacher competencies with the demands of the current era.

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Ethical approval

Ethical approval for the study was granted by the Istanbul University-Cerrahpaşa Social and Human Sciences Ethics Committee with the number 2022/267 and all participants gave informed consent for the publication of their data.

Competing interests

The author declares there are no competing interests.

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