Connection Levers: Supports for Building Teachers' Confidence and Commitment to Teach Mathematics and Statistics through Inquiry

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Gaps between teaching practices and research recommendations have been well documented. One challenge for research is in understanding the processes and systems that promote a bridging of these gaps. A year-long study with four primary teachers documented ten support mechanisms, or connection levers, that the teachers raised as important for building their expertise, commitment, and confidence in teaching mathematics and statistics through an inquiry-based approach. Their words provide insight into how support helps teachers to take on and commit to innovative practices.

For decades, teachers have been both the vehicle and target of school reform (Wilson & Berne, 1999), yet despite millions of dollars spent annually on professional development, pedagogies in mathematics and statistics have changed little (Cohen & Hill, 2001; Hollingsworth, Lokan, & McCrae, 2003; Moore, 1997; Stigler & Hiebert, 1999). There is fairly strong consensus in the field about what constitutes effective professional development (e.g., Elmore, 2002; Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2003), however difficulty remains in understanding elements that support teachers in *sustaining* innovative practices. This paper sought to understand, from *teachers' perspectives*, the support mechanisms that they indicated as critical in developing their expertise, confidence and commitment to teaching mathematics and statistics with inquiry.

Learning and Teaching Mathematics and Statistics with Inquiry

Inquiry-based approaches to teaching are a significant departure from traditional pedagogies in mathematics that typically focus on solving closed-ended problems. In inquiry, rather than work with problems that have a single, known correct answer, students pose and investigate meaningful, ill-structured problems and engage in the epistemological processes of coming-to-know used by experts. Ill-structured problems are those for which the initial conditions and perhaps the goals of the problem are ambiguous, or have a large number of open constraints (Reitman, 1965). Nearly all problems of everyday life are ill-structured. The goal of inquiry is both knowledge-building and building understanding of the processes of knowledge-building (Oliver & Herrington, 2001), that is, learning how to learn. In an era where life-long learning is highly valued, this is critical.

Understanding the process of solving an ill-structured problem is not as straightforward as it may appear. For example, the solution phase (where nearly all teaching is focused in schools) is a relatively small part of the problem-solving effort when compared to the effort required in conceptually establishing an overarching structure, potential directions to investigate, and an appropriate set of representations of the problem. The open constraints which create the difficulty in solving ill-structured problems generally emerge *during* the process of solving the problem rather than being known at the beginning of the problem (Reitman, 1965). It is the ability to envision a framework for solving an ill-structured problem and seek appropriate data that requires an extensive level of domain-specific knowledge and experience.

In one model of classrooms that utilise inquiry in science (Magnusson & Palincsar, 2005), learners enter investigations via engagement with a topic, exploring the topic to generate questions of interest. The teacher at this point may guide discussion in ways that will likely support her desired learning goals. Through further discussion and interaction with the topic, students define a more specific question to explore and consider possible approaches to investigate their question. The investigation deepens understanding of the complexity of the context under investigation and will likely generate new questions to explore, opening the potential for further investigations. Inquiry has been shown to foster deeper understanding, provide more challenging work, assist students in building on their natural problem solving abilities and see greater connection between classroom learning and their own lives, promote student ownership of knowledge, and build greater enthusiasm for learning (Boaler, 1997; Bransford, Brown, & Cocking, 2000; Diezmann, Watters, & English, 2001).

Research on transfer is clear that students' understanding in one context may not transfer to other contexts (Bransford et al., 2000). Likewise, research has suggested that performance on the well-structured problems typically given to students in school mathematics often do not carry over to performance in everyday, less-structured problems or even to similar, well-structured problems (Schoenfeld, 1991). Conversely, students may demonstrate proficiency of concepts in everyday contexts yet may not be successful solving similar problems embedded in the cultural syntax and semantic structure in a school-like setting (e.g., Nunes, Schliemann, & Carraher, 1993; Warren, Ballenger, Ogonowski, Rosebery, & Hudicourt-Barnes, 2001). Particular challenges exist in inquiry-based learning in statistics. These elements are typically ignored in school: creating measurable conjectures, collecting and recording appropriate data, and relating findings back to the original research questions (Hancock, Kaput, & Goldsmith, 1992; Konold & Higgins, 2002; Marshall, Makar, & Kazak, 2002). It is not only students who find these aspects difficult, but teachers as well (Confrey & Makar, 2002).

The current problem with the teaching of statistical topics in schools is an imbalanced focus on calculations and graphing skills. This emphasis lacks the kind of critical thinking needed for statistical literacy (Watson, 2006). Many of the elements critical to solving ill-structured problems mirror characteristics articulated by Wild and Pfannkuch (1999) as central to statistical thinking. For example, the recognition of the need for data, an ability to capture and transform measures and representations in order to seek meaning (*transnumeration*), and the integration of statistical and contextual information. Because "the ultimate goal of statistical investigation is learning in the *context* sphere" (Wild & Pfannkuch, 1999, p. 225), a key element of developing and assessing statistical thinking and reasoning depends on one's ability to engage with problem contexts plagued by uncertainty, ambiguity, and complexity.

Teaching Mathematics and Statistics with Inquiry

The teaching of inquiry requires a classroom culture which values challenging, complex, real-life applications and a focus on discussion of processes. How well do classrooms in Australia provide these experiences for students compared to classrooms in other industrialised countries? From an analysis of Year 8 classrooms in the TIMSS 1999 Video Study (Hollingsworth et al., 2003), Australia had the highest percentage of time spent on very basic content (55%) compared to other industrialised countries including the USA (40%), Switzerland (30%), the Netherlands (30%), Czech Republic (15%), and Hong Kong (5%). In working on complex problems, Australia had one of the lowest percentages of time spent on solving complex problems (problems requiring multiple decisions) per lesson (8%), compared with Japan (39%). On the other hand, Australia (27%) was second only to the Netherlands (42%) in the time spent on solving problems embedded in real-life situations although even these problems tended to be very basic. Finally, Australian students had far fewer experiences (2%) than their international counterparts (median 12%) publicly discussing the processes and mathematical connections in solving problems. Although these data were derived from older middle years classrooms (Year 8), it is likely that the classroom culture is not too different in early middle years classrooms (Years 4 and 5).

The teaching of mathematical inquiry necessitates an ability to embrace uncertainty, capacity to support student decision-making — balancing collaboration and independence, the experience to recognise opportunities for learning in unexpected outcomes, the commitment to flexible thinking, a deep understanding of disciplinary content, and tolerance of periods of noise and disorganisation (National Research Council, 2000). These often go against teachers' expected learning trajectories for students, expectations of neat and orderly classrooms, and beliefs about mathematics as a discipline of certainty.

Because an inquiry approach is such a marked departure from teachers' experiences in teaching and learning mathematics, the learning curve for this approach is steep — even if the teachers are already 'on board' in believing that students will gain from this approach to learning (Nelson, 1997). Difficulty in envisioning what an inquiry-based mathematics classroom would look like is one important hurdle (Fosnot, 1996). Beliefs about the nature of mathematics teaching and learning can also be a hurdle (Beswick, 2007; Olson & Barrett, 2004; Philipp, 2007; Thompson, 1992). Logistical and contextual issues have been reported as barriers to reform-based teaching more broadly (Kennedy, 2005) and specifically to teaching inquiry in science. These include inadequate space and materials, inadequate time, and limited instructional freedom (Songer, Lee, & Kam, 2002).

Low content knowledge has also been repeatedly named as a barrier to improved practice (Department of Education, Science and Training (DEST), 2004; Hill, Rowan, & Loewenberg Ball, 2005; Hill, Sleep, Lewis, & Loewenberg Ball, 2007; Ma, 1999; Songer, et al., 2002). However, this notion has been challenged by others (e.g., Kennedy, 2005) who argue that the category of 'content knowledge' is overly broad and issues raised under the banner of content knowledge may be more likely attributed to other (possibly related) factors such as experience in managing high intellectual engagement with ideas (e.g., unanticipated questions or ideas raised by students), and unrealistic expectations or poor direction provided by institutional documents (e.g., tests, curriculum documents, textbooks). Kennedy (2005) also points out that a problem with reformists is that they rarely take into consideration the teachers' perspectives and frequently marginalise the broad range of factors that teachers must consider in their teaching.

Other problems in moving towards reform point to the professional development itself that aims to instigate these changes. It is well accepted among teacher educators that an isolated workshop approach to professional development is ineffective in changing teacher practice or sustaining teachers in learning innovative approaches to teaching, yet this is still the primary source of professional development for teachers (Cohen & Hill, 2001; Loucks-Horsley et al., 2003; Sykes, 1996). Even if there is a match between teachers' interests and the topic of a workshop, teachers require sustained support in learning new approaches to teaching. This comes as no surprise given what we know about students' learning. Contemporary theories of learning support students in building their knowledge through inquiry within a community of practice (Goos, 2004; Lave & Wenger, 1991). Therefore, if the aim is "to prepare teachers who can teach mathematics for understanding, [we] must create opportunities for teachers to build connections with mathematics, not just as teachers but as learners themselves" (Ball, 1996, p. 39). If schools are to move students away from isolated procedural knowledge in mathematics and statistics towards an ability to manage a complex and uncertain world, teachers must develop experience in the use of mathematics and statistics towards this perspective. Teachers need specific support when they are attempting to develop entirely new cognitive structures as foundations for their teaching practices (Olson & Barrett, 2004). Guskey (2002) further contends that professional development too often tries to impact change in teachers' knowledge and beliefs about teaching *before* they implement new practices rather than the other way around. He argues that until teachers see that change in teaching practices can have an effect on student learning, it is unlikely that a change will be sustainable. "The crucial point is that it is not the professional development *per se*, but the experience of successful implementation that changes teachers' attitudes and beliefs. They believe it works because they have seen it work, and that experience shapes their attitudes and beliefs" (Guskey, 2002, p. 383).

A challenge for research is in understanding how teachers can, over time, successfully develop and sustain expertise, confidence, and commitment in teaching with reform-oriented teaching practices. The ultimate goal is the development of *adaptive expertise* (Schwartz, Bransford, & Sears, 2005), which entails a balance of the dimensions of innovation and efficiency (Figure 1). Erring too far on the innovation of curriculum will exhaust and hence frustrate teachers, unable to meet the demands of their own expectations. On the other hand, an overly efficient approach hardens teachers into a routine that can impede creativity and encumber a commitment to lifelong learning.

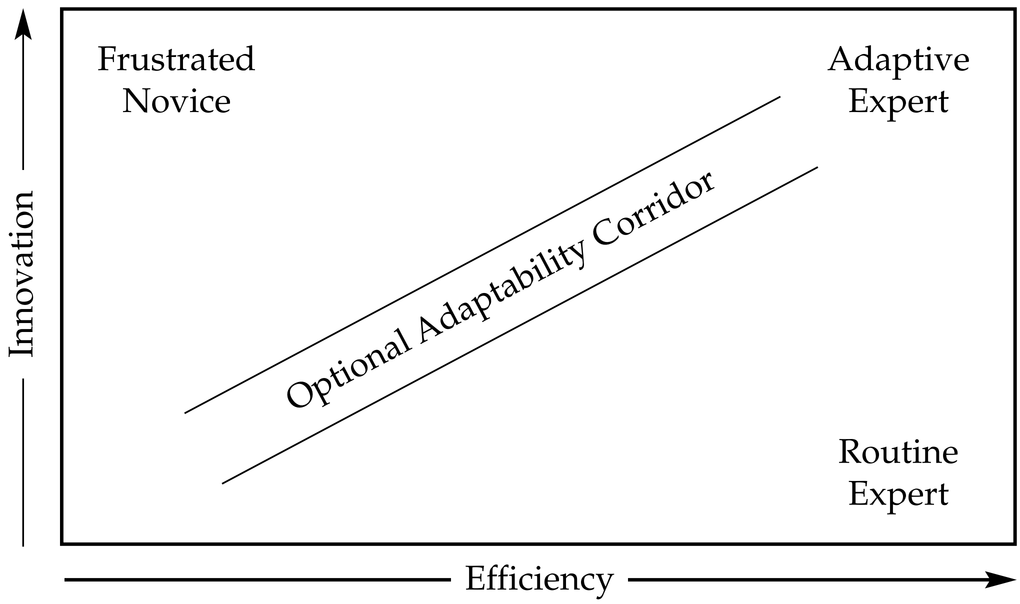


Figure 1. Dimensions of adaptive expertise (Schwartz et al., 2005)

Inquiry-based teaching requires a high level of innovation on the teacher's part. Without the skills to adapt to this innovation, teachers can become like the frustrated novice — an experience particularly difficult for an experienced professional. Research has indeed provided evidence that these initial experiences of teaching and learning in mathematics and statistics with an inquiry approach can be both challenging and frustrating (Diezmann et al., 2001; Hancock et al., 1992; Makar & Confrey, 2007). Because mathematics is not envisioned as a field requiring inquiry, it is not routine for teachers to conduct inquiry in mathematics. If they do attempt to move outside of their comfort zone and try teaching mathematics through inquiry, the frustration and challenge in the initial experience can be sufficient to dissuade them from persevering, pushing them back towards the 'routine expert' dimension. A goal, therefore, is to support them in building their expertise with the innovation and encouraging them to persist.

Motivation for the Study

A 4-year project conducted in Queensland aims to understand how teachers come to develop inquiry-based pedagogies by developing a preliminary theoretical model of learning to teach mathematics and statistics through iterative cycles of inquiry. Cycles of inquiry is a pedagogical approach to teaching and learning which immerses the learner in investigating an open-ended problem with repeated phases of investigating, reporting, and continued refinement of understanding of the problem through reflection and improved knowledge tools (e.g., statistical concepts, technology support). The study examines teachers' processes of coming to know how to teach inquiry through the use of *iterative* cycles over an extended period. By incorporating multiple experiences in teaching over an extended period, and because "a constructivist learning environment must incorporate reflection" (Goodell, 2000, p. 57), the researcher had opportunities to utilise teachers' reflections on their experiences in teaching inquiry units, and observe and listen to how they constructed their evolving understanding about inquiry-based teaching in subsequent teaching.

Initial experiences with inquiry presented unique challenges (Makar, 2004; Makar & Confrey, 2007). The researcher conjectured that with support, the teachers' abilities to envision, plan, implement, reflect on, and commit to effective inquiry-based units would change and develop over time. This paper reports on elements that supported the teachers in the study to build confidence, competence, and commitment to teaching mathematics and statistics through inquiry. The researcher was interested in locating particular levers that supported the teachers' ongoing professional learning about inquiry in mathematics and statistics, their ability to reflect on and connect their learning in subsequent experiences of teaching with inquiry, and their growing expertise and confidence in teaching with this approach. These support mechanisms are the focus of this paper.

Method

The goal of the study was to gain insight into teachers' experiences in teaching mathematics and statistics with inquiry, particularly in the initial stages of that experience. This paper reports on findings in the first year of the study in response to the question: *What are support mechanisms for teachers as they develop expertise in teaching mathematics and statistics with inquiry?*

The overall study was developed using a design experiment framework (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003), where the researcher simultaneously and iteratively studies and works to improve the study context. Its strength is its ability to capitalise on the practicality and complexity of authentic classroom contexts, where "in contrast to most research methodologies, the theoretical products of design experiments have the potential for rapid pay-off because they are filtered in advance for their instrumental effect. They also speak directly to the types of problems that practitioners address in the course of their work" (Cobb et al., 2003, p. 11).

According to Cobb et al. (2003), five features are common to design studies:

- *An overarching focus on the development of theories about the process of learning and the means to support it*. In this case, the research project aimed to develop a theoretical framework for understanding teachers' evolving experiences as they gained familiarity in teaching mathematical inquiry over time. The researcher conjectured that in a supported context (under investigation in this study), teachers would generally improve their confidence and commitment to teaching with this approach. The overall project aimed to develop a theoretical model to describe this evolution (Makar, under review) and elements that supported teachers in persisting through this evolution.

- *The implementation of an innovative intervention designed to understand factors that contribute to targeted educational improvement*. The intervention in the project included full day professional development and planning sessions, once per term (about 10-12 weeks), where the teachers worked as *learners* on various aspects of addressing open-ended, ill-structured problems. Time was also set aside for the teachers during the day for discussion, sharing of experiences and ideas, and planning units for their classrooms. Other intervention elements also formed a part of the design, for example involving multiple teachers in the study (usually working in pairs) so that they could mutually support one another as learners in a community of practice; expecting that units would be developed and implemented at regular intervals (to ensure that the teacher experienced multiple iterations); and observing lessons to offer validation and technical support as needed. Although the researcher aimed to support the teachers in their learning, there was a conscious effort to limit this support to elements that could be repeated and scaled up under more typical circumstances.
- *Simultaneous prospective and reflective aspects that are continuously scrutinised, capitalising on contingencies that emerge as the design unfolds*. At any given time, the researcher revised designs to improve the learning environment and its supports in a way that put emerging and increasingly specialised conjectures to the test. Concurrently, these emergent conjectures were reflected on and scrutinised in light of previous conjectures and exposure. This look-forward/look-back approach allowed the researcher greater flexibility to respond quickly to contingencies that arose. For example, the researcher noticed that after a particularly challenging class, a teacher's response to her fifth iteration of teaching an inquiry unit had shifted from initial helplessness and self-blame in early units to a more proactive stance. The theory was then further assessed through targeted questioning during the interview and consideration of this difference as a potential measure of change (Makar, under review).
- *Dependence on an iterative process that allows for refutation, revision, or refinement of conjectures, learning environments, and measures being inspected*. The prospective-reflective process gets its power in the opportunity to iterate. By putting fresh understandings to the test anew, the researcher can trial tentative ideas to work as they emerge, assess

their strength, reflect on their generalisability, and re-assess on subsequent iterations until concepts stabilise.

- *Design studies acknowledge that theories developed by this process are humble, intermediate, and specific to the design*. The great advantage of this process is that, "in contrast to most research methodologies, the theoretical products of design experiments have the potential for rapid pay-off because they are filtered in advance for their instrumental effect. They also speak directly to the types of problems that practitioners address in the course of their work" (Cobb et al., 2003, p. 11).

Four teachers of Years 4 and 5 (ages 8-10) at a government school in Queensland participated in the initial phase (Year 1) of the study. The teachers volunteered for the study after being approached by school administration.

Over the course of the year, each teacher committed to plan and implement one inquiry-based unit in their classrooms per term (see Table 2). The focus in the first year was on integrating data into their units because of the natural connection of statistics to contextual problems. However, all units also included other strands in mathematics. Units were videotaped by the researcher in Terms 1, 2, and 4 (about half of teachers' lessons in each unit) to capture the flavour and content of the classroom environments, enculturate the researcher into teachers' classroom practices, record the teachers' experiences 'in the trenches' during the units, determine ongoing professional development needs, and gather detailed and episodic evidence of teaching and learning issues that arose while teaching the units. Some artefacts of student work were also collected. In addition, teachers were interviewed at the beginning and end of the units to gather data on their intent, the anticipated and actual challenges that arose, unexpected outcomes and opportunities, and elements of the teachers' experiences that changed each time they taught a unit. Indicators of particular support mechanisms were recorded and collated.

A preliminary list of support mechanisms relevant to the study (Table 1) was developed using previous research recommendations made by the author (Makar, 2004) regarding inquiry (developing content knowledge in a reform-based environment, time, feedback and support, validation, multiple iterations, and reflection), as well as additions from professional development research literature recommending the use of reform curriculum, sustained involvement, and a collaborative environment (e.g., Elmore, 2002; Loucks-Horsley et al., 2003). This list was used as an initial framework to mark episodes in the transcripts where the teachers discussed these support mechanisms or raised additional possibilities. Special focus was given to supports articulated by the teachers that helped them to connect their learning from one unit to apply to subsequent units and their evolving practice. These interviews were coded in more detail to describe the elements that emerged from the teachers' words and to locate excerpts that best captured these concepts, such as excerpts that demonstrated strong links to the development of the teachers' evolving practice. Due to the role these support mechanisms had in helping teachers apply learning from one teaching experience to subsequent ones, they were called *connection levers*.

Table 1
Initial List of Support Mechanisms

• Developing content knowledge in a reform-based environment	• Time
• Reform curriculum	• Feedback & Support
• Collaborative environment	• Validation
• Sustained involvement	• Multiple iterations
	• Reflection

An Example of a Unit

One unit is briefly described here to give the reader a feel for the kind of units the teachers designed (Table 2). No single unit was 'perfect' (if there is such a thing), but all provided students with rich learning opportunities with a spirit of developing students' lifelong learning attributes (Pendergast et al., 2005) and mathematical understanding. For example, Kaye, a Year 4 teacher developed an inquiry unit lasting 2-3 weeks in which the class addressed the question, "How many commercials does a typical Year 4 student watch in a year?" In this unit (adapted from TERC, 1998), students discussed issues related to advertisements on television. Although the task appeared straight-forward initially, students needed to resolve issues in defining the question before they could develop more measurable questions. For example, an early discussion engaged students in a discussion of "What counts as a commercial?" In supporting discussions such as these, the teacher needs to consciously step back and carefully listen to the students' contributions and reinforce the connections between the ideas they raise and the question under investigation. Getting the balance right between student and teacher input is critical.

Table 2
Units Designed or Adapted by the Teachers in Year 1 of the Study

TERM	Year 4 Units (Kaye & Carla)	Year 5 Units (Naomi & Josh)
1	Investigating hereditary traits	Are athletes getting faster over time?
2	What's in your lunchbox?	A predator-prey simulation (Describing cyclic phenomenon, Naomi)
		Understanding our class blue-tongue lizard (Designing a class-negotiated experiment, Josh).
3	Tibia mystery (estimating height from a bone)	Is there a "typical" Year 5 student? (Naomi)
		[Josh did not complete a unit this term]
4	How many commercials does a typical Year 4 student watch in a year? (Kaye)	Investigating factors that influence flight (Naomi)
	How does my age compare to my family members? (Carla)	Designing a parachute for an egg (Josh)

Further issues were discussed as students generated and mathematised their ideas into questions. For example, (a) Are there differences in the number of commercials on public (e.g., ABC, SBS), free commercial television (Seven, Nine, and Ten Networks), and subscription television (e.g., Discovery Channel, Cartoon Network, ESPN)? (b) Do the number and type of commercials differ at various times of the day (morning, daytime, primetime, night-time)? The teacher probed the class on how they would be able to decide on these differences, and (with some strategic questioning) it was agreed that data were needed to be able to answer these questions. The class agreed that each student would collect information on the commercials in four to six 30-minute slots over a week. Besides the number of commercials in the half-hour, students recorded the channel, time of day, and types of commercials that aired. The students collated their data and within small groups, addressed different aspects of the overarching question. For example, one group compared the number of commercials per half-hour between the most commonly watched channels. By comparing rather than simply describing the data, students gained experience drawing inferences from data as well as isolating important aspects of distributions, like variability and central tendencies (Watson & Moritz, 1999).

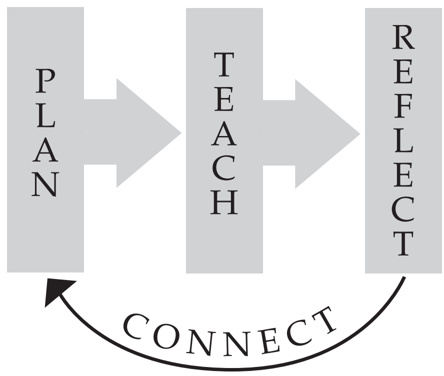
These units appear straightforward, but as articulated earlier, implementation often presents challenges. The next section details the kinds of support mechanisms, or connection levers, that the teachers named as helping them to develop their confidence and expertise in teaching mathematics through inquiry-based learning.

Connection Levers

In a simplistic model for developing and implementing curriculum units, teachers plan, implement, and reflect on the units that they teach. Although reflection is identified again and again as a key component of teacher learning, this aspect of teaching is often cut out as schools and teachers cope with increasing work loads. Even where teachers take it upon themselves to make time to reflect, they often do so in isolation with little or no support. In an effort to be more explicit about the ways in which teachers can be supported to reflect on and utilise their learning from their teaching as they plan subsequent units, *connection levers* were identified (Figure 2) by the teachers and researcher as key elements in scaffolding teachers as they undertake new ways of teaching. What follows are episodes from interviews conducted throughout the year where the teachers raised and discussed these connection levers.

Inquiry Experiences as a Learner

One of the most compelling experiences for the teachers in learning to teach using inquiry was having the opportunity to work through inquiry problems themselves during professional development sessions. Three times over the course of the year, the teachers took part in an inservice activity to give them an opportunity to experience what an inquiry feels like as a learner, and time to talk



CONNECTION LEVERS		
• Inquiry experiences as a learner • Multiple iterations • Validation • Resources	• Sustained support and feedback • Collegiality • Development of deep disciplinary knowledge	• Time and support for reflection • Relevance • Accountability

Figure 2. Connection levers to support teachers' learning to teach with innovative pedagogies.

about and reflect on these experiences. The first inservice focused on experiencing and learning to cope with the ambiguity and uncertainty associated with ill-structured problems. In a task adapted from the *Investigations* series (TERC, 1998), the teachers were asked to work together to design an ergonomic chair for their students. They found the task difficult, but valued the opportunity to struggle with the ambiguity of trying to solve an ill-structured problem. They drew on this experience throughout the year and commented on the importance of the experience in interviews at the end of the year:

Naomi: Doing it yourself first before you try to teach it [was useful], yeah, definitely.

Kaye: I thought it was helpful to do actually physically throw us into the deep end and say 'I want you to investigate chairs'. And for a lot of us that's very different to what we've done before and for us, even as a group, it was quite a hard task for us to maintain some sort of focus and to have a direction moving forward. And I think putting us in that situation was good because I think it showed us some of the things the kids can do — it gave us a little bit of an insight as to where we might need to help kids move forward.

Carla: You know it made you see sort of phases [of an inquiry process] didn't it? It made you see well, perhaps you need to just brainstorm this part first.

Although they believed in principle that inquiry was a beneficial approach for student learning, the teachers generally didn't know how to envision or apply inquiry to mathematics. The experience with the open-endedness of the initial activity raised a number of concerns as they considered how they would design and scaffold an appropriate unit for their students. Among the challenges, they talked about the difficulty in teaching students to work collaboratively, managing student diversity in dealing with an inquiry, and coming up with a good problem. Josh expressed his concern about managing the time constraints:

Josh: If you have a one-hour period, how immersed are they going to be in [the problem]? ... Because I'm very personally as a teacher, I'm very results-oriented. So I want to feel that we've come to some result. [What] if you spend an hour and nothing's happened?

For Kaye and Carla, the open-endedness of the inquiry process concerned them and they chose to carefully structure the first unit for their students.

Kaye: We've seen how difficult it is for us, [so] that we'll try to make the introductory process less stressful for them.

Carla: Yeah, it might have to be more constrained the first time.

Kaye: Or they would probably need more teacher input or adult input or someone just to sit and focus them. Like you had to come back ... [and] focus on trying to subtly pulling us back to where you want us to go without dominating our investigation but you would hope that we even now, given the same task or a different task after lunch, would be more focused.

After teaching their first unit, Carla and Kaye again commented on their experiences of managing the open-endedness of the problem in the first inservice and how they used that experience to scaffold the first unit:

Carla: Really you've just got to do a great model at the beginning. Guided modelling all the time and then they can do it by themselves and even I was going to do less guidance or less modelling at the beginning but I'm glad I haven't. Otherwise, yeah, I could foresee that my kids would just go, 'oh well, I don't know what I'm supposed to do so, oh well, why bother doing it?' Those kinds of questions.

Kaye: It's not only children. Let's have a look at four of us up a few, two or three weeks ago when we were given "Do an investigation on a chair". How much time did we spend, really without any direction? We were going off in all different planets.

Carla: Yeah, that's right.

Kaye: But we, as adults, we found it difficult to do, so children will find it difficult to do. I guess even as adults we like structure and we like a scaffold — I guess that's why Carla and I went for a scaffold and we're pleased that we did.

KM: Do you think that experience of having a really open ended [problem] and struggling with that with the chairs has made you think 'Gosh, maybe I should really try and restructure this'?

Kaye: I think if there's no structure, everything starts to swim. And I know with me, if things start to swim, well I shut down and I think well I can't cope with this so unless I can focus on something ... And I think that if we give kids a positive experience the first time they're more inclined to feel comfortable with it to have a go, whereas if we put them in and they feel threatened or, uh, they haven't been able to achieve what we've asked them to do then they will be less inclined to risk take next time.

Kaye and Carla's reaction to the inservice activity was quite different from the other pair of teachers. Naomi and Josh were excited by the opportunity to encourage students to take risks and manage an open-ended question, and they designed their unit to incorporate these experiences for their students.

Naomi: I think to a large extent this is how it does work in the world ... It's not as if the boss is standing there saying, 'well this is what the end product has to look like and these are the steps you're going to take', which is what we do in the classroom. ... I don't mean we don't scaffold it and we certainly are scaffolding but I think [inquiry is] approaching much more of a real life issue than the way traditional teaching is.

Although the teachers had different responses to the initial experience with the ambiguity of an open-ended task in planning their first unit, it was clear that it was an important opportunity for them to think back to during the year.

Multiple Iterations

Regardless of whether they were structured or open-ended, all of the teachers experienced a number of difficulties the first time they taught their units. Since this was their first time teaching this way, they were unsure what to expect.

Naomi: The first one, we were more uncomfortable with it. We wanted something that was absolutely wonderful ... We wanted something that was absolutely, you know, out of this world and we didn't, we didn't plan properly where it was going and whether or not we had the tools to get it to go in the right direction ... *that* was a *steep* learning curve!

Carla: In the first one, because we're learning as well [as the students] where it's all going ... you're not sure where that data is going to go just yet ... So it's, everyone is learning at the same time so you're not as confident what you're going to do with the data once you've got it.

Kaye: Even us as adults found it a difficult thing to do to start with. It's like all things that we introduce to kids to start, we think the results you get on the first thing you do are probably not going to follow what we want, but probably the more that we do the better they get at it.

In the second unit, both pairs of teachers designed units that were more balanced between structure and open-endedness than their first unit had been. Over the course of the year, the teachers experimented with focusing on different phases of the inquiry cycle, sometimes concentrating on data planning and collection and other times spending more time interpreting findings and communicating results. When a unit revealed an unexpected outcome, sometimes their next unit would focus on addressing that outcome. For example in her first unit, Naomi mentioned the difficulty that students had with seeing trends in data; her second unit incorporated skills in learning to describe and identify trends in periodic time series data. At the end of the year, she reflected on how, by going through multiple iterations, both she and her students came away with a robust sense of what statistical inquiry could do:

Naomi: In the first unit, we looked straight at data collection, really and the interpretation of that data.

KM: And probably in the second unit as well?

Naomi: [The second unit] was, yeah, just collecting data and having a look at the data. Then the third unit we extended it a little bit further and we looked at devising our own [survey] forms with which to collect data. And then, interpreting the data to the extent of saying, well, you know, 'What was a typical Year 5 student?' But the last one is by far my favourite one, because it went right from collecting the data all the way through using that data. And then creating something from that data then using, um, taking more measurements and using that data to see what could be improved and keeping a cycle going. So the children could actually then look at the data and say, 'Ok, well, this is what we can realise from it and this is what we need to do next time'. It was so much more of a practical use in how we would really use that sort of data in the outside world.

Naomi's discussion was indicative of comments from the other teachers as well. Over the course of the year, the teachers experimented with focusing on different aspects of the inquiry cycle. In most cases, the final unit was their most complex and well-designed. A major part of this was the teachers' abilities to learn from previous units, from each other, and available resources in refining their expertise.

Validation

Having the support of the researcher and the other teachers in the study helped the teachers to build their confidence and perseverance. In the beginning of the project, when the teachers were designing and teaching their first units, they had a lot of concern about whether they were "doing it right". When things didn't go well, they often blamed themselves for not anticipating issues in advance, but were able to learn from the experience.

Naomi: I didn't completely anticipate how the children were going to go and what that would mean for data collection and in terms of them analysing the data and the underlying skills that they needed to analyse that data ... They're all things I should have seen ahead of time which is what frustrated me because that all came back on me as a teacher. I should have accounted and adjusted for each one of those ... So I was blaming myself. You have lessons where something goes wrong and it's outside your control that's one of those things, but this was well within my control and I didn't account for it.

Josh: And I tell you what if um I hadn't have had previous life experience, I probably would have crashed and burned at that moment. Because I was, you know the blood pressure went up and I thought 'Right, think quick' ... It was challenging but it was good because it made me think 'Well hang on, I've got to now readjust the process and get them immersed in the activity' ... I mean I think if you had had a first year graduate out there, it would have been really difficult for them.

When he was asked what helped him persist through the units, Josh commented that the validation that their experiences were 'normal' was important to his ability to carry on when things didn't go as anticipated.

Josh: Well, to start with ... you were always there saying, 'look, this is a normal classroom'.

Besides support from the researcher, the students also validated the teachers' efforts through their enthusiasm and learning. Naomi recounted one day that was particularly challenging for her, but when she reflected on the kind of lifelong skills the students had gained from the unit, she felt validated.

Naomi: There was one day I could have thrown my hands up and said 'I'm not doing this' but I could see that the children were enjoying it and to be honest, it's more in a way that I enjoy teaching. It's great to be able to plan something two weeks ahead and say, 'ok this is where we're going' two weeks ahead. That's about a comfort zone for a teacher [but] with the inquiry learning, at the end of every lesson I was having to sit down that afternoon and say 'ok, well this is where they've gone with it, this is what I have to prepare and have ready for tomorrow'. But at the same time it was exciting for me as well as for the children. Just to see where they were going to go with it and what they were going to do and just see them getting so involved. And it did, it carried over to so many other areas, just in the way that they think because, um, one of the things that we did at the end of the [first] unit was a little bit of a wrap up

> and we said, 'Can we say that people have got faster?' And the children were looking at, 'well, we've also got to look at technology, we've got to look at, you know, different types of running shoes, different types of swimsuits, you know, with those Ian Thorpe full body swimsuits and those sorts of things ... How much has that contributed towards increasing, err, decreasing times?', and things like that. So the children were then able to sit down and say 'ok, well the facts tell us this, the mathematical numbers tell us this'. But, and I've seen that travel over into other things now — totally unrelated units where the children will approach it and say 'ok, that's what it looks like, but ___'. It goes on to the point where sometimes I feel like strangling them! (laughs) Not literally. Just because I try and teach a fact and the children go 'well, maybe if' (laughs). But really as a teacher that's a fantastic thing to say about the children — just the way they're now approaching things and saying 'yes, but — what if ___? Could it be that ___?' and that's just wonderful.

The validation of their experiences, both from the researcher and from the learning of their own students, were strong supports to help the teachers in getting through those first few units, when things didn't go as planned. Finding an interesting problem was another challenge the teachers faced in the initial attempts. They talked about the role that resources played in providing them with support in initial units they designed.

Resources

Several times during the study, the researcher asked the teachers what they would suggest to someone attempting to teach mathematical inquiry.

> Kaye: Give good quality resources that we've worked on before they even start to do something. I know the resource we used was fairly structured but at least it gave us groundwork to start a structure. And I do believe where teachers feel a bit threatened or are doing something new, they work better if they've got a structure to work from. They're more inclined to have a go at it. Like I don't know if we would have gone down the path that we have or had the ideas to go down the path that we have without the resource that we've used. And even with a fairly structured resource, it has still taken quite a bit of time, of our time, outside to prepare, to do this.

Providing an initial structure to follow was one role that the resources played, particularly before the teachers had developed a vision of what inquiry would look like. Naomi and Carla talked about how they used the resources for inspiration and guidance.

> Naomi: The other thing that really helped is that *TeachStat* book [Gideon, 1996] because just flipping through there was a really good place to start to get ideas. Because right from the start, it was well, 'Ok, this is a great principle, great in theory. How do I do it? ... What do I *do*?

How do I come up with ideas?' So that *TeachStat* book was actually full of some really good ideas. And one of them gave me the idea for 'The Typical Year 5 Student' [her third unit]. So even though I didn't actually follow through that unit, it gave me the idea and actually spelled out what was some of the really important things to watch out for when the children were designing [survey] forms, you know, the open-ended questions and categories ... The ideas are hard to generate sometimes.

Carla: I'm sure that if Kaye and I didn't have that resource we'd be racking our brains trying to think of a good [question] that's going to try and interest as many people as possible.

The resources were mentioned several times during the year as an important way to generate ideas. But resources alone are insufficient (Helme & Stacey, 2000). Feedback and technical support from the researcher were also raised by the teachers as important to their learning.

Sustained Support and Feedback

Over the course of the study, the researcher made a concerted effort not to direct the teachers. Any suggestions that were made were done in the spirit of collaborative inquiry, when the context seemed to invite input. It was vital for the researcher to maintain the teachers' trust and suggestions were made sparingly and in a positive light. This also minimised the issue of the teachers looking to the researcher as an expert for confirmation of 'correct' ways of teaching through inquiry. However, there were instances where a friendly and non-judgemental suggestion was made or when the researcher probed the teachers' thinking to spark reflection.

In an interview at the end of the year, Kaye recalled such a discussion with the researcher about a lesson she was about to teach during her fourth unit. In the lesson, Kaye had planned to have students plot data they had collected about the number of commercials observed in several 30 minute segments. Before the lesson, Kaye talked to the researcher about what she had planned and the researcher suggested that she consider having students create *stacked* plots instead of using just a *single* distribution as this would allow students to compare, not just describe, the distributions of their data. Kaye was able to modify the lesson on the spot and commented on the difference that made:

Kaye: And actually the support, the throwing in of things that we could do, I appreciated it. A couple of times when you came in, [and suggested] 'this is how you can do this'. ... A classic example was stacked line plots, which was something that, you know, I hadn't even registered that stacked line plots made it so easy for the students to interpret the data. And from there that's something that they have been able to a lot easier, doing it that way rather than putting it [directly] on their presentation. Yet in all the books I read through, it hadn't *mentioned* stacked line plots, so without your input there, I wouldn't have been able to fly the way I did.

Having that suggestion within the context of the lesson Kaye was about to teach was tremendously relevant to her. She may not have made that connection if she had been told in a more general sense that comparisons can provide richer opportunities for students to interpret data. Equally, the suggestion could have shaken Kaye's confidence or implied disapproval if a strong rapport wasn't already well established after months of working together.

One way that the rapport was built was by having the researcher frequently present in the classroom, thinking with the teacher about optimising student learning. This collaborative inquiry enabled the researcher to participate in the teachers' classroom culture, to become 'part of the furniture', and to discuss insights together about what the children were learning. Other kinds of support included help with terminology or requests for technological assistance with the kid-friendly data investigation software *Tinkerplots* (Konold & Miller, 2004) that had been purchased for the project.

Collegiality

The teachers relied on one another during the project and expressed how important it was for them to spend time together, interacting, sharing ideas and concerns, and developing a community together.

Josh: I think one of the most beneficial things about today, has just been listening to each other. Because I think sometimes when even, you know, we're running around and we might have a half an hour meeting here or a half an hour meeting there, and you don't tend to get, um, unless you really work very closely with somebody, you don't get a lot of interaction ... So that's been good for me, to get other people's opinions.

Naomi: We all had problems, it was ok because we could learn from each other's problems.

Kaye: Professionally it has made us more aware of what we should be doing and a way that we can go about doing it. It has made us professionally look at preparation in a different way.

Carla: Yeah, that's true, definitely yep. Yeah, I feel that. So that's improving my teaching.

Kaye: Professional sharing?

Carla: Yeah.

Kaye: I think that has been one of the major aspects of Carla and I just actually working, and bouncing off, 'oh well, this is what we can do, let's try it with this' or 'let's use this resource' or, so that has been professionally very good for us.

Development of Deep Disciplinary Knowledge

Another connection lever that the teachers said helped them to sustain and develop expertise in teaching inquiry-based mathematics was their new understanding of what it means to do statistics. During the teaching of their units, the teachers found that they needed more robust understandings of statistical processes or more connected understanding of mathematics in order to anticipate the knowledge and skills involved in the units they designed. They were quite surprised how their conception of statistics had changed and developed over the year.

> Carla: Now at the end of the year, I know what it might mean to understand a statistical investigation or working with data, where at the beginning of the year [I only considered] 'can they draw that graph'? ... [But now we know] what to look for to say this child understands what working statistically means. [To the others] Wouldn't you say?
>
> Naomi: Oh, definitely. I'll be honest, I used to look at chance and data and say, yeah, 'if they can draw a graph — good, if they can work out the probability of tossing a head when tossing a coin — that's done. Chance and data's out of the way'.
>
> Carla: But now you can say, 'Wow, this person can interpret that data and make this assumption'.
>
> Naomi: Can they design an investigation? Can they manipulate the data? Can they (unfinished response)
>
> Carla: Can they represent it in different ways?

Time and Support for Reflection

The project immersed the teachers in thinking about and reflecting on what it means to teach mathematics with inquiry. Time was set aside several times a year for the teachers to spend a day away from school at the university to develop their skills and talk about their experiences. This was time for them to reflect on what they had learned within a supported environment with others sharing in the same experience. The importance of this time to think, reflect, plan, discuss, and generate ideas away from school was talked about by the teachers.

> Naomi: Once you're out at the university, or anywhere else that's away from school, you stop thinking about what's going on at school ... If you're at school, you're thinking, 'Oh, that's right, when I get a cup of coffee, I'll book the buses for the school camp' or whatever. Out there [at the university], we could just shut out school completely, and just sit and talk and focus completely on maths. And that was really valuable. The other thing that was really valuable was ... you showed us so many practical ways that we could incorporate things. And they were often the bouncing point

> for, ok, well, we're not going to design chairs back in our classroom, but this is the kind of thing we could do, we could look at, I think Carla and Kaye picked up on something like that, where they were they were measuring parts of the body to work out relative heights and things. But it gave a chance to see, 'Well, there's a practical way that it can be incorporated'. Again, it was a source of ideas, it was a model, 'this is something that can be done, this is what it could look like', because otherwise, just this whole concept of inquiry maths, you can discuss it, you can talk about it until the cows come home, but you needed to see how it can be valuable.

Through supported reflection, the teachers drew on their experience of each unit in planning subsequent units and to stand back, abstract from their experience, and consider how they would apply it to improving their practice.

> KM: What about for you professionally? What do you think that you gained?

> Naomi: Well a couple of things. First of all, I'd never actually thought to use an inquiry approach in mathematics before. We use it in science commonly but not in mathematics. So to see that there was a way that we could incorporate that into the classroom was wonderful. It was, uh, a learning curve for me though because I've realised now there's a lot more planning that I have to do in inquiry maths than I would in a normal maths unit. Simply because I have to try and anticipate now where the unit could go to make sure the children have those underlying skills.

Relevance

The ways that the professional learning opportunities were directly linked to the teachers' classroom practice (Cohen & Hill, 2001) and were sustained throughout the year became important support mechanisms for the teachers. Taken together, the inquiry experiences they had during professional development, the opportunity to participate in a community of learning about what they were doing in their classroom, and knowing that others were thinking through the unit with them as they were teaching it all contributed to their ability to build their expertise. The opportunity to integrate their learning with their teaching was relevant to their classroom work and day-to-day practice. They were excited when they saw that their work was at the forefront of teaching mathematics and that the inquiry approach was being promoted as well by state and local initiatives:

> Kaye: We've had to really look deeply at what an investigation really is and investigations really do form a major part of the new maths syllabus ... One of the new [mathematics] outcomes ... was about children creating and interpreting and analysing data, which is all what we've been doing the whole year! So I guess this whole thing we've been doing has been excellent for us getting a handle on the sorts of things that we can do.

Josh: There was a classroom magazine that a friend of mine had the other day and there was a big [article on] inquiry. ... I looked at it and I thought, "Oh! That's what we did!"

One aspect of their teaching that added to their commitment was the ability they developed in seeing the multiple opportunities to design their mathematical inquiry unit that connect with other units in other subject areas. This reemphasised the relevance of the work for them. Some of them commented that they 'suddenly saw mathematical inquiry everywhere'.

Accountability

A critical issue for these teachers was not motivation, but juggling the demands on their time. Although with good teachers there is often intent to try new approaches, sometimes the best intentions get buried. Naomi spoke about the fact that she wouldn't have gone beyond the first iteration had I not been there expecting a unit to watch each term.

KM: You mentioned ... the fact that you were expected to do a unit every term was helpful.

Naomi: Yes, because it kept me going. Otherwise ... you go to the conference, you sit there and you write it all down, you say 'this looks wonderful', and you go back and you drop it on your desk. And about six months later when you sort out the pile of things that's built up on your desk. You go, 'oh, that looks interesting, I'll put it in a file and I'll try and read that later'. And that's kind of it. Whereas this was good. The first one [unit], yep, we did it. We did what we were supposed to do. It was good, I can see some value in it and I can honestly say, that I probably would have then said, 'ok, well, I'll try that next year'. Maybe! And then probably forgotten. Whereas because there was an expectation to do one every term, by the time you got to the last one, you felt comfortable with it, the unit was great, the kids took it to places that I just, and showed understandings that I didn't think they would be capable of. ... So, I'm completely sold, but it would have taken more than one to do that. And now that I've seen this unit and I've seen just how you could bring in measurement and space, and chance and data, and number and just all of those skills, it was, even though it was a technology/science/maths unit, really, it was, you could almost draw a whole term's work on that ... The accountability, and the fact that you had to rehearse it, effectively, over and over, kind of solidified the skills.

Discussion and Implications

Research has been clear that knowledge developed from closed tasks is unlikely to transfer to ill-structured inquiry tasks. Responding to an open-ended inquiry requires learners to draw on skills not taught in traditional mathematics —

tolerance for ambiguity, ability to manage uncertainty, collaboration, negotiation and debate of ideas, and integration of mathematical and contextual knowledge. Indeed, over the past two decades, there has been a paradigm shift in the teaching and learning of mathematics. In this shift, the ideal for mathematical instruction transforms from an emphasis on skills, facts, and procedures towards greater stress on developing children's mathematical conceptions and proficiency at applying mathematical tools to new situations — in particular, open-ended, complex and everyday problems. Ill-structured problems are different from traditional problems in mathematics — they naturally challenge an epistemology of correct answers, yet teachers can understand the value of them.

In order for teachers to create these kinds of learning experiences for their students, they must develop their capability with this approach; be able to envision it and embrace it. Yet there are distinct challenges for teachers in their first time teaching mathematics through inquiry. This project worked to document the processes of teaching and learning in an environment which employs multiple cycles of open-ended inquiry in learning mathematics and statistics. The preliminary results presented here, meant to be neither sufficient nor exhaustive, suggest that particular support mechanisms, or connection levers, were viable ways to enable the teachers to reflect on their iterative experiences in teaching mathematical inquiry towards building their capability. Beyond just a list, the teachers described how these *connection levers* sustained their ability to persist beyond the challenges encountered during the initial teaching experiences, as well as *how* they continued to support them towards building their expertise, confidence, and commitment.

A network of supports worked together to build a deeper understanding of what it means to do inquiry in mathematics. For example, work during the professional development sessions provided the teachers with important experiences with open-ended inquiry and content-rich investigations; the collegiality that developed in discussions about their learning allowed them to validate and strengthen their understanding and to develop as a community of practice; the time and opportunity for reflection supported them in connecting their learning across their experiences; the resources supported them in generating ideas and giving them structure when needed in planning their units; the ongoing mentoring role of the researcher provided feedback and technical support in a collegial environment; the unexpected content issues that arose during their teaching were reported as contributing to their deeper understanding; and the expectations from the researcher (accountability) through multiple iterations of teaching their units helped them persist through their initial discomfort and frustrations.

The teachers in this study developed a great deal of expertise in the course of the year, more than was anticipated by the researcher. It should be cautioned, however, this is partly due to the fact that they already possessed beliefs about learning that were consistent with inquiry. Quite possibly progress would be slow unless teachers first committed to an inquiry-based, constructivist epistemology. Similar work in research on middle schooling suggests that unless teachers' philosophy is consistent with the reform, any short-term change in practice is not sustainable (Pendergast et al., 2005).

Although these findings are tentative and preliminary, many of the connection levers named by the teachers were consistent with research on good professional development needing to be: purposeful; focused on improved student learning of specific content; research-driven; embedded in school contexts; aligned with theories of adult learning; collaborative; active in participating teachers; sustained over a long period of time; one that models effective practices; and inclusive of feedback (Ball, 1996; Cohen & Hill, 2001; Elmore, 2002).

Importantly, the research reported in this paper was conducted in authentic classroom contexts. The challenges the teachers faced and the supports they named were in the context of work in real classrooms with diverse student needs. There was no magic in these levers; none of them is beyond the reach of schools and districts with creative leadership. The practicality of the approaches lends potential to their sustainable use by schools. This implies that these are realistic approaches for schools and can be strengthened through university-school partnerships. These partnerships are not always available to schools, however, and the availability of mentors may not always be feasible. Like any good inquiry, this study therefore raises more questions than it answers. Although the findings reported here *suggest* that the connection levers the teachers named as support mechanisms for their learning, helped to move them towards increasingly more stable levels of confidence and commitment to teaching mathematics through inquiry, clearly there is more research needed to better understand sustainability issues (e.g., longitudinal or follow-up research) and how the research outcomes reported here can be replicated and scaled up. Likewise, the effect of this approach on student learning is another area needing research.

The excerpts from the teachers and the support mechanisms they listed point to the complexity of moving teachers from a stage of orientation about teaching mathematical inquiry towards a commitment to teaching with this approach. These kinds of changes are unlikely to occur as a result of traditional one-off workshops; yet these supports are consistent with moves in education to support more collaborative engagement of teachers throughout their careers in the learning profession.

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